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Adaptive Decoupling for Open Chain Planar Robots

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Abstract. *Decoupling of dynamic equations in robotic mechanisms has attracted many researchers in the recent years. This kind of decoupling can be achieved by modifying the original kinematic structure through the use of counterweights attached to the moving links. Therefore, the robotic control becomes easier due to reducing the coupling disturbances. In this paper, different methods of decoupling including the static balancing, Coriolis and centripetal force eliminating and dynamic balancing are introduced and their concepts are described based on Lagrange-Euler equations. The systematic adaptive approach is proposed for any open-chain planar robots whose links are connected by revolute joints. The method is tested for two-link and three-link manipulator. The results indicate that the system is fully-decoupled and simple classical approach is sufficient to control it.*

Keywords: *Decoupling; Balancing; Planar mechanism; Robotic; Static balancing; Dynamic balancing.*

INTRODUCTION

Practically, serial robots are extensively used for materials handling, pick and place applications, welding, cutting, painting and other repeatedly industrial processes. So a small improvement in their operations leads to more economical usage. In these manipulators, coupling forces generated among the joints provides some complex and nonlinear dynamics [1]. Diken [2] showed that simple feedback controller (like PD-controller) is ineffective for unbalanced manipulator due to highly nonlinear dynamics. Decoupling can reduce the nonlinear terms considerably. When decoupling take places, each actuator is responsible for its link. Therefore, many researchers tried to apply decoupling in manipulators.

Paul reviewed some decoupling methods. The first conceivable method is simplification based on neglecting Coriolis and centripetal forces in the robots which provides big errors in high-speed motions [3]. Another approach to overcome this problem is introducing the inverse dynamics controller that gives totally a decoupled system [4]. Performance index approach is another choice for improving the mechanism behavior. This method is based on introducing the nonlinear performance index (npi) for identifying and quantifying nonlinear effects during the manipulator motion [5]. In this field, Herman addressed quasi-velocity terms to define the acceptably approximate decoupling scheme [6,7].

Besides these three methods, to provide a good performance using structural mechanism modification, one should ensure that magnitudes and directions of shacking forces and moments do not change significantly. If the former approach is considered, some minimization method should be applied on the mechanism structure [8]. Although application of optimization method is very intelligent method for balancing, it generally has not a high performance [9]. In the later approach, for structural modification based decoupling of robot, the model dynamic is really simplified without neglecting the nonlinear terms. Therefore, the control becomes easier even in the high speed motions. Structural modification is achieved by adding mass or spring to the mechanism. Selecting the proper additional masses can eliminate gravity and all of the coupling terms from the dynamical equations [9]. Many efforts have been applied on the structural balancing of planar

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robots until now. Bagci [10] showed that the revolute joint planar mechanisms are only linkages that they can be fully balanced. Toumi and Asada [11] proposed a method based on conditions that completely decoupled the mechanisms based on theoretical concepts.

Two types of balancing can be considered, statical and dynamical. Mechanism is statically balanced if it has a stationary centroid or equivalently constant potential energy in each configuration. Although static balancing eliminates the shaking force, it cannot essentially remove the shaking moment, so dynamical balancing is also necessary. In this case, Kochev [12] reviewed the moment balancing methods, and also provided a general theory for the moment balancing based on adding some members. Another review has been prepared by Arakelian and Smith [13]. Kolarski and Vukobratović show that regardless of balancing method, the direct balancing by masses or springs may have positive effects on the dynamic characteristics of the mechanism [14]. Herder and Gosselin [15] used Counter-Rotating-Counter-Weight method (CRCW) to complete balancing and then optimized additional masses. But this method cannot been applied on balancing in cause of complication [16]. Wijk et al. [17] compared the various dynamic balancing method regarding additional mass and additional inertia. In most of the works dealing with balancing, close chain manipulators are considered.

Recently Coelho proposed adaptive balancing method for open chain two-link manipulator in which the whole nonlinear effects are removed by adding two rotary and movable masses [18]. The applied method is based on annulling the coefficients of the cross inertia, centripetal and Coriolis. Such coefficients for a two-link manipulator are obtained after deriving the dynamic equations by Lagrange formulation.

In the current paper the adaptive balancing done by Coelho for two-link manipulator is developed for n-link serial robot. Instead of balancing-based dynamic equations, energy-based index method is used. For this purpose, Lagrange equation for n-link serial robot manipulator is considered and equations dealing with different balancing conditions are derived. In this paper two balancing conditions are introduced. Static balancing in which the terms associated with gravity forces is eliminated. Dynamic balancing in which Coriolis, centripetal and cross inertia terms are eliminated.

The paper is organized as follows. At First, the energy-based decoupling approach based on Lagrange equations is presented to obtain the decoupling criteria. Then, this method is applied on proposed structure of manipulator and the unknown parameters for complete decoupling are achieved. After that, the proposed method is applied on two-link and three-link manipulator. Finally, discussion and conclusion about the method and results are given.

ENERGY-BASED DECOUPLING APPROACH

Lagrange dynamics method is appealing for extracting of many degrees of freedom systems. For each dynamical system, it is possible to extract a kinetic energy and potential energy as defined in [3]. In this section, we obtained the balancing in Lagrange form [19]. Euler-Lagrange relation for open-loop robots can be derived from variational concept of Lagrangian as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = u_i, \qquad i = 1, \cdots, n, \tag{1}$$

where $L = T - P$, T denotes kinetic energy, and P denotes potential energy of system. Also the dynamical model of a robot manipulator with n degree of freedoms is described as:

$$\mathbf{M(q)\ddot{q}} + \mathbf{C(q,\dot{q})} + \mathbf{G(q)} = \mathbf{u}, \tag{2}$$

where $\mathbf{u} \in \mathbf{R^n}$ is the vector of input torques, $\mathbf{\ddot{q}}$, $\mathbf{\dot{q}}$ and $\mathbf{q}$ are vectors of joint angular accelerations, velocities and positions, respectively. $\mathbf{M(q)} \in \mathbf{R^{n\times n}}$ is the inertia matrix, and $\mathbf{C(q,\dot{q})} \in \mathbf{R^n}$ represents the centripetal/centrifugal and Coriolis forces and $\mathbf{G(q)} \in \mathbf{R^n}$ describes the gravity effects.

