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Transaction E: Industrial Engineering
Vol. 17, No. 1, pp. 36–47
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SCIENTIA IRANICA

A Heuristic Algorithm and a Lower Bound for the Two-Echelon Location-Routing Problem with Soft Time Window Constraints

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Abstract. *The location-routing problem is one of the most important location problems for designing integrated logistics systems. In the last three decades, various types of objective function and constraints have been considered for this problem. However, time window constraints have received little attention, despite their numerous real-life applications. In this article, a new 4-index mathematical model, an efficient and fast heuristic and a lower bound for the two-echelon location-routing problems with soft time window constraints are presented. The proposed heuristic tries to solve the problem via creating an initial solution, then improving it by searching on six neighborhoods of the solution, and using the Or-opt heuristic. At the end, computational results show the efficiency of the proposed heuristic, using the proposed lower bound.*

Keywords: *Location-routing; Location; Routing; Soft time window; Heuristic algorithm.*

INTRODUCTION

Integration of supply chain management activities is an important step toward the success of a supply chain. One of the main types of supply chain integration involves integrating the physical material flow between suppliers, manufacturers, distribution centers and customers [1]. During the last three decades, this type of integration, known as an integrated logistics system, has become one of the most important aspects of logistics and supply chain management. This concept considers the interdependence between facility location, transportation and routing structures, inventory control systems and production planning and scheduling systems for various parts of the supply chain. This comprehensive approach and simultaneous solving of logistics problems prevents the local optimization of dependent problems. Integrated logistics problems include different problems, such as location-routing [2], inventory-location [3], queuing-location [4] and inventory-routing [5] problems.

The Location-Routing Problem (LRP) tries to jointly solve the problem of finding the optimal number, the capacity, the location of facilities serving more than one supplier/customer and optimal routing structure and scheduling [6]. Applications of LRPs range from bill delivery, postal system, dairy distribution and communication network designs to waste/hazardous material collection. Two main subproblems of LRP are the Location-Allocation Problem (LAP) and the Vehicle Routing Problem (VRP). Since both of these problems are *NP-Hard* [7,8], LRP may be considered *NP-Hard* as well.

The first steps of creating LRPs dates back to the 1960s [9,10]. However, the created models were not the same as LRP, since they did not consider the trip from the last customer of the route back to the starting facility. The first real LRP models were developed in the late 1970s and early 1980s through the efforts of various researchers [11-15].

Despite extensive applications of time window constraints in the more realistic modeling of LRP, researchers have paid little attention to LRPs with time window constraints beginning only in the early 1980s. Jacobsen and Madsen solved a one-echelon LRP with hard time windows (LRPTW) for a newspaper distribution network, using a combination of a location-allocation-first, route-second heuristic with a tree-tour method [11] and a saving method [12]. In a two-echelon LRP model for rubber collection,

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Received 4 September 2008; received in revised form 27 July 2009; accepted 29 September 2009

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Nambiar et al. [14] considered the maximum allowed time for delivering commodities as the time window for the depots. In recent years, two researchers have considered the application of multi-echelon LRPTW in military theater distribution. Cox [16] considered the possibility of multiple trips for each and solved a mixed integer programming model using CPLEX. In addition, Burks [17] solved multi-echelon LRPTW with pick-up and delivery using a tabu search algorithm.

In recent years, researchers have considered other types of LRP. Lin et al. [18] solved a two-echelon LRP with a capacitated vehicle fleet, using a three-phase heuristic based on simulated annealing, branch and bound and the traveling salesman problem. Wu et al. [19] considered a one-echelon LRP with a capacitated heterogeneous vehicle fleet, and solved it with a hybrid heuristic based on decomposition and a simulated annealing algorithm. Chan et al. [20] considered a LRP with a maximum route duration constraint and solved it via a saving/insertion heuristic. In addition, they presented an upper and lower bound for the number of facilities and the vehicle fleet size in each facility. Albareda-Sambola et al. [21] modeled a one-echelon LRP and presented an upper bound, using a tabu search and a lower bound using a saving/insertion heuristic.

More recently, Alumur and Kara [22] modeled a two-echelon LRP for hazardous material with multiple objectives as a mixed integer programming problem, and solved it via CPLEX. Özyurt and Asken [23] presented a branch and bound scheme based on a tabu search heuristic, Lagrangian relaxation and a minimum spanning forest problem. Albareda-Sambola et al. [24] considered a LRP with stochastic customers and applied it, using a neighborhood search heuristic and a lower bound, based on decomposition of the objective function to location and m-TSP subproblems. Schwardt and Fischer [25] proposed a neural network algorithm, based on self-organizing maps for a single facility LRP in the continuous space. Finally, Ambrosino et al. [26] proposed a two-phase heuristic with a large neighborhood search, based on path and cyclic exchanges of customers among routes, for the single facility LRP.

The main reason for neglecting time window constraints in LRP literature can be attributed to different planning levels and horizons of location and routing decisions; strategic and tactical, respectively. However, researchers have shown that initial simultaneous and joint decision making for these two problems leads to lower costs in the long run, even though the routes change in the course of time [27,28]. Hence, one can conclude intuitively that in the long run solving LRPTW can also lead to lower costs

The main purpose of this article is to model and solve a two-echelon location-routing problem with soft time window constraints (2ELRPSTW). In this problem, serving each customer is possible in two consecutive time intervals. The second interval differs from the first in the fact that serving the customer is only possible via paying a fixed penalty cost. In addition, the vehicle fleet capacity for each regional distribution center is considered limited. Due to working regulations and/or the requirements of the commodity being delivered, it is assumed that each vehicle can only be used for a limited duration in each working day.

For this problem, a new 4-index mathematical model and a heuristic algorithm method are presented. The proposed heuristic algorithm method is based on location-first, allocation-routing second, for an initial solution construction and a neighborhood search and *Or-opt* heuristic for solution improvement. Then, a lower bound for 2ELRPTW based on objective function decomposition is presented. For the computation of lower bound routing subproblems, existing methods for using the minimum spanning forest problem are improved via a binary programming problem. The efficiency of the proposed heuristic is shown via computational results. Finally, research results and future research opportunities are discussed.

