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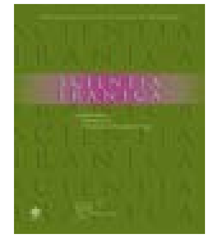

Research note

Utilization of percolation approach to evaluate reservoir connectivity and effective permeability: A case study on North Pars gas field

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Abstract Reservoir characterization, especially during early stages of reservoir life, is very uncertain, due to the scarcity of data. Reservoir connectivity and permeability evaluation is of great importance in reservoir characterization. The conventional approach to addressing this is computationally very expensive and time consuming. Therefore, there is a great incentive to produce much simpler alternative methods. In this paper, we use a statistical approach called the percolation theory, which considers a hypothesis wherein the reservoir can be split into either permeable (i.e. sand/fracture) or impermeable flow units (i.e. shale/matrix), and assumes that the connectivity of permeability contrasts controls the flow. We developed master curves for the reservoir connectivity and its associated uncertainty, as well as the effective permeability and its associate uncertainty curves. To validate the approach, we have used the data set of the North Pars gas field located at the North of the South Pars field in the Persian Gulf. We have shown that the percolation approach gives reliable predictions when compared with results from a conventional reservoir modeling approach. Moreover, the first approach, as obtained from algebraic manipulation, is very fast, whereas the latter is very costly and time consuming.

1. Introduction

The nature of fluid flow in hydrocarbon reservoirs is very complex because of the complicated sedimentary processes that control the spatial distribution of the heterogeneities. Reservoir characterization, especially during early stages of reservoir development, is very uncertain due to the limited availability of certain data. The conventional approach to investigating the impact of geological uncertainties on the prediction of reservoir performance is to build a detailed geological model (with around 10^7 grids) using geophysical, geological and petro physical data, upscale it to around 10^4 or 10^5 simulation grids and finally perform flow simulation. Because of uncertainties in the data, it is necessary to construct a number of possible geological models and then run flow simulations many times to get a reliable model. This approach is computationally very expensive. Furthermore, during the early stages of field development, when data are scarce, the conventional deterministic based approach cannot be used. As the data, as well as the models, are all uncertain, it is necessary to construct a number of possible geological models using a stochastic modelling approach to account for underlying uncertainties. Thus, there is a great incentive to produce a much simpler, physically-based methodology to predict uncertainties in the reservoir performance very quickly, especially for engineering purposes.

Reservoir connectivity and effective permeability evaluation is of great importance to reservoir forecasting, which is used for decision making in various possible development scenarios. This can be addressed by using a percolation approach. The percolation theory is a mathematical model of the connectivity of randomly distributed objects in complex geometries. The global geometrical and physical properties of such a system are related to the density of objects placed randomly in a domain through some universal laws [1]. These laws turn out to be independent of the detail of the system, i.e. local geometries. This statistical approach considers a hypothesis

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wherein the reservoir can be split into either permeable (i.e. sand/fracture) or impermeable zones (i.e. shale/matrix), and assumes that the connectivity of permeability contrasts controls the fluid flow. There are many cases where this is a very good approximation. For example, the reservoir may have been deposited by meandering river belts in which case good sand occurs as sand bodies on an impermeable background. Another heterogeneity that one can consider is fractures, where the flow is essentially through the interconnected fracture network.

Percolation has many applications, from the spread of diseases [2] and forest fires [3] to polymers materials [4] and porous media [5]. Moreover, there is an extensive literature discussing various applications of the percolation theory at both pore and core scale levels [6–12]. However, in this paper we concentrate on the field scale applications of the percolation theory in order to make an estimation of global properties, such as reservoir connectivity and conductivity. There are number of good reviews of the percolation theory [1,13,14]. This theory is particularly well adapted to describe global physical properties, such as the connectivity and conductivity behaviour of geometrically complex systems. For example, Watson and Leath [15] and Pike and Seager [16] studied the conductivity behavior of two dimensional site percolation problems. King [17] considered the connectivity and conductivity of overlapping sand bodies in low to intermediate net-to-gross reservoirs. Lee and Torquato [18] worked on correlated continuum percolation models and found the master curves and percolation threshold of such systems. Baker et al. [19] found the percolation threshold of continuum systems for interpenetrating objects in two and three dimensions. Berkowitz [20] used the percolation theory to characterize the flow behavior of fractured geological media. Nurafza et al. [21] used the percolation approach to model connectivity of low to intermediate net-to-gross reservoirs, and validated the developed curves against a realistic field dataset. Masihi et al. [22] worked on the connectivity of fractured reservoirs using the percolation theory. They in [23] worked on the facies' connectivity and conductivity of reservoirs in two and three dimensions. Sadeghnejad et al. [24] analyzed the conductivity behavior of two-dimensional isotropic continuum systems in petroleum reservoirs. They in [25] incorporated the effects of anisotropy on the finite-size scaling of connectivity and conductivity of continuum percolation in three dimensions.