The unique cause of potential energy in manipulator is gravity, and the mechanism is statically balanced if its potential energy is constant for any configurations. Static balancing is typically achieved by attaching additional mechanical elements to the system, such as counter-weights or springs. The use of counter-weight is more practical and has been applied to the design of planar linkages [15]. Static balancing can be achieved by applying:

$$\mathbf{G(q)} = 0. \tag{3}$$

By this way, the equation of motion is reduced to:

$$\mathbf{u} = \mathbf{\hat{M}\ddot{q}} + \mathbf{\hat{C}(q,\dot{q})}, \tag{4}$$

where $\mathbf{\hat{M}}$ is altered inertia matrix, and $\mathbf{\hat{C}}$ is the altered vector of centripetal and Coriolis forces of the balanced manipulator. When Equation 3 retains, the potential energy becomes constant for any configuration, therefore:

$$\frac{\partial P}{\partial q} = 0. \tag{5}$$

By considering Equation 5, Equation 1 for static balanced system becomes:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = u_i, \qquad i = 1, \cdots, n. \tag{6}$$

In the second step of decoupling, it is important to eliminate the Coriolis and cross inertia couple forces.

If the kinetic energy is only a function of velocity terms ($T = T(\dot{q})$) or:

$$\frac{\partial T}{\partial q} = 0, \qquad i = 1, \cdots, n. \tag{7}$$

Equation 6 simplifies to:

$$\frac{d}{dt}\left(\frac{\partial T(\dot{q})}{\partial \dot{q}_i}\right) = u_i, \qquad i = 1, \cdots, n. \tag{8}$$

In this case, it is possible to show that the value of $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ in Equation 2 becomes zero, and dynamical system is simplified as:

$$u_i = \frac{d}{dt}\left(\frac{\partial T(\dot{q})}{\partial \dot{q}_i}\right) = \sum_{j=1}^{n} M_{ij}\ddot{q}_j, \quad i = 1, 2, \cdots, n, \tag{9}$$

where $M_{ij} = \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_i}$. This equation can be expressed as:

$$\mathbf{u} = \tilde{\mathbf{M}}\ddot{\mathbf{q}}, \tag{10}$$

where $\tilde{\mathbf{M}}$ is altered symmetric non-diagonal inertia matrix. However, there are some coupling forces among actuators. At the end, for diagonalization of the inertia matrix or equivalently completely decoupling of the system, the non-diagonal elements of $\tilde{\mathbf{M}}$ or cross inertia terms in dynamical equation must be zero. Using symmetry property of inertia matrix, one can write:

$$\frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_i} = 0, \quad i \neq j \text{ and } i, j = 1, 2, \cdots, n. \tag{11}$$

This can be presented as dynamic balancing that is to achieve the complete decoupling of dynamic equations. Thus Coriolis, centripetal, gravitational and cross inertia terms are eliminated and dynamic Equation 2 is simplified to:

$$\mathbf{u} = \tilde{\mathbf{M}}\ddot{\mathbf{q}}, \tag{12}$$

where $\tilde{\mathbf{M}}$ is a diagonal matrix represents the inertia terms. Therefore, the criteria for any balancing approach can be presented in Lagrange form in Table 1.

APPLYING THE ENERGY-BASED METHOD ON ROBOT MANIPULATORS

In this section, the theory presented in the previous section is applied on n-link planar open chain robot.

Table 1. Balancing criteria in Lagrange form.

Equation	The Kind of Balancing
Equation 5	Static balancing
Equation 7	Eliminating Coriolis and centripetal terms
Equation 11	Completely decoupling

Proposed Mechanism for Decoupling

Figure 1 shows a manipulator containing rotary masses as adaptive balancing, and counter-weight masses as static balancing. General mechanism contains revolute joints, and has assumed to be a planar open kinematic chain. The joints are numbered from 1 to n, and the link i is between joint i and $i+1$. In addition, the displacement of link i is denoted by θ_i which is the angle of rotation about the joint. In this figure, m_p, m_i, m_{ci} $(i = 1, 2, \cdots, n)$ denote payload mass, link mass and counter-weight mass, respectively. Also l_i, r_{gi}, r_i denote link length, centroid position of link, and counter-weight length, respectively. The proposed mechanism has additional rotary masses more than static balanced mechanism denoted by m_{bi}. Generally g is used for gravitational acceleration constant. Also m_b denotes rotary mass and r_b denotes the rotary length.

There is a ratio between pulleys denoted by r_{ti} for ith link. The angular position of ith rotary mass can be stated as:

$$\theta_{bi} = \sum_{j=1}^{i} \theta_i - r_{ti}\theta_{i+1}. \tag{13}$$

The detail of transformation has been presented in Figure 2.

In the next sections procedures of static balancing, complete decoupling and payload adaptive mode of balancing is presented.