MATHEMATICAL MODEL

The 2ELRPSTW logistic system is defined on an undirected graph, $G = (N, E)$. The nodes of this graph (N) consist of Central Depot Centers (CDC), Regional Depot Centers (RDC) and Customers (C). The undirected edges of this graph (E) are composed, of edges linking CDCs to RDCs, RDCs to customers and customers to customers. Triangle inequality is assumed to be valid for edges linking RDC to customers and customers to customers. It is assumed that the capacities of the CDCs, RDCs and the homogeneous vehicle fleet are deterministic and known parameters. The vehicle fleets assigned to RDCs are assumed homogeneous. Finally, customer demand is also known and deterministic and cannot be split. The 2ELRPSTW parameters are as follows:

- I: set of central depot center nodes,
- J: set of regional depot center nodes,
- C: set of customer nodes,
- N_{1j}: set of all nodes belonging to the set, $C \cup j$, $\forall j \in J$
- F'_i: capacity of the ith CDC,
- FC_j: fixed cost of opening jth RDC,
- VC_j: variable cost of operating the jth RDC for a unit of the commodity,
- CR_{ij}: cost of transportation for a unit of the commodity between the ith CDC and the jth RDC,
- F_j: capacity of the jth RDC,

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CV: fixed cost of a vehicle,
σ: vehicle capacity,
$n\nu_j$: maximum number of vehicles assignable to the jth RDC ($n\nu_j = \lceil F_j/\sigma \rceil$),
τ: maximum allowable route duration,
D_k: demand of the kth customer,
t_{km}: travel time between the nodes k and m, $k \& m \in N_{ij}$, $j \in J$,
α_{km}: travel cost between the nodes k and m, $k \& m \in N_{ij}$, $j \in J$,
$[a_k, b_k]$: acceptable time interval for serving the kth customer with no penalty,
$[b_k, b'_k]$: acceptable time interval for serving the kth customer with penalty,
PC_k: penalty cost for serving the kth customer in the interval $[b_k, b'_k]$,

The 2ELRPSTW variables are as follows:

x_{ij}: amount of commodity to be transported between the ith CDC and the jth RDC,
y_j: binary variable for opening the jth regional depot center,
u_{jl}: binary variable for assigning the lth vehicle to the jth RDC, $j \in J, l = 1, ..., n\nu_j$,
ν_{km}^{jl}: binary variable for traveling the link (k, m) by the lth vehicle of the jth RDC, k & $m \in N_{1j}, k \neq m, j \in J, l = 1, ..., n\nu_j$,
z_{kj}: binary variable for assigning the kth customer to the jth RDC,
w_k^{jl}: arrival time of the lth vehicle of the jth RDC to the kth customer, $k \in C$, $j \in J$, $l = 1, ..., n\nu_j$,
r_k^{jl}: binary variable for serving the kth customer with the lth vehicle of the jth RDC in the penalized interval $[b_k, b'_k]$, $k \in C, j \in J$, $l = 1, ..., n\nu_j$,

The 4-index mathematical model for the 2ELRPSTW based on models proposed by Daskin [29] and Daskin et al. [30] is as follows:

$$\min \sum_{j\in J} FC_j y_j + \sum_{i\in I}\sum_{j\in J} CR_{ij}x_{ij} + \sum_{j\in J} VC_j \sum_{k\in C} D_k z_{kj} + CV \sum_{j\in J}\sum_{l=1}^{n\nu_j} u_{jl} + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in N_{1j}}\sum_{k\in N_{1j}} \alpha_{km}\nu_{km}^{jl} + \sum_{k\in C} PC_k \sum_{j\in J}\sum_{k=1}^{n\nu_j} r_k^{jl}. \tag{1}$$

Subject to:

$$\sum_{j\in J} x_{ij} \leq F'_i \qquad \forall i \in I, \tag{2}$$

$$\sum_{i\in I} x_{ij} \leq F_j y_j \qquad \forall j \in J, \tag{3}$$

$$\sum_{k\in C} D_k z_{kj} - \sum_{i\in I} x_{ij} < 0 \qquad \forall j \in J, \tag{4}$$

$$\sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in N_{1j}} \nu_{km}^{jl} = 1 \qquad \forall k \in C, \tag{5}$$

$$\sum_{k\in C} D_k \sum_{m\in N_{1j}} \nu_{km}^{jl} \leq \sigma \quad \forall j \in J, \quad l =, 1, ..., n\nu_j, \tag{6}$$

$$\sum_{m\in N_{1j}} \nu_{km}^{jl} - \sum_{m\in N_{1j}} \nu_{mk}^{jl} = 0 \qquad \forall j \in J, \quad l = 1, ..., n\nu_j, \quad \forall k \in N_{1j}, \tag{7}$$

$$\sum_{m\in N_{1j}} \nu_{mk}^{jl} + \sum_{h\in N_{1j}} \nu_{jh}^{jl} - z_{kj} \leq 1 \qquad \forall k \in C, \quad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{8}$$

$$\sum_{k\in C} \nu_{kj}^{jl} = u_{jl} \qquad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{9}$$

$$\sum_{k\in C} \nu_{jk}^{jl} = u_{jl} \qquad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{10}$$

$$w_m^{jl} \geq w_k^{jl} + t_{km} - M(1 - \nu_{km}^{jl}) \qquad \forall k \ \& \ m \in C, \quad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{11}$$

$$\max\{t_{jm}, a_m\} \leq w_m^{jl} \leq \min\{(\tau - t_{mj}), b'_m\} \qquad \forall m \in C, \quad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{12}$$

$$w_k^{jl} \leq b_k + M r_k^{jl} \qquad \forall k \in C, \quad \forall j \in J, \quad \forall l = 1, ..., n\nu_j, \tag{13}$$

$$w_k^{jl} > b_k - M(1 - r_k^{jl}) \qquad \forall k \in C, \quad \forall j \in J, \quad \forall l = 1, ..., n\nu_j, \tag{14}$$

$$x_{ij} \geq 0 \qquad \forall i \in I, \quad \forall j \in J, \tag{15}$$

$$y_j \in \{0, 1\} \qquad \forall j \in J, \tag{16}$$

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$$u_{jl} \in \{0,1\} \qquad \forall j \in J, \quad l = 1, ..., n\nu_j, \tag{17}$$

$$\nu_{km}^{jl} \in \{0,1\}$$

$$\forall j \in J, \quad l = 1, ..., n\nu_j, \quad \forall k \ \& \ m \in N_{1j}, \tag{18}$$

$$z_{kj} \in \{0,1\} \qquad \forall k \in C, \quad \forall j \in J, \tag{19}$$

$$r_k^{jl} \in \{0,1\} \quad \forall k \in C, \quad \forall j \in J, \quad l = 1, ..., n\nu_j. \tag{20}$$

In the above model, Objective Function 1 includes RDCs opening fixed costs, CDCs to RDCs commodity transportation costs, RDCs variable costs, vehicle fleet acquisition costs, routing costs and soft time windows violation penalty costs. Constraint 2 limits the outgoing commodity from each CDC to its capacity, while Constraint 3 limits the incoming commodity into each RDC to its capacity. Constraint 4 balances the incoming and outgoing commodity volume at each RDC. Constraint 5 requires each customer to be on just one route belonging to one vehicle of one of the RDCs. Constraint 6 imposes a capacity restriction on each vehicle.