To develop the methodology, an object based technique is used to model the spatial distribution of isotropic sand bodies in two dimensions. We extend the previously published approach in two directions. We reevaluate the percolation results for the mean reservoir connectivity and extend the technique to find the effective permeability of the reservoir. The main contribution is to develop the master curves for the reservoir connectivity and its associated uncertainty, as well as effective permeability and its associate uncertainty.

To validate the approach, we used the data set of the North Pars gas field, which is located 85 km north of the giant South Pars gas field in the Persian Gulf. We have shown that reservoir connectivity and effective permeability predictions from the percolation approach give reliable results once compared with exact results obtained numerically from the conventional modeling of real field data. Moreover, the first approach, as obtained from algebraic manipulation, is very fast, whereas the latter is very costly and time consuming.

2. Percolation theory approach

A simple model, with which we can describe the percolation theory, is a continuum system, in which different overlapping objects with net-to-gross, p, are distributed randomly (Figure 1(c)). Geologists have used the idea that a reservoir consists of geometrically complex connected and disconnected sand bodies. Because of complicated sedimentary processes, the reservoir can be assumed to be defined by connected flow units. An example of this is a meandering river, which deposits layers of sand (sand bodies) in its flowing impermeable bed (Figure 1(a) and (b)). Due to upstream events, the river path is changed and new sand bodies may deposit over previous ones. This is a simple representation of the complicated process of sedimentation over millions of years.

This simple model of overlapping sand bodies, as shown in Figure 1(c), was previously used within the framework of continuum percolation [17]. In this isotopic model, all bodies are represented by squares of the same size in a reservoir with size of L. The sand bodies, which are distributed independently in an impermeable background, are assumed to be permeable in the entire reservoir. Furthermore, it is assumed that there is a perfect hydraulic contact between the connected sand bodies. Clusters are formed when neighboring sand bodies are overlapped. As the net-to-gross, p, which is the probability that a point in space lies within the sand bodies, is increased, the clusters grow in size and, at percolation threshold, p_c, one large cluster spans the whole reservoir. A standard algorithm [17] is used to identify various clusters.

One fundamental percolation property is the reservoir connectivity, $P(p)$, which is defined as a probability that any point in space belongs to the spanning cluster. For a given value of p, this is defined as [1]:

$$P(p) \propto (p - p_c^{\infty})^{\beta}, \tag{1}$$

where p_c^{∞} is the percolation threshold of an infinite reservoir and the exponent β is called the connectivity exponent. Above the threshold, p_c, reservoir connectivity, $P(p)$, increases rapidly. The second property is the reservoir effective permeability. From the percolation theory, the effective permeability of an infinite system has power law behavior near the threshold as [1]:

$$K(p) \propto (p - p_c^{\infty})^{\mu}, \tag{2}$$

where μ, a universal exponent, is called the conductivity exponent.

These simple scaling laws are valid for an infinite system size. For finite size reservoirs, if we plot reservoir connectivity, P, as a function of net-to-gross p, over a large number of realizations for a particular reservoir size, we get a scatter of points. The statistical analysis gives the mean reservoir connectivity, $P(p, L)$, and its associated standard deviations, $\Delta_P(p, L)$ (the fluctuations about the mean value) as shown in Figure 2.

A similar analysis can be done for effective permeability results, which leads to mean effective permeability, $K(p, L)$, and its associated uncertainty, $\Delta_K(p, L)$.