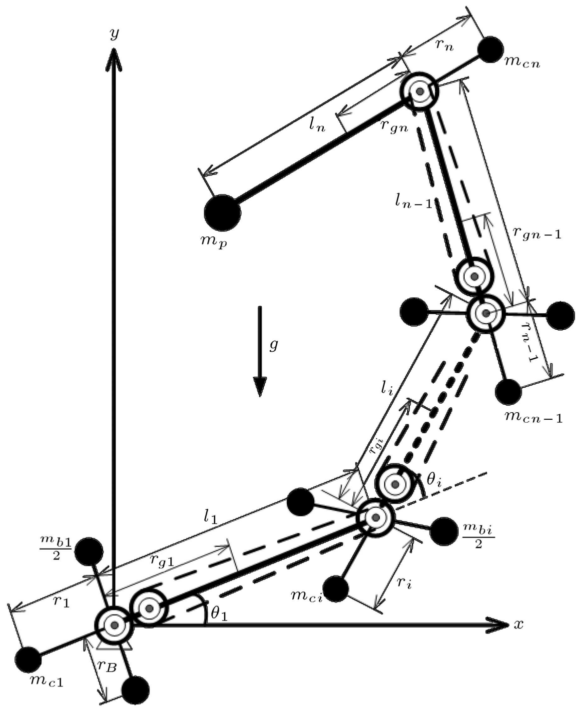

Figure 1. n-link open chain manipulator including counter-weights and rotary masses.

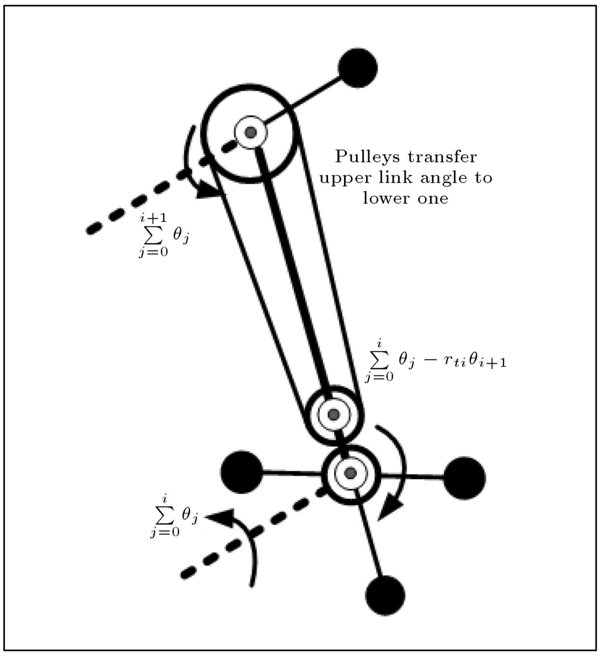

Figure 2. Angular transformation of rotary masses.

Static Balancing of Robot Manipulator

As a first step, due to Equation 5, the static balancing of the proposed mechanism can be achieved by elimination of the potential energy. Potential energy of each mass is $P = mgh$, and the total energy of manipulator is equal by:

$$P = \sum_{i=1}^{n} \left(m_{ci} g h_{ci} + m_i g h_i + m_p g h_p + m_{bi} h_b \right), \quad (14)$$

where:

$$h_{bi} = \sum_{j=1}^{i-1} \left(l_j S_{12\cdots j} \right),$$

$$h_i = h_{bi} + r_{gi} S_{12\cdots i},$$

$$h_{ci} = h_{bi} - r_i S_{12\cdots i},$$

$$h_p = h_{b(n+1)}.$$

In Equation 14 h_{bi}, h_i, h_{ci} and h_p denote height of center of rotary rod link, height of centroid of link, height of counter-weight mass, and height of payload mass, correspondingly. Also $S_{12\cdots j} = \sin(\theta_1 + \theta_2 + \cdots + \theta_j)$ and $C_{12\ldots j} = \cos(\theta_1 + \theta_2 + \cdots + \theta_j)$. As shown in Figure 1, nth link has not the rotary mass, so the m_{bn}=0. By substituting of values in Equation 14, potential energy becomes:

$$P = \sum_{i=1}^{n} \left(m_{ci} g (h_{bi} - r_i S_{12\ldots i}) + m_i g (h_{bi} + r_{gi} S_{12\ldots i}) + m_p g h_{bn} + m_{bi} h_b \right). \quad (15)$$

Due to static balancing form, Equation 5 should be applied, so Equation 15 becomes:

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{i-1} \left((m_{ci} + m_i + m_{bi} + m_p)\, l_j S_{12\cdots j} \right) + m_i r_{gi} - m_{ci} r_i) S_{12\cdots i} + \sum_{j=1}^{n} (m_p l_j S_{12\cdots j}) \right) \equiv 0,$$

$$\Rightarrow \sum_{i=1}^{n} \Big\{ [l_i \times [m_p + \sum_{j=i}^{n} (m_{cj} + m_j + m_{bj})] - m_{ci} r_i + m_i r_{gi}] \times S_{12\cdots i}) \equiv 0 \Big\}. \quad (16)$$

Since $S_{12\cdots i}$ is not generally zero, all of the coefficients must be zero. So there are n equations that can be pulled out from Equation 16 to obtain the counter-weights as follow:

$$m_{ci} = \frac{1}{r_i} \Big\{ l_i \times \Big[m_p + \sum_{j=i+1}^{n} (m_{cj} + m_j + m_{bj}) \Big] + m_i r_{gi} \Big\}, \quad i = 1, 2, \cdots, n. \quad (17)$$

These values of counter-weights and positions satisfy Equation 5 due to static balancing of manipulator.