Constraint 7 ensures the flow conservation. Constraint 8 assigns a customer to a RDC, if a vehicle from that RDC enters that customer node and leaves the RDC node itself at the beginning of its trip. Constraints 9 and 10 ensure that if a vehicle is assigned to a RDC, it both enters and leaves that RDC. Constraint 11 calculates the arrival time of vehicles to customers and also eliminates subtours [31]. Constraint 12, a generalization of Kontoravdis and Bard's model for a time window constraint [32], defines the soft time window domain for each customer. Its lower bound tightens the time window lower bound with the earliest direct arrival time of a vehicle to a customer from the origin RDC, if possible. In addition, its upper bound tightens the time window upper bound with the route maximum allowed travel time, if possible.

Constraints 13 and 14 determine if a vehicle has passed the maximum allowed time for arrival to a customer without paying a penalty (b_k). Finally, Constraints 15 to 20 define the variables types. The complexity of the above mixed integer programming model is due to the existence of soft time window constraints and the *NP-Hard* nature of the problem. Hence, solving medium and large-sized instances of this problem via exact methods is a challenging and difficult task.

The proposed mathematical model for the 2EL-RPSTW is based on 4-index routing variables (source node, destination node, source RDC and the vehicle number belonging to the RDC) for on routing RDC-to-customer and customer-to-customer edges. Since this 4-index model considers as many variables for each RDC as the maximum number of vehicles assignable to that RDC, this model has fewer variables used in each constraint compared to common 3-index models [29,30]. In addition, this model allows for merging maximum route duration constraints with time window constraints, which has been described previously.

HEURISTIC ALGORITHM

In this section, a two-phase heuristic algorithm based on the neighborhood search proposed by Albareda-Sambola et al. [24] and *Or-opt* heuristic [33], is presented (Figure 1). In the construction phase, an initial solution is created with a location-first, allocation-routing-second algorithm and then improved with an *Or-opt* heuristic. Then, in the improvement phase, the final solution is obtained by searching six neighborhoods of the initial solution and *Or-opt* heuristic.

Construction Phase

1. **Location**: In this step, RDCs to be opened are found sequentially based on the ratio of their fixed cost to their capacity. An unopened RDC with a minimum ratio is selected for allocating customers to it.
2. **Allocation-Routing:** In this step, customers

Construction phase{
- 1. Find an unopened RDC with minimum fixed cost to capacity ratio,
- 2. Assign previously unassigned customers to the last opened RDC and create intial rutes,
  - 2.1. If all customers are assigned goto step 4,
  - 2.2. Else goto step 1,
- 3. Improve intial route with *Or-opt* heuristic.**}**

Improvement Phase{
- 1. Update stopping criteria,
- 2. While stopping criteria are not met:
  - 2.1. Until no feasible move for improvement is found, repeat:
    - 2.1.1. Search on $N_2(x)$ and update x,
    - 2.1.2. Search on $N_1(x)$ and update x,
    - 2.1.3. Search on $N_5(x)$ and update x
    - 2.1.4. Search on $N_6(x)$ and update x,
  - 2.2. Until no feasible move for improvement is found, repeat:
    - Improve routes with *Or-opt* heuristic and update x.
  - 2.3. Search on $N_3(x)$ and update x,
  - 2.4. Search on $N_4(x)$ and update x,
  - 2.5. Improve routes with *Or-opt* heuristic and update x.
- 3. Goto step 1.}

Figure 1. Heuristic solution algorithm.

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are added to the last opened RDC and inserted into routes using the modified Balakrishnan algorithm [34], based on the minimum weighted sum of the routing time, amount of soft time window violation and customer priority. In each customer allocation, none of the Constraints, 3, 6 and 12 can be violated. If all customers are not allocated to the current opened RDCs, the location step will be repeated until all customers are allocated to the opened facilities. In this step, the maximum allowed soft time window constraints violation penalty cost is restricted to 20% of its total sum.

3. **Initial Route Improvement:** In this step, the initial routes of step 2 are improved via the *Or-opt* heuristic.

Improvement Phase

In this phase, the initial solution derived from phase one is improved by searching on six neighborhoods of the initial solution. In addition, the *Or-opt* heuristic is used to improve each route with intra-route exchanges. All moves are performed with respect to the amount of total saving due to changes in the RDCs fixed cost, RDCs variable costs, transportation between CDCs and RDCs, routing costs and soft time windows violation penalty costs. Also, in this phase, the same feasibility conditions as the feasibility conditions of step 2 of phase one are controlled.

Six neighborhoods of the improvement phase are an extension of the algorithm proposed by Albareda-Sambola et al. [24] for the single-echelon LRP. Albareda-Sambola et al. [24] proposed four neighborhoods $(N_1(x), N_2(x), N_3(x)$ and $N_4(x))$ for a local search in a LRP with a stochastic customer presence. The new neighborhood N_5(x) exchanges the starting RDC of two routes. In each solution belonging to $N_5(x)$, the starting RDCs of two routes belonging to ith RDC and jth RDC are set to jth RDC and ith RDC, respectively. In addition, the new neighborhood $N_6(x)$ adds a new route to a RDC with the least used capacity ratio, and then, some customers from other routes are removed and inserted into the new route. A search on the four neighborhoods of step 2.1 of Figure 1 improves the current solution with respect to current opened RDCs. In addition, two neighborhoods of steps 2.3 and 2.4 of Figure 1 diversify the search space with opening new RDCs.

Stopping Criteria

If no feasible move is performed while searching neighborhoods $N_3(x)$ and $N_4(x)$, then the algorithm stops. In addition, if the amount of saving between two main algorithm iterations is less than a specified value, ε, then the algorithm stops.