Plotting the mean reservoir connectivity and mean effective permeability, obtained from the different reservoir sizes, as a function of net-to-gross, gives different curves, which can be collapsed on top of each other through the following finite size scaling transformation [1,17]:

$$P(p, L) = L^{-\beta/\nu}\mathcal{F}[(p - p_c^{\infty})L^{1/\nu}], \tag{3}$$

$$K(p, L) = L^{-\mu/\nu}\kappa[(p - p_c^{\infty})L^{1/\nu}], \tag{4}$$

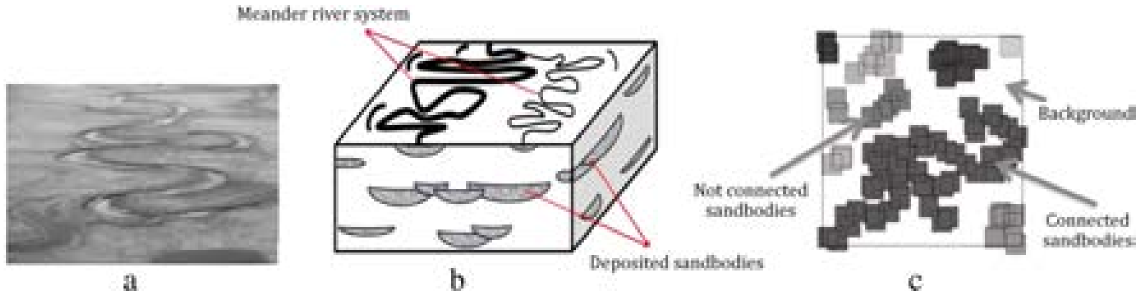

Figure 1: (a) A meandering river system. (b) Overlapping sand bodies deposited by a meandering river. (c) Simplified meandering river modeling by continuum percolation.

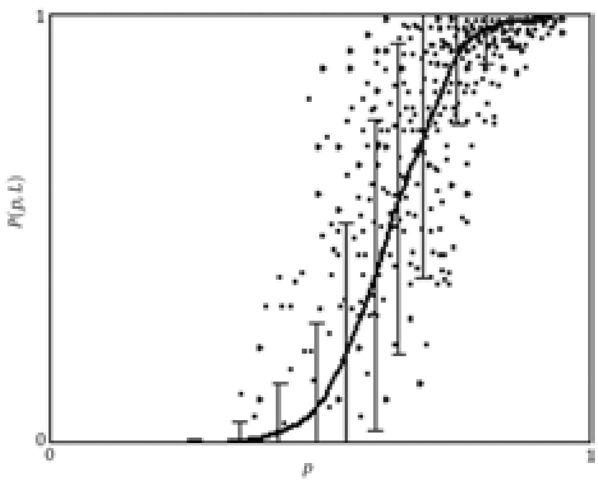

Figure 2: A typical scatter obtained for reservoir connectivity of a finite size reservoir. The curve and lines represent average and error bars, respectively, determined over all realizations at the same net-to-gross.

where $\mathcal{F}$ and κ are two master functions for mean reservoir connectivity and mean effective permeability, respectively, and ν is a universal exponent called the correlation length exponent.

Similarly, the results of the standard deviation of reservoir connectivity and effective permeability can be related to each other by applying the following scaling [1,17]:

$$\Delta_P(p, L) = L^{-\beta/\nu}\mathcal{R}[(p - p_c^{\infty})L^{1/\nu}], \quad (5)$$

$$\Delta_K(p, L) = L^{-\mu/\nu}\xi[(p - p_c^{\infty})L^{1/\nu}], \quad (6)$$

where $\mathcal{R}$ and ξ are two master functions for the standard deviation of reservoir connectivity and effective permeability, respectively.

3. Reservoir connectivity and effective permeability prediction

We used standard algorithms [17] to generate the realization of overlapping sand bodies. We have generated nearly 10^3 realizations for different system sizes. System sizes are considered in dyadic form, from 2^2 to 2^7. First, we reevaluate the connectivity master curve of square sand bodies in two dimensions, which is in good agreement with previous published work [17]. However, this is the first time that we have evaluated the conductivity curve of overlapping sand body systems using a high number of realizations and relatively large system sizes. Then, we plot $PL^{\beta/\nu}$ against $(p - p_c^{\infty})L^{1/\nu}$ for different reservoir sizes. The numerical results obtained from the simulation of various system sizes from which the master curves for the mean connectivity and its associated fluctuations (standard deviation) are discovered, and shown in Figure 3.

The scaling results of the reservoir connectivity show a good data collapse from which the connectivity curve, $\mathcal{F}$, and its standard deviation, $\mathcal{R}$, can be extracted. From these two master curves, one can predict the mean reservoir connectivity and its associated uncertainty for any system size very quickly, without performing any explicit realization.