Kinetic Energy of the Mechanism

In this section, second condition is applied and the Coriolis and centripetal forces are eliminated using Equation 7. Kinetic energy of each link can be written as Equation 18 by defining the virtual payload (m_{pi}) applied instead of the eliminated link masses. For instance, virtual payload of nth link is equal by the actual payload. However, for $(n-1)$th link, the virtual payload becomes the sum of all masses in nth link with its actual payload. Therefore, the ith link kinetic energy becomes:

$$T_i = \frac{1}{2} \left(m_{pi} + m_i + m_{ci} + m_{bi} \right) v_i^2 + \frac{1}{2} \left(m_{pi} l_i^2 + m_{ci} r_i^2 + m_i r_{gi}^2 + I_{ci} \right) \dot{\theta}_{1\cdots i}^2 + \frac{1}{2} m_{bi} r_{bi}^2 \left(r_{ti} \dot{\theta}_{i+1} - \dot{\theta}_{1\cdots i} \right)^2, \quad (18)$$

where v is centroid velocity of link, I_{ci} is the inertia of link around the centroid of link, $\dot{\theta}_{1\cdots i} = \dot{\theta}_1 + \dot{\theta}_2 + \cdots + \dot{\theta}_i$, and m_{pi} is the virtual payload applied on joint between ith and $(i+1)$th link. In the static balanced link, the centroid of ith link coincides on joint between ith and $(i-1)$th links as shown in Figure 3. Therefore, we can

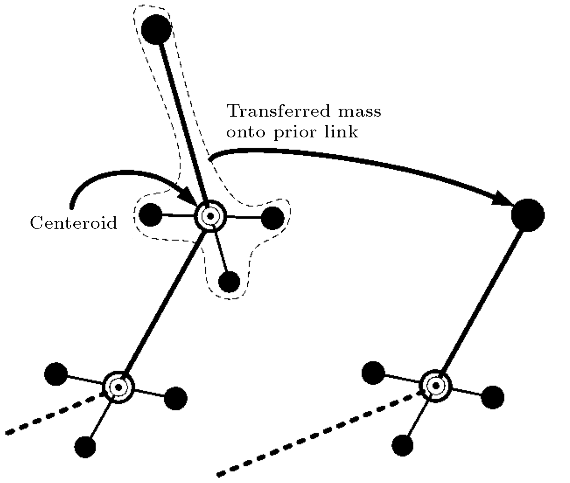

Figure 3. Transferring the mass in balanced planar manipulator onto prior link.

assume that the ith link masses, as virtual payload, are on $(i-1)$th link, and the ith link has only the inertia momentum. Then, all the ith link masses are transferred to $(i-1)$th link. Considering previous transferred masses on link, Equation 18 in this case becomes:

$$T_i = \frac{1}{2}M_i v_i^2 + \frac{1}{2}I_i\dot{\theta}_{1\cdots i}^2 + \frac{1}{2}m_{bi}r_{bi}^2\left(r_{ti}\dot{\theta}_{i+1} - \dot{\theta}_{1\cdots i}\right)^2$$
$$i = 1, 2, \cdots, n, \tag{19}$$

where M_i and I_i denote the total mass and inertia accumulated on the ith link as:

$$M_i = \sum_{j=i}^{n-2} m_{bi} + \sum_{j=i}^{n-1}(m_j + m_{cj}) + m_p,$$
$$I_i = I_{ci} + m_i r_{gi}^2 + m_{ci}r_i^2 + (M_i - m_i + m_{ci})l_i^2. \tag{20}$$

If we apply the transferring procedure on the ith link, the kinetic energy of link becomes:

$$T_i = \frac{1}{2}I_i\dot{\theta}_{1\cdots i}^2 + \frac{1}{2}m_{bi}r_{bi}^2\left(r_{ti}\dot{\theta}_{i+1} - \dot{\theta}_{1\cdots i}\right)^2. \tag{21}$$

This procedure can be applied for the whole mechanism starting from the nth link to the first link. Then, it is possible to write the total energy mechanism as:

$$T_{\text{total}} = \sum_{i=1}^{n} T_i = \frac{1}{2}M_1 v_1^2 + \frac{1}{2}\sum_{i=1}^{n}\left(I_i\dot{\theta}_{1..i}^2\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{n-1}\left[m_{bi}r_{bi}^2\left(r_{ti}\dot{\theta}_{i+1} - \dot{\theta}_{1..i}\right)^2\right], \tag{22}$$

where M_1 is the total mass of manipulator transferred on joint 1, and v_1 is the velocity of manipulator's base. Since the platform is usually stationary, the total kinetic energy is only a function of angular velocities.

$$v_1 = 0 \Rightarrow T_{\text{total}} = \frac{1}{2}\sum_{i=1}^{n}\left(I_i\dot{\theta}_{1..i}^2\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{n-1}\left[m_{bi}r_{bi}^2\left(r_{ti}\dot{\theta}_{i+1} - \dot{\theta}_{1..i}\right)^2\right]. \tag{23}$$

Equation 22 coincides with Equation 7, which means that the nonlinear terms are crossed out. Therefore, the dynamical equation becomes:

$$u_l = \sum_{k=1}^{n} M_{lk}\ddot{\theta}_k, \quad M_{lk} = \frac{\partial^2 T}{\partial\dot{\theta}_k\partial\dot{\theta}_l}. \tag{24}$$

So applying the static balancing not only completely eliminates the potential gravitational effects, but it also eliminates the Coriolis and centripetal terms in the dynamic equation. Thus Equation 7 is satisfied.