LOWER BOUND COMPUTATION

For computing the lower bound of the objective function of Problem (1-20), Z_{LB}, the set of constraints linking the location and routing subproblems (Constraint 8) are relaxed and the customers-to-RDCs assignment variables are omitted. Then, each of the location (Z_1) and routing (Z_2) subproblems are minimized, regarding their respective constraints. The first subproblem consists of the fixed and variable costs of RDCs and the costs of CDCs to RDCs commodity transportation. The second subproblem consists of the vehicle acquisition fixed costs, routing costs and soft time windows violation penalty costs. The first subproblem is a location problem. The second subproblem can be converted to a degree-constrained capacitated Minimum Spanning Forest Problem (MSFP) with maximum route duration, after the necessary modifications.

Lower Bound Location Subproblem

Before obtaining the first subproblem solution, implementation of the following modification is necessary. Since the Minimum Spanning Forest Problem (MSFP) neglects the cost of returning from the last customer in each route to the RDC, the term $\min\limits_{m\in C}\{\alpha_{mj}\}.n\nu_j.sr_c$ is added to the fixed cost of each RDC, FC_j, as an estimate of the return cost to the RDC. In the aforementioned term, sr_c is used to compensate for the possible overestimation caused by $n\nu_j$, and it is calculated as the average of the ratio of vehicles assigned to each opened RDC to the maximum number of vehicles assignable to that RDC in the final solution of the proposed heuristic for each test problem size. The objective function and constraints of the lower bound location subproblem are as follows:

$$\min \ \ Z_1 = \sum_{j\in J} FC_j y_j + \sum_{i\in I}\sum_{j\in J} CR_{ij}x_{ij} + \sum_{j\in J} VC_j \sum_{i\in I} x_{ij}, \tag{21}$$

Subject to:

$$\sum_{j\in J} x_{ij} \leq F_i' \qquad \forall i \in I, \tag{22}$$

$$\sum_{i\in I} x_{ij} \leq F_j y_j \qquad \forall j \in J, \tag{23}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ij} = \sum_{k\in C} D_k. \tag{24}$$

In the above problem, Constraints 22 and 23 are the same as Constraints 2 and 3. Constraint 24 makes the total amount of the commodity transported from CDCs to RDCs equal the total demand.

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Lower Bound Routing Subproblem

The application of MSFP with the degree, vehicle capacity and route duration constraints, in solving the LRP and obtaining its solution, using Prim's algorithm [35], has received little attention from researchers [23,36]. This problem in its simplest case (relaxing vehicle capacity and route duration constraints) on a connected graph is a degree-constrained minimum spanning tree problem that is known to be NP-Complete [37]. Hence, obtaining its optimal solution in polynomial time using exact optimization techniques is not possible. The objective function of the second subproblem of Objective Function 1, after relaxing soft time window constraints, is as follows:

$$Z_2 = CV\sum_{j\in J}\sum_{l=1}^{n\nu_j} u_{jl} + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in N1j}\sum_{k\in C}\alpha_{mk}\nu_{mk}^{jl}. \tag{25}$$

For transforming Objective Function 25 into a degree-constrained capacitated MSFP with route duration constraints, the vehicle acquisition fixed cost term must be omitted via adding the vehicle fixed cost to all of the edges emerging directly from RDCs to customers. This modification also restricts the irregular emerging edges from the RDCs selection and hence, an unrealistic customer service scheme. Then, the remaining constraints must be modified so that a degree-constrained capacitated MSFP with route duration constraints is obtained. The objective function and constraints of this problem are as follows:

$$\min \quad Z_2' = \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{k\in C}(\alpha_{jk} + CV)\nu_{jk}^{jl} + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in C}\sum_{k\in C}\alpha_{mk}\nu_{mk}^{jl}. \tag{26}$$

Subject to:

$$\sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in N_{1j}}\nu_{mk}^{jl} = 1 \qquad \forall k\in C, \tag{27}$$

$$\sum_{k\in V}\sum_{m\in V}\sum_{j\in J}\sum_{l=1}^{n\nu_j}\nu_{km}^{n\nu_j} \le |V| - 1$$

$$2 \le |V| \le |C|, \quad \forall V \subseteq C, \tag{28}$$

$$\sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in C}\nu_{jm}^{jl} + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{k\in C}\sum_{\substack{m\in C\\ m>k}}\nu_{km}^{jl} = |C|, \tag{29}$$

$$\sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in C}\nu_{jm}^{jl} \ge \left\lceil \sum_{k\in C} D_k/\sigma \right\rceil, \tag{30}$$

$$\sum_{l=1}^{n\nu_j}\sum_{m\in C}\nu_{jm}^{jl} \le n\nu_j \qquad \forall j\in J, \tag{31}$$

$$\sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in C}\nu_{km}^{jl} \le 1 \qquad \forall k\in C, \tag{32}$$

$$\sum_{m\in N_{1j}}\sum_{k\in C} t_{mk}\nu_{mk}^{jl} \le \tau \quad \forall j\in J, \quad l = 1, ..., n\nu_j, \tag{33}$$

$$\sum_{k\in C} D_k \sum_{m\in N_{1j}}\nu_{mk}^{jl} \le \sigma \quad \forall j\in J, \quad l = 1, ..., n\nu_j. \tag{34}$$

In the above problem, Constraint 29 sets total number of selected edges to the number of customers. Constraint 30 guarantees the selection of at least as many edges as the minimum required number of vehicles to emerge from RDCs. Constraint 31 limits the number of outgoing edges at each RDC node to the maximum number of vehicles allowed to assign to that RDC. Constraint 32 limits the maximum number of outgoing edges at each customer node to one. The remaining constraints are the same as the similar constraints of Problem (1-20).