We shall now consider a simple model for determining the effective permeability of the reservoir. Consider the same permeability (say equal to one) for the entire occupied regions. The background section is considered to have zero permeability. We can determine the effective permeability of the reservoir by solving the single phase pressure equation, $\nabla K \nabla P = 0$, numerically, within the overlapping sand bodies. This, however, requires a lot of computer time and storage, and becomes prohibitively expensive when many realizations are required to determine the mean behavior. Instead, we may use a computationally efficient up-scaling method, known as real space renormalization [26]. The renormalization technique is based on the principle of calculating the permeability of a small region of space in order to define a coarser grid. We, then, repeat this procedure on the coarse grid and so on, until a fixed value is found. To do this, we first place a fine grid of points over the reservoir and assign a grid point permeability of 0 or 1, depending on whether the points are occupied or not. As observed for reservoir connectivity, there is a scatter in the results from finite size effects. Here, we shall only discuss the mean behavior. We have generated nearly 10^3 realizations for different system sizes. Again, system sizes are considered in dyadic form, from 2^2 to 2^6. We plot $KL^{\mu/\nu}$ against $(p - p_c^{\infty})L^{1/\nu}$ for different reservoir sizes. The numerical results obtained from the simulation of various system sizes, from which the master curves for the mean effective permeability and its associated fluctuations (standard deviation) are discovered, and shown in Figure 4.

As can be seen, the result of finite size scaling of the effective permeability shows a good data collapse, from which we can determine both the master effective permeability curve, κ, (Figure 4(a)) and the master standard deviation of effective permeability, ξ (Figure 4(b)).

4. Comparison with real field data

To validate the percolation approach, we have used the North Pars field dataset. The North Pars gas field is located 85 km north of the giant South Pars gas field in the Persian Gulf. It was discovered in the 1960s. The main reservoir fluid is gas and condensate, and the estimated reserve is about 47 trillion cubic meters of gas. The main production of this field is supposed to

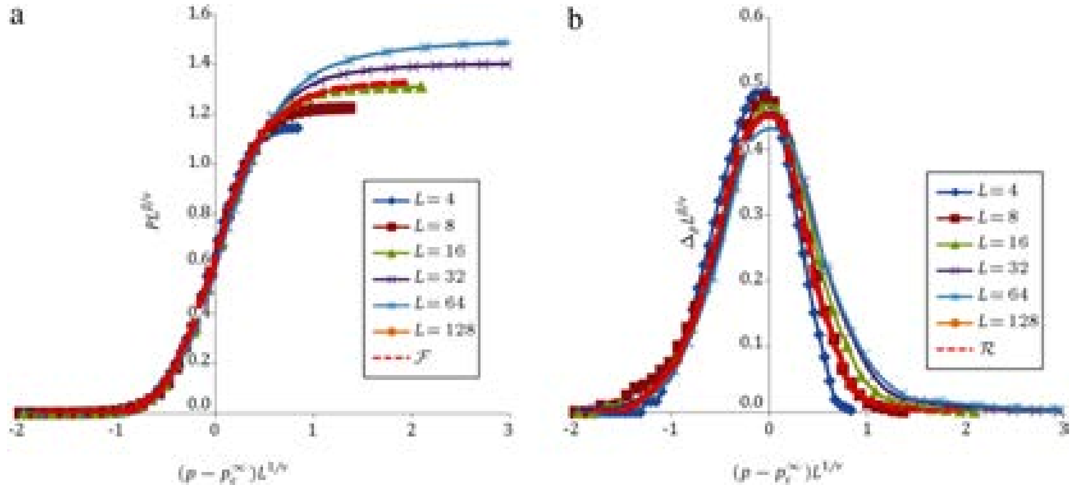

Figure 3: Illustration of data collapse of reservoir connectivity. (a) Master curve for mean reservoir connectivity; and (b) master curve for standard deviation of reservoir connectivity.

