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Research note

Maximum power and efficiency of an irreversible thermoelectric generator with a generalized heat transfer law

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Abstract An advanced model of irreversible thermoelectric generator with a generalized heat transfer law is established based on finite time thermodynamics. The generalized heat transfer law represents a class of heat transfer laws including Newtonian heat transfer law, linear phenomenological heat transfer law, radiative heat transfer law, Dulong-Petit heat transfer law, generalized convective heat transfer law and generalized radiative heat transfer law. The inner effects including Seebeck effect, Fourier effect, Joule effect and Thomson effect, and external heat transfer are taken into account in the model. The Euler–Lagrange functions at maximum power output and maximum efficiency are established. Applying the model to a practical example in engineering, it is found that the external heat transfer law does affect the characteristics and optimal performance of the thermoelectric device, and the maximum power output and maximum efficiency with Newtonian heat transfer law are the maximum among the several typical heat transfer laws. The results can offer principles for the power and efficiency optimization of practical thermoelectric generators at various external heat transfer conditions.

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1. Introduction

The recent worldwide energy consciousness has focused attention on unconventional means of producing electrical energy. The two of these – the photovoltaic process and the thermoelectric process – are the only processes that convert solar energy directly into electrical energy [1–4]. Thermoelectric generation technology, due to its several kinds of merits, especially its promising applications to waste heat recovery, is becoming a noticeable research direction [5–7]. Considerable efforts have been developed to identify new classes of thermoelectric materials. In addition to the improvement of thermoelectric materials, the system modeling and optimization of thermoelectric generators are equally important in improving the performance of thermoelectric generators [8–10].

So far, most of the researches on thermoelectric generators are mainly based on the non-equilibrium thermodynamics [11–14] and the finite time thermodynamics (FTT) [15–22]. Researches based on the non-equilibrium thermodynamics paid attention to the inner structure of thermoelectric elements. A significant increase in the power output from a module can be achieved by modifying the size of the thermoelectric elements [23]. Burshtein [24], Chen et al. [25], Omer and Infield [26], Moizhes et al. [27,28], Seifert et al. [29,30], Drabkin and Ershova [31] and other researchers investigated the power output or efficiency of single-element thermoelectric generator. Actually, a commercial thermoelectric generator is a multi-element device, which is composed of many fundamental thermoelectric elements. Rowe [32], Sisman and Yavuz [33], Mayergoyz and Andrel [34] investigated the power output and efficiency of a multi-element thermoelectric generator in various working conditions. However, the external heat transfer irreversibility was not considered in the literature hereinbefore.

Thermoelectric devices should be connected with heat exchangers to absorb and dissipate heat [1–10]. Much work has shown that the heat transfer irreversibility between an engine and its external reservoirs affects the thermodynamic processes obviously. The theory of finite time thermodynamics is a powerful tool for the performance analysis of thermodynamic processes. Some authors have investigated the performances

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Nomenclature

A	Area (m^2)
C	Coefficient
eJ	Electrical current density (A/m^2)
f	Ratio of thermal conductance allocation
I	Working electrical current (A)
K	Thermal conductance (WK^{-1})
k	Thermal conductivity ($Wm^{-1}K^{-1}$)
L	Length (m)
N	Number
P	Power output (W)
$\dot{Q}$	Heat flow rate (W)
R	Electrical resistance (Ω)
T	Temperature (K)

Greek letters

α	Seebeck coefficient (VK^{-1})
Δ	Difference
μ	Thomson coefficient (VK^{-2})
ρ	Electrical resistivity (Ωm)
σ	Electrical conductivity ($\Omega^{-1}m^{-1}$)

Subscripts

c	Cold junction
H	Heat source
h	Hot junction
L	Heat sink
n	N-type semiconductor leg
p	P-type semiconductor leg
P	At maximum power
T	Total
η	At maximum efficiency

of thermoelectric generators using a combination of finite time thermodynamics and non-equilibrium thermodynamics. Gordon [35,36], Yan and Chen [37], Agrawal and Menon [38], and Chen and Wu [39] analyzed the performance of single-element thermoelectric generators with finite rate heat transfer between the thermoelectric device and its external heat reservoirs. Further, Chen et al. [40], Yu and Zhao [41], Hsiao et al. [42], Astrain et al. [43], and Meng et al. [44] investigated the characteristics of multi-element thermoelectric generation system with external heat transfer.