Since regular algorithms for the MSFP such as Prim and Kruskal [38], cannot consider maximum route duration and vehicle capacity constraints, Constraints 33 and 34 are relaxed using Lagrangian relaxation [39]. Hence, the right-hand side of these constraints is subtracted from their left-hand side, and then, the difference is multiplied with nonnegative Lagrangian multipliers, λ and μ. Finally, these terms are added to the objective function to create the new Lagrangian function, $Z_2'(\lambda,\mu)$.

$$\begin{aligned} Z_2'(\lambda,\mu) = {} & \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{k\in C}(\alpha_{jk}+CV)\nu_{jk}^{jl} \\ & + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\sum_{m\in C}\sum_{k\in C}\alpha_{mk}\nu_{mk}^{jl} \\ & + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\lambda_{jl}\left(\sum_{m\in N_{1j}}\sum_{k\in C} D_k\nu_{mk}^{jl} - \sigma\right) \\ & + \sum_{j\in J}\sum_{l=1}^{n\nu_j}\mu_{jl}\left(\sum_{m\in N_{1j}}\sum_{k\in C} t_{mk}\nu_{mk}^{jl} - \tau\right). \end{aligned} \tag{35}$$

It is known that the maximum of the minimum of the Lagrangian Function 35 ($Z_2^{'*}(\lambda,\mu)$) regarding λ and μ, subject to Constraints 27 to 32, is a lower bound for Objective Function 25 [40]. For solving this maximization problem, a subgradient search method (Figure 2) [41] is used for finding the Lagrangian

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Step 1. $\lambda^{(0)} \leftarrow 0$, $\mu^{(0)} \leftarrow 0$, $u \leftarrow 2$, $Z_2^{'(\text{best } 1)} \leftarrow -\infty$, and $Z_2^{'(\text{best } 2)} \leftarrow 0$.

Step 2. Solve $Z_{2LR}^{'(k)}\left(\lambda^{(k)}, \mu^{(k)}\right)$.

Step 3. If $Z_{2LR}^{'(k)}\left(\lambda^{(k)}, \mu^{(k)}\right) > Z_2^{'(\text{best } 1)}$, then $Z_2^{'(\text{best } 2)} \leftarrow Z_2^{'(\text{best } 1)}$ and $Z_2^{'(\text{best } 2)} \leftarrow Z_2^{'(k)}$.

Step 4. If $u \leq 3^*10^{-5}$ or best known solution gap, $\Delta Z_2^{'(\text{best})} = Z_2^{'(\text{best } 1)} - Z_2^{'(\text{best } 2)}$, is less than $\varepsilon = 0.0015$ for three successive interations, then stop and report $Z_2^{'(\text{best } 1)}$.

Step 5. If $m = 5$ successive iterations have passed since the last improvement of $Z_2^{'(\text{best } 1)}$, then $u \leftarrow u/2$.

Step 6. Update Lagrangian multipliers using Equations 36 and 37.

Step 7. Goto step 2.

Figure 2. Lagrangian multipliers search algorithm.

multipliers maximizing objective function, $Z_2'(\lambda, \mu)$. In addition, an extended Kruskal algorithm (Figure 3) is used for solving $Z_2'(\lambda, \mu)$. In each of the subgradient search method iterations, a step is taken along the Lagrangian function subgradient, regarding the Lagrangian multipliers values. Let the Lagrangian multipliers vector of Lagrangian Function 35, consisting of λ and μ, be shown by vector η and matrix A be the technology matrix of two Lagrangian-relaxed constraints. Also, let θ be the right-hand side vector of these constraints. Lagrangian multipliers values and a Lagrangian multiplier step-size modifier are updated using the following equations:

$$\eta^{(k+1)} = \max\{0, \eta^{(k)} + t_k.(AV^{(k)} - \theta)\}, \tag{36}$$

$$t_k = u\frac{(Z_2^{'*}(\lambda, \mu) - Z_2^{'(k)}(\lambda^{(k)}, \mu^{(k)}))}{||AV^{(k)} - \theta||^2}. \tag{37}$$

In Equation 37, $Z_2^{'*}(\lambda, \mu)$, the upper bound of Lagrangian Function 35, is estimated using the routing objective function of the proposed heuristic. The Lagrangian multiplier step-size modifier, u, in Equation 37 is used for correcting the error of overshooting the $Z_2^{'*}(\lambda, \mu)$ upper bound.

The initial edge finding Problem (38-42), in step 2 of the extended Kruskal algorithm (Figure 3), tries to minimize the total weight of the selected edges. Constraint 39 results in selecting exactly the same number of edges as the minimum number of required vehicles. Constraint 40 limits the number of derived edges from each RDC to the maximum number of vehicles allowed to assign to that RDC. Constraint 41 assigns each customer to at most one RDC. Finally, Constraint 42

Step 1. Create an empty set named F.
Step 2. Insert n (minimum-required number of vehicle) minimum-cost intial edges into set F by solving Problem (38-42).

$$\min \sum_{j \in J} \sum_{k \in C} t_{kj} z_{kj} \tag{38}$$

Subject To:

$$\sum_{j \in J} \sum_{k \in C} z_{kj} = \left\lceil \sum_{k \in C} D_k / \sigma \right\rceil, \tag{39}$$

$$\sum_{k \in C} z_{kj} \leq n\nu_j \qquad \forall j \in J, \tag{40}$$

$$\sum_{j \in J} z_{kj} \leq 1 \qquad \forall j \in J, \tag{41}$$

$$z_{kj} \in \{0, 1\} \qquad \forall k \in C, \qquad \forall j \in J. \tag{42}$$

Step 3. Sort remaining edges based on travel cost between nodes increasingly.
Step 4. Sequentially, insert $|C| - n$ edges into set F if no constraint (not creating a subtour and each node maximum degree) is violated.

Figure 3. Extended Kruskal algorithm.

defines the assignment variable of the kth customer to the jth RDC as a binary variable. The solution of this problem is an improvement on the methods available in the literature [23,36], resulting in better solutions by not eliminating shorter edges because of node degree constraints. This binary programming problem can be solved optimally by means of common optimization software.

For adapting Lagrangian Function 35 to the standard Kruskal algorithm objective function, the 3rd and 4th terms of this function are integrated with the cost coefficient of the 1st and 2nd terms, as in Equation 43.

$$\begin{aligned} Z_2'(\lambda, \mu) = & \sum_{j \in J} \sum_{l=1}^{n\nu_j} \sum_{k \in C} (\alpha_{jk} + CV + \lambda_{jl}.D_k \\ & + \mu_{jl}.t_{jk})\nu_{jk}^{jl} + \sum_{j \in J} \sum_{l=1}^{n\nu_j} \sum_{m \in C} \sum_{k \in C} (\alpha_{mk} \\ & + \lambda_{jl}.D_k + \mu_{jl}.t_{mk})\nu_{mk}^{jl} \\ & - \sum_{j \in J} \sum_{l=1}^{n\nu_j} (\lambda_{jl}\sigma + \mu_{jl}\tau_j). \end{aligned} \tag{43}$$

Validation of the Lower Bound ZLB

In the last two previous subsections, the minimum values of location subproblem, Z_1^*, and routing, $Z_2^* = Z_2'^*(\lambda, \mu)$, were derived using an exact algorithm and a heuristic algorithm, respectively. It can be shown that

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$Z_{LB} = Z_1^* + Z_2^*$ is a valid lower bound for Objective Function 1 using the following axiom and lemma.