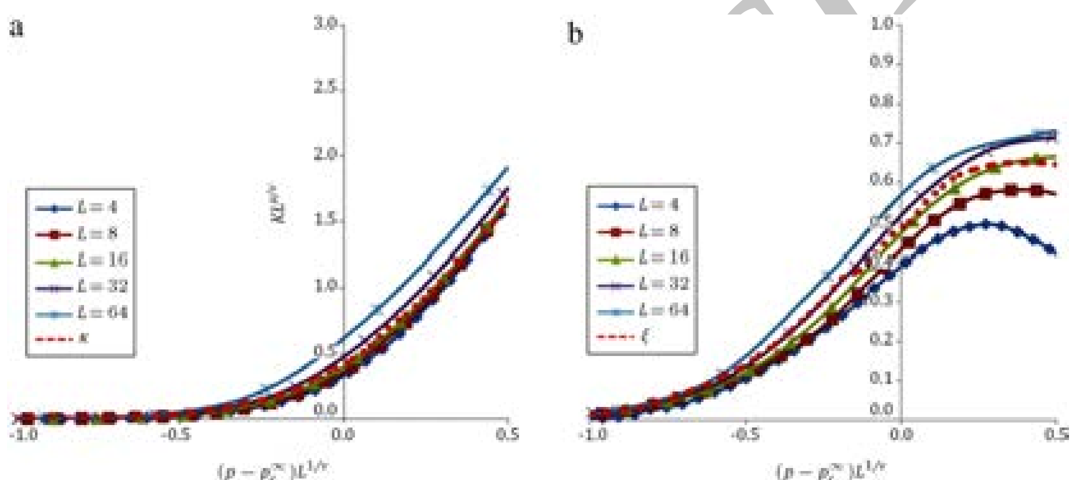

Figure 4: Illustration of data collapse of effective. (a) Master curve for mean effective permeability; and (b) master curve for standard deviation of effective permeability.

be used as a source for injection in onshore oil fields. The main lithology consists of limestone, dolomite, dolomitic limestone and anhydrite. The reservoir compartment is occurred due to the development of Nar member evaporates, which divide the Dehram group into two separate reservoirs. Because of its dome shaped anticline and a closure of 1700 m, it is known as a "C-Structure". North Pars covers an area of about 21 km × 17 km with relatively low dips (seldom exceeding 20°).

The aim of the validation section is to find both reservoir connectivity and effective permeability as a function of net-to-gross ratio, p, by using both the percolation method and the conventional modeling approach.

Conventional modeling uses data from different sources (e.g. geology, geophysics, petro physics, etc.) through a deterministic or stochastic framework to build a fine grid reservoir model. This fine grid reservoir model can then be used to evaluate the 2-D reservoir connectivity and permeability.

To evaluate the reservoir connectivity and effective permeability of the reservoir under study (as its 3-D permeability map is shown in Figure 5), two dimensional cross sections of the permeability map of the reservoir in different locations were extracted (see Figure 6(a) for a typical 2-D permeability map). The extracted cross sections have different sizes, in the range of 1600 up to 12,800 m. Then, based on the permeability distribution histogram for each cross section of the map, a permeability threshold is determined. The permeability of grids with values lower than this threshold is set to zero and values greater than this are set to one. This is for preserving the similarity of the problem to the percolation approach. The realization of such a transformation, from a full to a black-and-white permeability map, is shown in Figure 6(b). It is emphasized that we need this transformation, as the permeability calculations are performed on these black-and-white maps.

Then, for these cross section maps, we computed the net-to-gross ratio, p, and the reservoir connectivity, P, by applying standard methods [17]. Net-to-gross ratio, p, is simply calculated by summing up all occupied grids in the equivalent black-and-white permeability map of a cross section (e.g. Figure 6). After finding the spanning cluster, as a specific cluster that connects the two opposite boundaries of the given cross section, the connectivity, P, of this cross section is calculated as the ratio of occupied grids in the spanning cluster to the total number of occupied grids. For permeability computation of each cross section, we used an

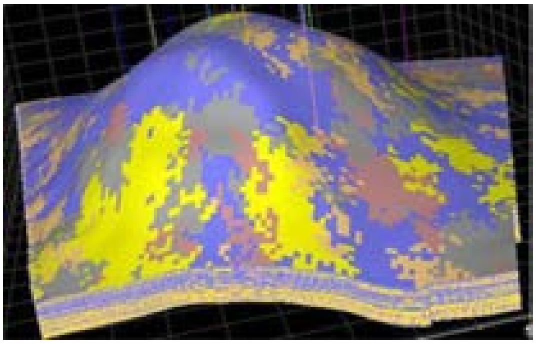

Figure 5: Geological model of North Pars gas field.

upscaling method, based on the renormalization theory [26]. The outcome of this part based on numerical results is a graph of scatter points of computed reservoir connectivity and effective permeability versus the net-to-gross ratio, p, as shown in Figure 7. Each point on this graph represents a specific cross section of the reservoir under study. To compare these numerical results of real field data with the predictions of the percolation theory, we study the reservoir connectivity and effective permeability of the facies type, with average sizes of 4500×800 m^2, in the X and Y directions, which show a reasonable match. Although the fit may not be perfect for a small amount of cross sections, the CPU time required for the percolation method is performed in fractions of seconds on a spreadsheet, whereas the conventional reservoir modeling approach is computationally very intensive. The numerical value of different percolation parameters for generating percolation curves is summarized in Table 1 [1,19].