Inertia Tensor Diagonalization Process

Static balancing may influence the dynamic unbalance of the mechanism (it may even increase it [1,2]). Therefore, the dynamic balancing may be carried out in conjunction with the static balancing. Complete decoupling can be satisfied if Lagrangian of balanced system is applied on Equation 11. Total kinetic energy was derived in Equation 22. Let $I_{bi} = m_{bi}r_{bi}^2 r_{ti}^2$ as the ith rotary-link inertia. Now according to the Equation 11 for completely decoupling, derivative of T with respect to $\dot{\theta}_l$ and $\dot{\theta}_k$ can be presented as:

$$\frac{\partial T}{\partial\dot{\theta}_l} = \sum_{i=l}^{n}\left[I_i\left(\sum_{j=1}^{i}\dot{\theta}_j\right)\right] + \frac{I_{b(l-1)}}{r_{t(l-1)}}\left(r_{t(l-1)}\dot{\theta}_l - \sum_{j=1}^{l-1}\dot{\theta}_j\right)$$
$$- \sum_{i=l}^{n-1}\left[\frac{I_{bi}}{r_{ti}}\left(r_{ti}\dot{\theta}_{i+1} - \sum_{j=1}^{i}\dot{\theta}_j\right)\right], \tag{25}$$

$$\frac{\partial}{\partial\dot{\theta}_k}\left(\frac{\partial T}{\partial\dot{\theta}_l}\right) = \sum_{i=l}^{n}\left[I_i\frac{\partial}{\partial\dot{\theta}_k}\left(\sum_{j=1}^{i}\dot{\theta}_j\right)\right]$$
$$+ \frac{I_{b(l-1)}}{r_{t(l-1)}}\frac{\partial}{\partial\dot{\theta}_k}\left(r_{t(l-1)}\dot{\theta}_l - \sum_{j=1}^{l-1}\dot{\theta}_j\right)$$
$$- \sum_{i=l}^{n-1}\left[\frac{I_{bi}}{r_{ti}}\frac{\partial}{\partial\dot{\theta}_k}\left(r_{ti}\dot{\theta}_{i+1} - \sum_{j=1}^{i}\dot{\theta}_j\right)\right], \tag{26}$$

where $I_{bi} = m_{bi} r_{bi}^2 r_{ti}^2$. After applying some mathematical simplification presented in the Appendix, Equation 26 becomes:

$$k \geq l : \frac{\partial}{\partial \dot{\theta}_k}\left(\frac{\partial T}{\partial \dot{\theta}_l}\right) = \sum_{i=k}^{n} I_i + I_{b(k-1)} - \sum_{i=k}^{n-1} I_{bi}. \quad (27)$$

According to Equation 11 for complete decoupling, Equation 27 must be vanished. This leads to obtain complete decoupling inertia values as:

$$I_{b_{(k-1)}} = \sum_{i=k}^{n} I_i - \sum_{i=k}^{n-1} I_{bi}, \qquad k = 2, 3, \cdots, n. \quad (28)$$

Sequentially, direct recursive equation can be shown as:

$$I_{b(i-1)} = 2I_{b_i} + I_i,$$

$$I_{b_n} = 0,$$

$$i = 1, 2, \cdots, n. \quad (29)$$

Additionally, solution of this difference equation can be obtained as:

$$I_{bi} = \sum_{j=i+1}^{n-1} (2^{j-i} I_{j+1}), \quad (30)$$

that directly gives rotary value of added inertia value for decoupling criteria. Table 2 shows the rotary inertia for $n = 2, 3, 4$.

General dynamical Equation 2 is now completely decoupled and has been simplified to a single term as:

$$u_l = \left[\sum_{i=l}^{n}\left(I_i + \sum_{j=i}^{n-1}(2^{j-i} I_{j+1})\right)\right]\ddot{\theta}_l,$$

$$l = 1, \cdots, n. \quad (31)$$

This equation can be simplified as:

$$u_l = \left[\sum_{i=l}^{n}(2I_i + I_{bi})\right]\ddot{\theta}_l, \quad l = 1, 2, \cdots, n, \quad (32)$$

where I_{bn} is zero.

Table 2. Values of rotary inertia for n=2, 3 and 4.

n	i	I_{bi}
2	1	I_2
3	1	$I_2 + 2I_3$
	2	I_3
4	1	$I_2 + 2I_3 + 4I_4$
	2	$I_3 + 2I_4$
	3	I_4

Payload Adaption

In this section, the effect of payload is analyzed on the rotary and counter-weight masses and their positions. Using Equation 17, it is possible to obtain the positions of counter weights masses in term of payload as follow:

$$r_i = \frac{1}{m_{ci}}\Big\{l_i \times \Big[m_p + \sum_{j=i+1}^{n}(m_{cj} + m_j + m_{bj})\Big]$$

$$+ m_i r_{gi}\Big\}, \quad i = 1, 2, \cdots, n, \quad (33)$$

and using Equation 30, the positions of rotary masses in term of payload as follow:

$$r_{bi} = \sqrt{\frac{I_{bi}}{m_{bi} r_{ti}^2}}. \quad (34)$$

So the value of counterweight and rotary masses are considered to be constant, while their positions are adjusted to compensate the payload change using Equations 33 and 34.