Axiom

Function $H(X,Y)$ is composed of two functions, $H_1(X)$ and $H_2(Y)$. The minimum value of $H(X,Y)$ is greater than/equal to the summation of $H_1(X)$ and $H_2(Y)$ minimum values.

Lemma

Z_{LB} is a valid lower bound for Objective Function 1.

Proof

Objective Function 1 is composed of two subproblems, Z_1 and Z_2. Solving Objective Function 21, with respect to Constraints 22 to 24, minimizes subproblem Z_1 optimally. Also, the maximum value of the minimization problem, consisting of Objective Function 35, with respect to Constraints 27-32 is a lower bound for subproblem Z_2. Hence, according to the stated axiom, $Z_{LB} = Z_1^* + Z_2^*$ is a valid lower bound for Objective Function 1.

COMPUTATIONAL RESULTS

Test Problems and Algorithms Implementation

For evaluation of the efficiency of the proposed heuristic, 21 random test problems are designed and used. The graph nodes are selected randomly on a 1000*1000 Euclidean space. Table 1 contains the information regarding the size of the test problems created in 5 classes. In these problems, the distribution of customer demand, RDCs and CDCs capacities are $U[7,30]$, $U[30,140]$, and $U[100,600]$, respectively. Each vehicle fixed cost is set to 250, RDCs fixed and variable costs are set to $U[20,80] + U[100,110].F_j^2$ and $U[1.5,3] + U[0.01,0.03]^*F_j$, respectively.

Transportation costs between CDCs and RDCs nodes and routing cost between RDCs and customers nodes are 0.0075 and 0.5 per Euclidean distance unit, respectively. Total RDCs capacity to total demand ratios are set to be two and five. Vehicle capacity and maximum allowed route duration are designed, such that each route has between 5 and 10 customers.

Table 1. Problem instances class size.

Class	**Class Size**		
	$\|I\|$	$\|J\|$	$\|C\|$
C1	5	10	25
C2	5	25	50
C3	5	25	100
C4	5	50	100
C5	10	50	100

Finally, soft time window violation penalty costs, PC_k, are defined by $U[120,180] + D_k.U[3,5]$, and each customer's time window is defined by Equations 44 to 48:

$$c_k = U[0.15\tau, 0.85\tau], \tag{44}$$

$$l_k = N[0.1\tau, 0.04\tau], \tag{45}$$

$$a_k = \max\{0, c_k - l_k\}, \tag{46}$$

$$b'_k = \min\{c_k + l_k, \tau\}, \tag{47}$$

$$b_k = b'_k - l_k/2. \tag{48}$$

For evaluation of the proposed heuristic efficiency, six small-sized test problems (class 1), six medium-sized test problems (class 2) and nine large-sized test problems (classes 3, 4 and 5; each one three instances), are selected. From these 21 test problems, the small-sized ones are solved via the *LINGO* optimization package besides the proposed heuristic for the purpose of identifying the amount of closeness of the heuristic final solution and the lower bound to the global optimum. Based on the initial experimental results, the routing values of the proposed heuristic, sr_c, for small, medium and large-sized problems are set to 0.93, 0.88 and 0.81, respectively. All the proposed algorithms and classical optimization techniques are implemented using C# 2.0 and *LINGO* 8.0. These programs were run on a personal computer with a 2130 MHz CPU and one GB RAM.

Experiments Results

Regarding the computational results of the proposed heuristic (Tables 2 to 4), this method has an efficient and fast performance. The average lower bound gap for small, medium and large-sized test problems is 5.75%, 8.68% and 11.22%, respectively. The reason for the decline in the performance of the proposed heuristic along with the increase in the number of customers in the test problems from 25 to 100 (Figures 4 and 5) can be attributed to the growth of the solution space of the neighborhood search phase and the local search nature of the proposed heuristic. Also, the absence of considering soft time windows in the proposed lower bound can lead to an increase in this gap between the final solution and the lower bound. However, this effect is reduced and controlled via restricting the maximum allowed soft time window constraints violation penalty to 20% of its total sum. Also, by comparison of the proposed heuristic final solution to the optimal solutions obtained by *LINGO*, there is an average of a 1.86% gap with the optimal solution. Therefore, it seems that the heuristic solution is stronger from the lower bound and closer to the global optimum.

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Table 2. Computational results for small-sized problems.

Problem No.	Lower Bound	CPU Time (sec.)	Final Solution	CPU Time (sec.)	Lower Bound Gap	Optimal Solution	CPU Time (sec.)	Optimal Solution Gap
C101	10171.1	32	10814.63	96	6.33%	10537.77	27984	2.56%
C102	10786.27	23	11380.21	90	5.51%	11157.15	29761	1.96%
C103	9785.92	18	10297.81	103	5.23%	10297.81	32936	0.00%
C104	11143.82	24	11810.26	88	5.98%	11471.30	23521	2.87%
C105	11230.53	27	11756.64	92	4.68%	11756.64	31013	0.00%
C106	12326.12	25	13163.31	99	6.79%	12671.00	36709	3.74%
Average	10907.29	24.83	11537.14	94.67	5.75%	11315.28	30320.69	1.86%

Table 3. Computational results for medium-sized problems.

Problem No.	Lower Bound	CPU Time (sec.)	Final Solution	CPU Time (sec.)	Lower Bound Gap
C201	14948.45	663	16136.99	164	7.95%
C202	13223.91	613	14509.45	168	9.72%
C203	14021.72	666	15182.83	178	8.28%
C204	15751.13	573	17060.76	190	8.31%
C205	12290.65	634	13507.04	201	9.90%
C206	13837.21	574	14932.74	162	7.92%
Average	14012.18	620	15221.64	177.17	8.68%

Table 4. Computational results for large-sized problems.