Table 1: Value of parameters used in percolation calculations.

Percolation parameter	Value
p_c^{∞}	0.66
β	0.14
ν	1.33
μ	1.3

5. Conclusion and recommendations

The percolation theory is a powerful method that can be used for reservoir connectivity and effective permeability determination in geometrically complex systems, like porous media. We have briefly described and evaluated a theoretical method for reservoir connectivity and effective permeability prediction. Sand bodies were modeled by squares located in space, and the reservoir connectivity was modeled by a cluster of connected sand bodies. Also, the effective permeability of the reservoir was modeled by an upscaling method based on the renormalization theory. We have determined master curves

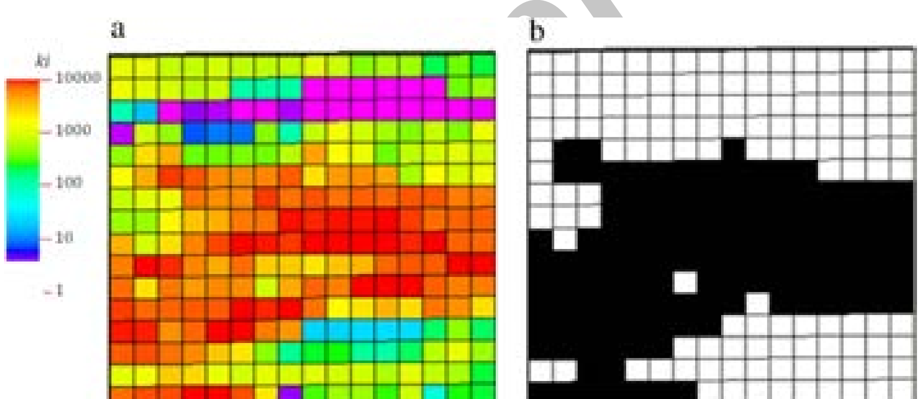

Figure 6: Typical cross section of permeability map of North Pars gas field (a) and a realization of black-and-white grids after applying the threshold value to the permeability map (b).

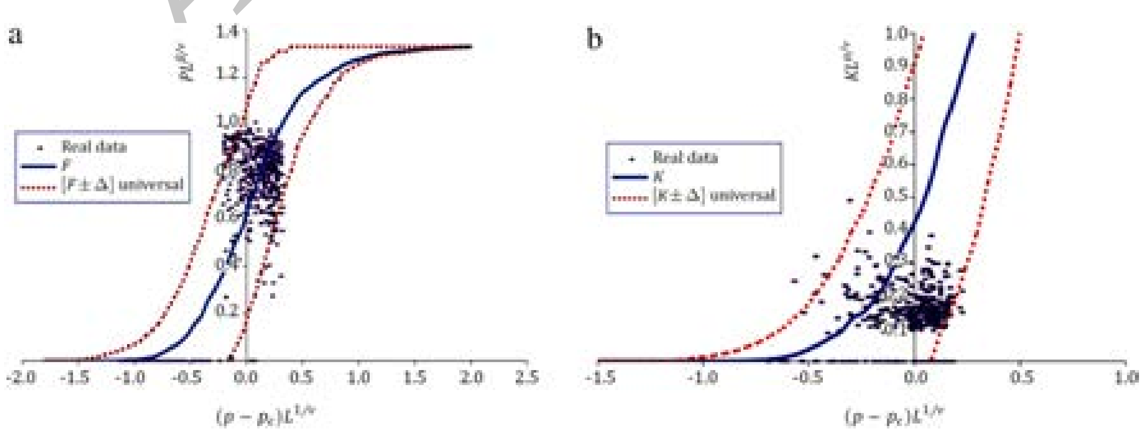

Figure 7: (a) Reservoir connectivity result from cross section maps of the North Pars field compared with percolation predictions of master reservoir connectivity. (b) Effective permeability result from cross section maps of the North Pars field compared with the percolation prediction of master effective permeability curves.

for reservoir connectivity, effective permeability and their associated uncertainties. Using master curves and simple algebraic manipulation, the connectivity and effective permeability of a given reservoir, and their related uncertainties, can be predicted in a very small amount of computational time, incomparable with the huge amount of human effort and CPU time necessary in the conventional reservoir modeling method. The results revealed promising and good matches.

This new approach can be extended in many ways, including 3-D analysis, various sand body sizes and orientations, and considering its spatial correlations.

Acknowledgments

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