ILLUSTRATIVE EXAMPLES AND SIMULATION

Two-Link Manipulator

In this section proposed method for adaptive balancing is demonstrated for two-link manipulator as shown in Figure 4. This model has been considered in [18]. As a first step, static balancing should be applied as mentioned in Equation 17. It can be written as:

$$m_{c2} = (m_2 r_{g2} + m_p l_2)/r_2,$$

$$m_{c1} = (m_p l_1 + m_1 r_{g1} + m_2 l_1 + m_{c2} l_1)/r_1, \quad (35)$$

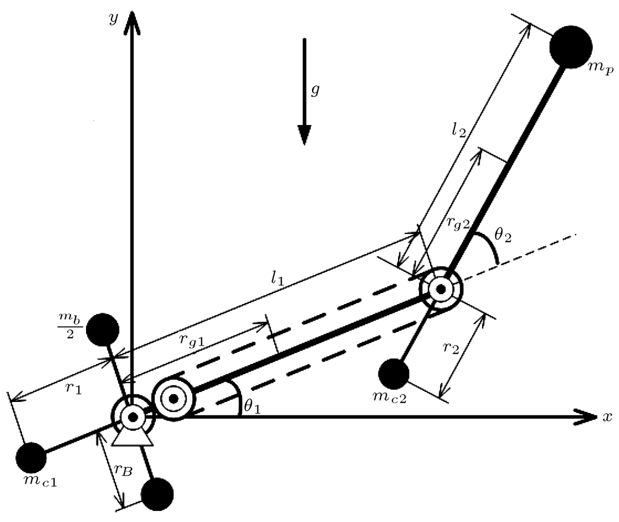

Figure 4. Dynamically balanced two-link manipulator.

and by using Equation 30, the rotary inertia can be driven as:

$$I_b = I_2, \tag{36}$$

where using Equation 20, I_2 is obtained as follow:

$$M_1 = m_1 + m_{c1} + m_2 + m_{c2} + m_p,$$

$$I_1 = I_{c1} + m_1 r_{g1}^2 + m_{c1} r_1^2 + (m_2 + m_{c2} + m_p)\, l_1^2,$$

$$M_2 = m_2 + m_{c2} + m_p,$$

$$I_2 = I_{c2} + m_2 r_{g2}^2 + m_{c2}^2 r_2^2 + m_p l_2^2$$

$$= I_{c_2} + m_2 r_{g2}^2 + (m_2 r_{g2} + m_p l_2)\, r_2 + m_p l_2^2. \tag{37}$$

Since $I_b = m_b r_b^2 r_t^2$, it is possible to derive rotary mass as:

$$m_b = \frac{m_2 r_{g2}(r_{g2} + r_2) + m_p l_2 (l_2 + r_2) + I_{c_2}}{(r_b r_T)^2}. \tag{38}$$

Using Equation 31, the equation of system can be presented as:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} M_{11} & 0 \\ 0 & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}, \tag{39}$$

where:

$$M_{11} = m_1 r_{g1}\,(r_1 + r_{g1}) + I_{c1}$$

$$+ m_2 \left(\frac{l_1 r_{g2}}{r_2}\,(r_1 + l_1) + l_1\,(l_1 + r_1) + 2 r_{g2}\,(r_{g2} + r_2) \right)$$

$$+ m_p \left(\frac{l_1 l_2}{r_2}\,(r_1 + l_1) + l_1\,(l_1 + r_1) + 2 l_2\,(l_2 + r_2) \right),$$

$$M_{22} = I_{c2} + 2 m_2 r_{g2}\,(r_{g2} + r_2) + 2 m_p l_2\,(l_2 + r_2)$$

$$= 2 m_b r_b^2 r_T.$$

This result is completely decoupled as same as decoupled manipulator given in [18].

Three-Link Manipulator Adaptivability Analysis

In this section, the numerical simulation has been shown for a three-link manipulator. Figure 5 exhibits practical three-link robot completely decoupled by using proposed method. The values of such mechanism are listed in Table 3.

Using Equations 17 and 31 the rotary inertia becomes:

$$\begin{cases} I_{b_2} = I_3 \\ I_{b_1} = I_2 + 2 I_3 \end{cases} \tag{40}$$

where:

$$M_1 = m_{b1} + m_1 + m_2 + m_3 + m_{c1} + m_{c2} + m_{c3} + m_p,$$

$$I_1 = I_{c1} + m_1 r_{g1}^2 + m_{c1} r_1^2 + (M_1 - m_1 - m_{c1})\, l_1^2,$$

$$M_2 = m_2 + m_3 + m_{c2} + m_{c3} + m_p,$$

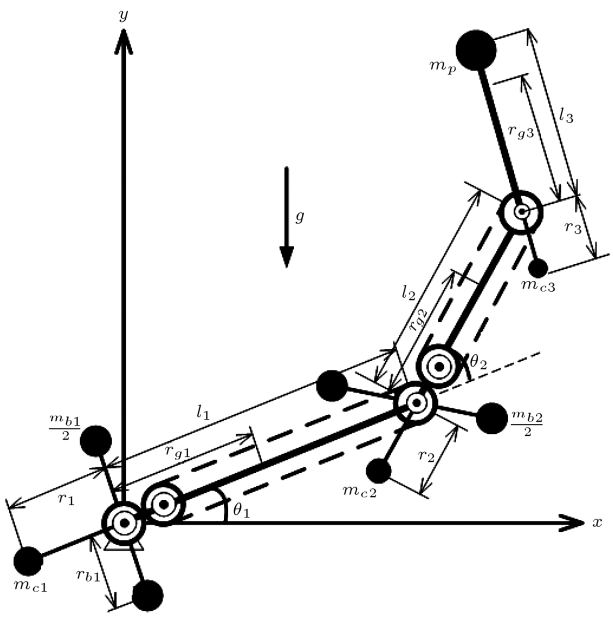

Figure 5. Three-link planar manipulator.

Table 3. The parameters of three-link manipulator.