Problem No.	Lower Bound	CPU Time (sec.)	Final Solution	CPU Time (sec.)	Lower Bound Gap
C301	20989.41	6362	23034.85	260	9.75%
C302	18190.95	6503	20380.6	286	12.04%
C303	19382.03	6356	21687.43	281	11.89%
C401	25270.54	6378	28209.48	277	11.63%
C402	19190.39	6031	21417.79	252	11.61%
C403	22835.8	6988	25067.38	265	9.77%
C501	32413.05	5950	36158.71	235	11.56%
C502	36821.07	6141	40598.12	269	10.26%
C503	36585.15	6342	41161.64	271	12.51%
Average	25742.04	6339	28635.11	266.22	11.22%

From a solution CPU time perspective, the proposed heuristic performs rapidly. This method solves small-sized problems in a small amount of time (averagely around 1.5 minute). It also solves medium and large-sized problems in a reasonable amount of time (averagely 3 and 4.5 minutes, respectively). It can be observed that CPU time has a direct relationship with the test problem size, and increases along with an increase in the number of customers because of the difficulty of solving the classical optimization problems. The lower bound CPU time in the largest problem instance is still less than 105 minutes, which is acceptable, considering the strategic nature of decision making problems. The main factor determining the proposed lower bound CPU time is solving the location subproblem optimally, which requires more CPU time along with an increase in the number of customers.

Altogether, the proposed heuristic has an 8.55% average lower bound gap and 179.35 sec average CPU

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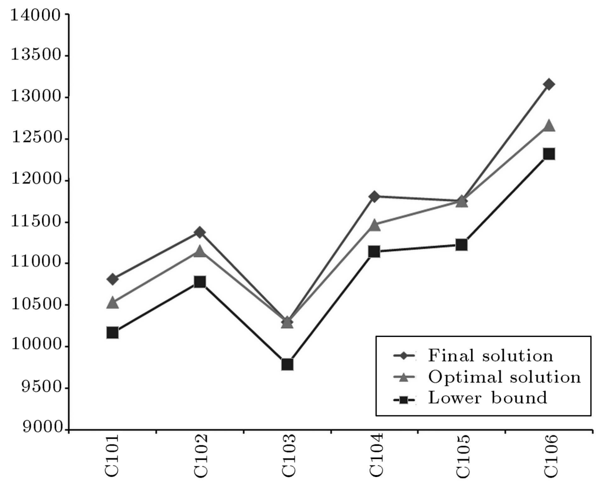

Figure 4. Comparison of the optimal solution/heuristic solution/lower bound for small-sized problems.

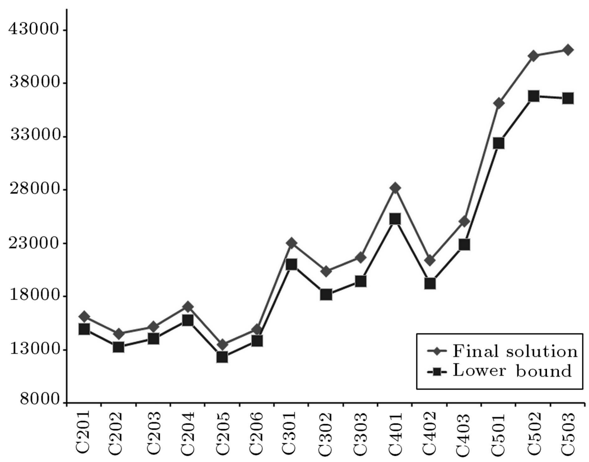

Figure 5. Comparison of the heuristic solution/lower bound for medium and large-sized problems.

time, which has an acceptable lower bound gap due to the fast performance and heuristic nature of the proposed solution algorithm.

CONCLUSION

In this article, a special case of integrated logistics problems, a two-echelon location-routing problem with time window constraints, was reviewed, modeled and solved. The location-routing problem is a barrier to the local optimization of location and routing decisions because of considering the interdependence between facility location, customer assignment and the structure of the routes. Despite the initial attention of researchers to location-routing problems with hard time window constraints, little attention has been paid to this practical problem during recent years. Therefore, the consideration of soft time window constraints is the distinguishing feature of this article.

For this problem, a new 4-index mixed integer programming model was developed. This model has fewer variables used in each constraint compared to common 3-index models. In addition, it allows for merging maximum route duration constraints with time window constraints. Then, a two-phase heuristic, based on location-first, allocation-routing second for initial solution construction and a neighborhood search for an initial solution improvement was developed. In addition, a lower bound for this problem was designed, based on objective function decomposition. For the routing subproblem of the lower bound, a Lagrangian relaxation scheme and the minimum spanning forest problem were used. Finally, the efficiency of the proposed heuristic was shown using the proposed lower bound.

Further research opportunities include the application of metaheuristics, such as tabu search and simulated annealing, improvement of the proposed heuristic via defining new neighborhoods for local search and consideration of other local search mechanisms, considering other types of time window violation penalty functions, and finally, integration of other logistics problems, such as inventory control with LRPTW.

ACKNOWLEDGMENT

The authors would like to thank the two anonymous referees for their constructive comments and suggestions, which have greatly enhanced the quality of this paper.

REFERENCES

1. Hosseini Baharanchi, S.R. "Investigation of impact of supply chain integration on product innovation and quality", *Scientia Iranica, Trans. E*, **16**(1), pp. 81-89 (2009).
2. Nagy, G. and Salhi, S. "Location-routing: issues, models and methods", *European Journal of Operational Research*, **177**(2), pp. 649-672 (2007).
3. Daskin, M.S., Coullard, C.R. and Shen, Z.-J.M. "An inventory-location problem: formulation, solution algorithm and computational results", *Annals of Operations Research*, **110**, pp. 83-106 (2002).
4. Shavandi, H. and Mahlooji, H. "Fuzzy hierarchical queuing models for the location set covering problem in congested systems", *Scientia Iranica*, **15**(3), pp. 378-388 (2008).
5. Baita, F., Ukovich, W., Pesenti, R., and Favaretto, D. "Dynamic routing-and-inventory problem: a review", *Transportation Research Part A*, **32**, pp. 585-598 (1998).
6. Min, H., Jayaraman, V. and Srivastava, R. "Combined location-routing problems: a research directions synthesis and future", *European Journal of Operational Research*, **108**, pp. 1-15 (1998).