In the analyses mentioned above, the heat transfer between the thermoelectric generator and the heat reservoirs was assumed to obey Newtonian (linear) heat transfer law $\dot{Q} \propto \Delta T$. Much work has shown that the heat transfer law between the device and its external reservoirs affects the performances of thermodynamic cycles strongly. However, when the external heat reservoirs obey nonlinear heat transfer laws, the analytical results are hard to obtain. Chen et al. [45] and Meng et al. [46] established a finite time thermodynamic model of a multi-element thermoelectric generator with linear phenomenological law and radiative law and, consequently, investigated the characteristics of the device.

Reviewing the former literature concerning thermoelectric generators, some features can be concluded as follows:

(1) Researches concerning the inner structure of a thermoelectric generator are without considering the external heat transfer irreversibility.
(2) Researches concerning the external heat transfer optimization of a thermoelectric generator are without considering the temperature dependence of thermoelectric properties, i.e. the Thomson effect of thermoelectric materials.
(3) Most of the researches assumed that the external heat transfer obeys Newtonian heat transfer law but not concerned nonlinear heat transfer law.

The heat transfer law represents the characteristic and regularity of the transfer. The laws would be complex and the well-known Newtonian heat transfer law is not the only law in practical heat transfer process. There are many other nonlinear heat transfer laws and the heat transfer law does affect the performance of the device. Many researchers have concerned the characteristics of conventional heat engines, refrigerators and heat pumps with linear phenomenological heat transfer law $\dot{Q} \propto \Delta T^{-1}$ [47,48], radiative heat transfer law $\dot{Q} \propto \Delta T^4$ [49,50], Dulong–Petit heat transfer law $\dot{Q} \propto (\Delta T)^{1.25}$ [51,52] generalized convective heat transfer law $\dot{Q} \propto (\Delta T)^m$ [53,54] and generalized radiative heat transfer law $\dot{Q} \propto \Delta(T^n)$ [55–57]. To establish a general, natural expression of various heat transfer laws, a generalized heat transfer law $\dot{Q} \propto (\Delta T^n)^m$, which represents a class of heat transfer laws including the heat transfer laws mentioned above, was put forward by Chen et al. [58] and extended by Li et al. [59–61]. The effects of heat transfer laws on the performance characteristics of the thermodynamic systems were investigated.

However, there is a lack of a model of thermoelectric generator, which can be applied to the analysis and optimization of a multi-element thermoelectric generator with generalized external heat transfer law, and can be applied to analyze the effects of heat transfer laws on the maximum power output, maximum efficiency and the optimal variables. This paper aims to establish an advanced model of an irreversible multi-element thermoelectric generator with generalized external heat transfer law $\dot{Q} \propto (\Delta T^n)^m$ by introducing the theory and method used in literature mentioned above. The Fourier heat leakage loss, Joule heat loss, temperature dependence of thermoelectric properties, especially, the Thomson heat loss are taken into account in the model. The maximum power output and maximum efficiency are explored. A practical example in engineering is proposed to analyze the effects of several typical heat transfer laws on the optimal variables, i.e. working electrical current and ratio of thermal conductance allocation of the heat exchangers.

2. Thermodynamic model and solution

2.1. An advanced finite time thermodynamic model

Figure 1 shows the advanced finite time thermodynamic model of an irreversible thermoelectric generator with external generalized heat transfer law established in this paper. The thermoelectric generator consists of P-type and N-type semiconductor legs. The number of thermoelectric elements is N. The junctions of the thermoelectric elements are fixed at a thermal conducting and electrical insulting ceramic plate. The thermoelectric elements are insulated, both electrically and thermally, from its surroundings. The internal irreversibility is caused by Joule loss and Fourier heat conduction loss through the semiconductor between the hot and cold junctions. The Joule loss generates an internal heat I^2R, where R is the total internal electrical resistance of the thermoelectric element and I is the electrical current generating from the thermoelectric elements. The conduction heat loss is $K(T_h - T_c)$, where K is the thermal conductance of the thermoelectric