Parameter	Value	Unit
Mass	$m_1 = 12, m_2 = 8, m_3 = 7$	kg
Length of link	$l_1 = 0.8, l_2 = 0.6, l_3 = 0.5$	m
Moment of inertia	$I_1 = I_2 = I_3 = 1/12$ ml^2	kg.m^2
Ratio of pulleys	$r_{t1} = 2, r_{t2} = 3$	rad/rad
Length of adjacent links	$r_1 = r_2 = r_3 = 0.5$	m
Centroid of links	$r_{g1} = 0.4, r_{g2} = 0.3, r_{g3} = 0.25$	m
Counter-weights masses	$m_{c1} = 85, m_{c2} = 35, m_{c3} = 15$	kg
Counter-rotating masses	$m_{b1} = 75, m_{b2} = 5$	kg

$$I_2 = I_{c2} + m_2 r_{g2}^2 + m_{c2} r_2^2 + (M_2 - m_2 - m_{c2})\, l_2^2,$$

$$M_3 = m_3 + m_{c3} + m_p,$$

$$I_3 = I_{c3} + m_3 r_{g2}^2 + m_{c3} r_3^2 + (M_3 - m_3 - m_{c3})\, l_3^2. \tag{41}$$

So the equation of system can be presented as:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix}, \tag{42}$$

where $M_{ll} = \sum_{i=l}^{n} (2I_i + I_{bi})$.

For this manipulator, the positions of rotary masses and counterweights are shown in term of payload variations in Figures 6 and 7, respectively. Figure 8 shows the value of inertia tensor of decoupled mechanism in term of payload mass.

DISCUSSION AND CONCLUSION

Completely decoupling of open-chain robot is presented in this article. Balancing is related to the Lagrange energy equation and by using this energy method, the general decoupling of planar serial robot is offered. Indeed, traditional method for balancing based on Newton representation is replaced with energy idea of balancing based on Lagrange representation. The obtained equations illustrate that the completely decoupling for n-link planar robot, using energy-based indexes, can be achieved and therefore, the control of mechanism becomes much easier. In fact, by decoupling the manipulator, the n-nonlinear equations are replaced with n-linear equation. Reduction of coupling in robot preserves energy and releases any coupling between actuators. This elimination decreases value of interference among joints and therefore it is possible to improve the precision of motion. Also an applicable and adaptive method is proposed to completely decouple the mechanism due to payload variations. The results of simulation confirmed this approach.

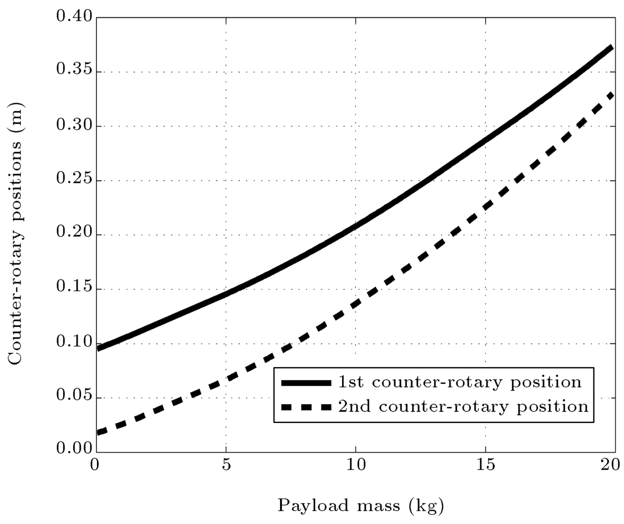

Figure 6. Counter-rotary mass positions.

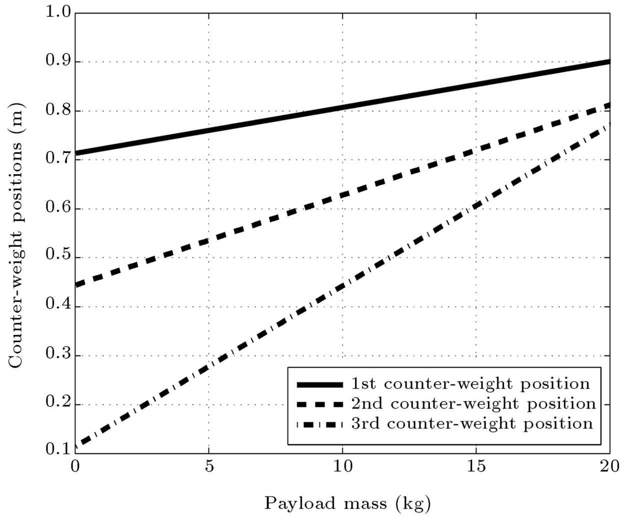

Figure 7. Counter-weight positions.

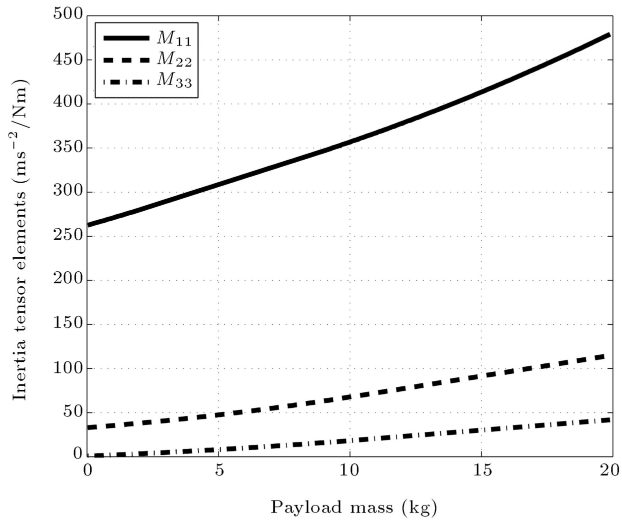

Figure 8. Inertia tensor changing for decoupled mechanism in terms of payload mass.