www.SID.ir

Archive of SID

7. Cordeau, J.-F., Laporte, G., Savelsbergh, M.W.P. and Vigo, D. "Vehicle routing", In *Handbook of Operations Research and Management Science - Transportation*, Barnhart, C. and Laporte, G., Eds., pp. 367-428, Elsevier, Amsterdam (2006).
8. Megiddo, N. and Supowit, K.J. "On the complexity of some common geometric location problems", *SIAM Journal on Computing*, **13**, pp. 182-196 (1984).
9. Maranzana, F.E. "On the location of supply points to minimize transport costs", *Operational Research Quarterly*, **15**, pp. 261-270 (1964).
10. Webb, M.H.J. "Cost functions in the location of depots for multiple-delivery journeys", *Operational Research Quarterly*, **19**(3), pp. 311-320 (1968).
11. Jacobsen, S.K. and Madsen, O.B.G. "On the location of transfer points in a two-level newspaper delivery system - A case study", *The International Symposium on Locational Decisions*, Banff, Alberta, Canada (1978).
12. Jacobsen, S.K. and Madsen, O.B.G. "A comparative study of heuristics for a two-level routing-location problem", *European Journal of Operational Research*, **5**, pp. 378-387 (1980).
13. Laporte, G. and Nobert, Y. "An exact algorithm for minimizing routing and operating costs in depot location", *European Journal of Operational Research*, **6**, pp. 224-226 (1981).
14. Nambiar, J.M., Gelders, L.F. and Van Wassenhove, L.N. "A large scale location-allocation problem in the natural rubber industry", *European Journal of Operational Research*, **6**, pp. 183-189 (1981).
15. Or, I. and Pierskalla, W.P. "A transportation, location-allocation model for regional blood banking", *AIIE Transactions*, **11**(2), pp. 86-95 (1979).
16. Cox, D.W. "An airlift hub-and-spoke location-routing model with time windows: Case study of the Conus-to-Korea airlift problem", Dissertation in Graduate School of Engineering, Air Force Institute of Technology, Ohio (2004).
17. Burks, R.E.J. "An adaptive tabu search heuristic for the location routing pickup and delivery problem with time windows with a theater distribution application", Dissertation in Graduate School of Engineering and Management, Air Force Institute of Technology, Ohio (2006).
18. Lin, C.K.Y., Chow, C.K. and Chen, A. "A location-routing-loading problem for bill delivery services", *Computers & Industrial Engineering*, **43**, pp. 5-25 (2002).
19. Wu, T.H., Low, C. and Bai, J.W. "Heuristic solutions to multi-depot location-routing problems", *Computers & Operations Research*, **29**, pp. 1393-1415 (2002).
20. Chan, Y., Carter, W.B. and Burnes, M.D. "A multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands", *Computers & Operations Research*, **23**, pp. 803-826 (2001).
21. Albareda-Sambola, M., Diaz, J.A. and Fernández, E. "A compact model and tight bounds for a combined location-routing problem", *Computers & Operations Research*, **32**, pp. 407-428 (2005).
22. Alumur, S. and Kara, B.Y. "A new model for the hazardous waste location-routing problem", *Computers & Operations Research*, **34**(5), pp. 1406-1423 (2007).
23. Özyurt, Z. and Aksen, D. "Solving the multi-depot location-routing problem with Lagrangian relaxation", In *Extending the Horizons: Advances in Computing, Optimization, and Decision Technologies*, Baker, K. et al., Eds., Springer, New York, NY (2007).
24. Albareda-Sambola, M., Fernández, E., and Laporte, G. "Heuristics and lower bound for a stochastic location-routing problem", *European Journal of Operational Research*, **179**, pp. 940-955 (2007).
25. Schwardt, M. and Fischer, K. "Combined location-routing problems - A neural network approach", *Annals of Operations Research*, **167**, pp. 253-269 (2008).
26. Ambrosino, D., Sciomachen, A. and Scutellà, M.G. "A heuristic based on multi-exchange techniques for a regional fleet assignment location-routing problem", *Computers & Operations Research*, **36**, pp. 442-460 (2009).
27. Salhi, S. and Nagy, G. "Consistency and robustness in location-routing", *Studies in Locational Analysis*, **13**, pp. 3-19 (1999).
28. Salhi, S. and Rand, G.K. "The effect of ignoring routes when locating depots", *European Journal of Operational Research*, **39**, pp. 150-156 (1989).
29. Daskin, M.S., *Network and Discrete Location: Models, Algorithms, and Applications*, Wiley, New York, NY (1995).
30. Daskin, M.S., Snyder, L.V. and Berger, R.T. "Facility location in supply chain design", In *Logistics Systems: Design and Operation*, Langevin, A. and Riopel, D., Eds., pp. 39-66, Springer, New York, NY (2005).
31. Desrochers, M. and Laporte, G. "Improvements and extensions to the Miller-Tucker-Zemlin subtour elimination constraints", *Operations Research Letters*, **10**, pp. 27-36 (1991).
32. Kontoravdis, G. and Bard, J.F. "A GRASP for the vehicle routing problem with time windows", *ORSA Journal on Computing*, **10**, pp. 10-23 (1995).
33. Or, I. "Traveling salesman-type combinatorial problems and their relation to the logistics of blood banking", Dissertation in Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, Illinois (1976).
34. Balakrishnan, N. "Simple heuristics for the vehicle routing problem with soft time windows", *The Journal of the Operational Research Society*, **44**(3), pp. 279-287 (1993).

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Archive of SID

35. Prim, R.C. "Shortest connection networks and some generalizations", *Bell System Technical Journal*, **36**, pp. 1389-1401 (1957).

36. Chan, Y. "Simultaneous location-and-routing models", In *Location, Transport and Land-Use - Modeling Spatial-Temporal Information*, pp. 210-333, Springer, Berlin (2005).

37. Garey, M.R. and Johnson, D.S. *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W.H. Freeman, New York, NY (1979).

38. Kruskal, J. "On the shortest spanning subtree of a graph and the travelling salesman problem", *Proceedings of the American Mathematics Society*, **7**(1), pp. 48-50 (1956).

39. Fisher, M.L. "The Lagrangian relaxation method for solving integer programming problems", *Management Science*, **27**(1), pp. 1-18 (1981).

40. Geoffrion, A.M. "Lagrangian relaxation and its uses in integer programming", *Mathematical Programming Study*, **2**, pp. 82-114 (1974).

41. Held, M. and Karp, R.M. "The traveling salesman problem and minimum spanning trees: part II", *Mathematical Programming*, **1**, pp. 6-25 (1971).

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