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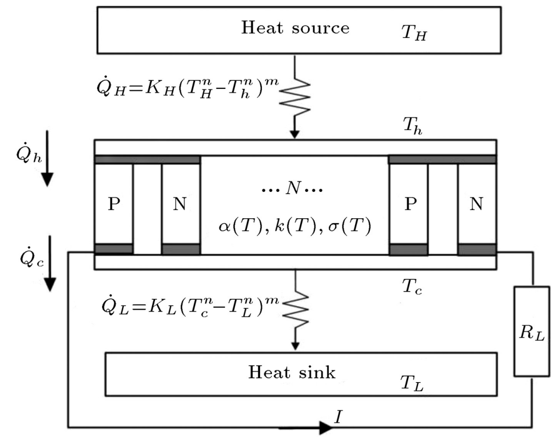

Figure 1: Finite time thermodynamic model of an irreversible thermoelectric generator with generalized heat transfer law.

element, T_h is the hot junction temperature and T_c is the cold junction temperature. The heat flow rates through the hot and cold junctions of the thermoelectric elements are $\dot{Q}_h$ and $\dot{Q}_c$, respectively.

The external irreversibility is caused by the finite rate heat transfer between the thermoelectric generator and its heat reservoirs. The temperatures of the heat source and heat sink are T_H and T_L, respectively. The thermal conductance of the heat exchangers are K_H and K_L, respectively. The total thermal conductance $K_T = K_H + K_L$ of the external heat transfer of the thermoelectric device is finite and fixed. The heat flow rate absorbed from the heat source to the thermoelectric generator is $\dot{Q}_H$. The heat flow rate dissipated from the thermoelectric generator to the heat sink is $\dot{Q}_L$.

As a commercial thermoelectric generation module is thermal insulation packaged, the heat leakage through the lateral face can be neglected. At a steady work state, the temperature distribution of the air gap is the same with the thermoelectric elements, so the heat transfer of the whole device can be treated as one dimensional heat transfer approximately. The increment rate of inner energy of an infinitesimal is zero at steady-state, so one can obtain the energy conservation equation of a semiconductor leg as follows:

$$\dot{Q}_{\mathrm{Kin}} - \dot{Q}_{\mathrm{Kout}} + \dot{Q}_\mu + \dot{Q}_J = 0, \tag{1}$$

where $\dot{Q}_{\mathrm{Kin}}, \dot{Q}_{\mathrm{Kout}}, \dot{Q}_\mu$ and $\dot{Q}_J$ are the Fourier heat input (heat input to the hot junction of the element by heat conduction), Fourier heat output (heat output from the cold junction of the element by heat conduction), generated Thomson heat (generated heat because of Thomson effect) and generated Joule heat (generated heat when electrical current flows through the conductor). Eq. (1) can be expressed as:

$$\frac{d}{dx}(T_p + dT_p)k_pA_p - \frac{dT_p}{dx}k_pA_p + \mu_p eJ_pA_p dT_p + (eJ_p)^2\frac{A_pL_p}{\sigma_p}\frac{dx}{L_p} = 0, \tag{2}$$

$$\frac{d}{dx}(T_n + dT_n)k_nA_n - \frac{dT_n}{dx}k_nA_n + \mu_n eJ_nA_n dT_n + (eJ_n)^2\frac{A_nL_n}{\sigma_n}\frac{dx}{L_n} = 0, \tag{3}$$

where $k_p, \sigma_p, \mu_p, A_p, L_p, T_p, eJ_p$ and $k_n, \sigma_n, \mu_n, A_n, L_n, T_n, eJ_n$ are the thermal conductivity, electrical conductivity, Thomson coefficient, cross section area, length, temperature and electrical current density of the P-type and N-type semiconductor leg. Reforming Eqs. (2) and (3) one can obtain the heat conduction differential equations of P- and N-type semiconductor legs (Eqs. (4) and (5)) with boundary conditions (Eqs. (6) and (7)) as follows [12]:

$$k_pA_p\frac{dT_p^2}{dx^2} + \mu_p I\frac{dT_p}{dx} + \frac{I^2}{\sigma_pA_p} = 0, \tag{4}$$