NOMENCLATURE

L	Lagrangian of system
t	time
P	potential energy
T	kinetic energy
n	degree of freedom
q	generalized position
$\dot{q}$	generalized velocity

$\ddot{q}$	generalized acceleration
u	input
$\mathbf{M}$	mass matrix
$\mathbf{C}$	Coriolis/centripetal matrix
$\mathbf{G}$	gravitational matrix
$\hat{\mathbf{M}}, \tilde{\mathbf{M}}$	altered Mass matrix
$\hat{\mathbf{C}}$	altered Coriolis/centripetal matrix
θ	angle of link
$\dot{\theta}$	rotational velocity of link
$\ddot{\theta}$	rotational acceleration of link
m	mass of link
m_p	mass of payload
m_c	mass of counter-weight
m_b	mass of rotary link
l	length of link
r_g	centroid position on link
r	counter-weight link's length
g	gravitational acceleration constant
r_t	pulley transition ratio
h	height of centroid of link
h_c	height of counter-weight
h_p	height of payload mass
h_b	height of rotary masse
I_c	inertia of link around the centroid
I	total inertia

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APPENDIX

According to Equation 23, it is possible to derive derivation as follow:

$$\frac{\partial T}{\partial \dot{\theta}_l} = \sum_{i=1}^{n} \left[I_n \dot{\theta}_{1..i} \frac{\partial \dot{\theta}_{1..i}}{\partial \dot{\theta}_l} \right] + \sum_{i=1}^{n-1} \left[\frac{I_{bi}}{r_{t(i+1)}} (r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i}) \frac{\partial}{\partial \dot{\theta}_l} (r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i}) \right]. \quad \text{(A1)}$$

It is obvious that:

$$\frac{\partial \dot{\theta}_i}{\partial \dot{\theta}_l} = \begin{cases} 1 & l = i \\ 0 & l \neq i \end{cases} \quad \Rightarrow$$

$$\frac{\partial \dot{\theta}_{1..i}}{\partial \dot{\theta}_l} = \begin{cases} 1 & l \leq i \\ 0 & l > i \end{cases} \quad \text{(A2)}$$

So by substituting Equation A2 in Equation A1, the following will be obtained:

$$\frac{\partial T}{\partial \dot{\theta}_l} = \sum_{i=l}^{n} \left(I_n \dot{\theta}_{1..i} \right) + \frac{I_{b(l-1)}}{r_{t(l-1)}} \left(r_{t(l-1)} \dot{\theta}_l - \dot{\theta}_{1..(l-1)} \right) - \sum_{i=l}^{n-1} \left[\frac{I_{bi}}{r_{ti}} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) \right]. \quad \text{(A3)}$$

The second derivation can be obtained as follow:

$$\frac{\partial}{\partial \dot{\theta}_k} \left(\frac{\partial T}{\partial \dot{\theta}_l} \right) = \sum_{i=l}^{n} \left[I_n \frac{\partial \dot{\theta}_{1..i}}{\partial \dot{\theta}_k} \right] + \frac{I_{b(l-1)}}{r_{t(l-1)}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{tl} \dot{\theta}_l - \dot{\theta}_{1..(l-1)} \right) - \sum_{i=l}^{n-1} \left[\frac{I_{bi}}{r_{ti}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) \right]. \quad \text{(A4)}$$

So:

$$\sum_{i=l}^{n} \left[I_i \frac{\partial \dot{\theta}_{1..i}}{\partial \dot{\theta}_k} \right] = \sum_{i=k}^{n} I_i, \quad k \geq l,$$

$$\frac{I_{b(l-1)}}{r_{t(l-1)}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{tl} \dot{\theta}_l - \dot{\theta}_{1..(l-1)} \right) = \frac{I_{b(l-1)}}{r_{t(l-1)}} \begin{cases} -1 & k \leq l-1 \\ r_{t(l-1)} & k = l \\ 0 & k > l \end{cases}$$

$$\frac{\partial \dot{\theta}_{1..i}}{\partial \dot{\theta}_k} = \begin{cases} 1 & k \leq i \\ 0 & k > i \end{cases}$$

$$\frac{\partial}{\partial \dot{\theta}_k} \left(r_{tl} \dot{\theta}_l - \dot{\theta}_{1..(l-1)} \right) = \begin{cases} -1 & k \leq l-1 \\ r_{tl} & k = l \\ 0 & k > l \end{cases}$$

$$\frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) = \begin{cases} -1 & k \leq i \\ r_{t(i+1)} & k = i+1 \\ 0 & k > i+1 \end{cases} \quad \text{(A5)}$$

and for a third term in Equation A4, one can write:

$$\sum_{i=1}^{n-1} \left[\frac{I_{bi}}{r_{ti}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \right) \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right] = \sum_{i=1}^{k-1} \left[\frac{I_{bi}}{r_{ti}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) \right] + \sum_{i=k}^{n-1} \left[\frac{I_{bi}}{r_{ti}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) \right]. \quad \text{(A6)}$$

After some simplification, it can be written as:

$$\sum_{i=l}^{n-1} \left[\frac{I_{bi}}{r_{ti}} \frac{\partial}{\partial \dot{\theta}_k} \left(r_{t(i+1)} \dot{\theta}_{i+1} - \dot{\theta}_{1..i} \right) \right] = - \sum_{i=k}^{n-1} I_{bi} + \begin{cases} I_{b(k-1)} & k > l \\ 0 & k = l \end{cases} \quad \text{(A7)}$$

Finally Equation 27 can be derived as:

$$k \geq l : \frac{\partial}{\partial \dot{\theta}_k} \left(\frac{\partial T}{\partial \dot{\theta}_l} \right) = \sum_{i=k}^{n} I_i + I_{b(k-1)} - \sum_{i=k}^{n-1} I_{bi}. \quad \text{(A8)}$$

BIOGRAPHIES

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