$$k_nA_n\frac{dT_n^2}{dx^2} - \mu_n I\frac{dT_n}{dx} + \frac{I^2}{\sigma_nA_n} = 0, \tag{5}$$

$$T_p(0) = T_n(0) = T_c, \tag{6}$$

$$T_p(L_p) = T_n(L_n) = T_h, \tag{7}$$

where $k_p, \sigma_p, \mu_p, A_p, L_p, T_p$ and $k_n, \sigma_n, \mu_n, A_n, L_n, T_n$ are the thermal conductivity, electrical conductivity, Thomson coefficient, cross section area, length and temperature of the P-type and N-type semiconductor leg. $I = eJ_pA_p = eJ_nA_n$ is the electrical current. In Eqs. (4) and (5), the first term (second derivative) is caused by thermal conduction of P- and N-type semiconductor legs; the second term (first derivative) is caused by Thomson effect; the third term is caused by Joule effect. The Seebeck effect is the surface effect, so it is not embodied in the equations but it causes the electrical current through the elements.

Taking effect of temperature dependence of thermoelectric properties into account, k_p, σ_p and μ_p are function of T_p; while k_n, σ_n and μ_n are function of T_n. However, such a differential equation cannot be solved analytically in general. Replacing k, σ and μ with the averaged coefficients $\overline{k}$, $\overline{\sigma}$ and $\overline{\mu}$ approximately [62] gives approximations of Eqs. (4) and (5) as follows:

$$\overline{k_p}A_p\frac{dT_p^2}{dx^2} + \overline{\mu_p}I\frac{dT_p}{dx} + \frac{I^2}{\overline{\sigma_p}A_p} = 0, \tag{8}$$

$$\overline{k_n}A_n\frac{dT_n^2}{dx^2} - \overline{\mu_n}I\frac{dT_n}{dx} + \frac{I^2}{\overline{\sigma_n}A_n} = 0, \tag{9}$$

where $\overline{k_p} \approx k_p|_{T=(T_h+T_c)/2}$, $\overline{\sigma_p} \approx \sigma_p|_{T=(T_h+T_c)/2}$, and $\overline{\mu_p} \approx \mu_p|_{T=(T_h+T_c)/2}$ for P-type semiconductor legs and $\overline{k_n} = k_n|_{T=(T_h+T_c)/2}$, $\overline{\sigma_n} = \sigma_n|_{T=(T_h+T_c)/2}$, and $\overline{\mu_n} = \mu_n|_{T=(T_h+T_c)/2}$ for N-type semiconductor legs. The total heat flow rates through the hot and cold junctions are:

$$\dot{Q}_h = N\left((\alpha_{ph} - \alpha_{nh})T_hI + \overline{k_p}A_p\left.\frac{dT_p}{dx}\right|_{x=L_p} + \overline{k_n}A_n\left.\frac{dT_n}{dx}\right|_{x=L_n}\right), \tag{10}$$

$$\dot{Q}_c = N\left((\alpha_{pc} - \alpha_{nc})T_cI + \overline{k_p}A_p\left.\frac{dT_p}{dx}\right|_{x=0} + \overline{k_n}A_n\left.\frac{dT_n}{dx}\right|_{x=0}\right), \tag{11}$$

where α_p and α_n are the Seebeck coefficients of the P- and N-type semiconductor legs, and the subscript h and c represent the hot and cold sides. In Eqs. (10) and (11), the first term is caused by Seebeck effect, and the second and the third terms are caused by thermal conduction of P- and N-type semiconductor legs. The Joule and Thomson effects are volume effect, so they are not embodied in the equations but they affect the temperature distribution of the elements.

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It is assumed that the heat transfer between the junctions of the thermoelectric elements and their respective reservoirs obeys a generalized heat transfer law $\dot{Q} \propto (\Delta T^n)^m$ (including Newtonian heat transfer law $\dot{Q} \propto \Delta T$, linear phenomenological heat transfer law $\dot{Q} \propto \Delta T^{-1}$, radiative heat transfer law $\dot{Q} \propto \Delta T^4$, Dulong–Petit heat transfer law $\dot{Q} \propto (\Delta T)^{1.25}$, generalized convective heat transfer law $\dot{Q} \propto (\Delta T)^m$ and generalized radiative heat transfer law $\dot{Q} \propto \Delta T^n$), as did the finite time thermodynamics for various thermodynamic processes and devices. According to the theory of finite time thermodynamics, one has:

$$\dot{Q}_H = K_H(T_H^n - T_h^n)^m \tag{12}$$

$$\dot{Q}_L = K_L(T_c^n - T_L^n)^m. \tag{13}$$

Eqs. (12) and (13), include various heat transfer laws. Specially:

(1) If $n = 1$, the law is generalized convective heat transfer law [53,54]. Further,
 (a) If $m = 1$, the law is Newtonian heat transfer law.
 (b) If $m = 1.25$, the law is Dulong–Petit heat transfer law [51,52].

(2) If $m = 1$, the law is generalized radiative heat transfer law [55–57]. Further,
 (a) If $n = -1$, the law is linear phenomenological heat transfer law [47,48].
 (b) If $n = 4$, the law is radiative heat transfer law [49,50].

The energy-balance equations, i.e. the control equations, are:

$$K_H(T_H^n - T_h^n)^m = N\left((\alpha_{ph} - \alpha_{nh})T_h I + \overline{k_p}A_p \left.\frac{dT_p}{dx}\right|_{x=L_p} + \overline{k_n}A_n \left.\frac{dT_n}{dx}\right|_{x=L_n}\right), \tag{14}$$

$$K_L(T_c^n - T_L^n)^m = N\left((\alpha_{pc} - \alpha_{nc})T_c I + \overline{k_p}A_p \left.\frac{dT_p}{dx}\right|_{x=0} + \overline{k_n}A_n \left.\frac{dT_n}{dx}\right|_{x=0}\right). \tag{15}$$

2.2. Maximum power output and maximum efficiency

The temperature distributions of the P- and N-type semiconductor legs can be solved by Eqs. (8) and (9) as follows:

$$T_p(x) = T_c - F_p x + \frac{T_h - T_c + F_p L_p}{\left(e^{-\omega_p L_p} - 1\right)}(e^{-\omega_p x} - 1), \tag{16}$$

$$T_n(x) = T_c + F_n x + \frac{T_h - T_c - F_n L_n}{\left(e^{\omega_n L_n} - 1\right)}(e^{\omega_n x} - 1), \tag{17}$$

where $\omega_p = \overline{\mu_p}I/(\overline{k_p}A_p)$, $F_p = I/(\overline{\sigma_p \mu_p}A_p)$, $\omega_n = \overline{\mu_n}I/(\overline{k_n}A_n)$ and $F_n = I/(\overline{\sigma_n \mu_n}A_n)$. T_h and T_c are not fixed because they can change if the current changes.

For maximum figure of merit of the thermoelectric element $Z = \alpha^2/(KR)$, the size of the thermoelectric element and the physical property of the material should satisfy the following equation:

$$\frac{A_p^2 L_n^2}{A_n^2 L_p^2} = \frac{k_n \sigma_n}{k_p \sigma_p}. \tag{18}$$

To reduce the cost of manufacture, the P- and N-type semiconductor legs are made with same sizes, i.e. $A_p = A_n = A$ and $L_p = L_n = L$. So one has $k_n\sigma_n/(k_p\sigma_p) = 1$ by Eq. (18). Similar doped alloys are adopted to make P- and N-type semiconductor legs. That is $\sigma_p = \sigma_n = \sigma$, $k_p = k_n = k$, $\alpha_p = -\alpha_n$, and $\mu_p = -\mu_n$. This choice is only due to the simplification of the calculations. According to Taylor's formula, when $|x| \ll 1$, $e^x \approx 1 + x$ holds true. Based on above assumptions, Eqs. (10) and (11) can be approximated and simplified by a polynomial expression of current Ias follows at the case of $\overline{\mu}I/K \ll 1$

$$\dot{Q}_h = N[\alpha_h I T_h + K(T_h - T_c) - 0.5I^2R - 0.5\overline{\mu}I(T_h - T_c)], \tag{19}$$

$$\dot{Q}_c = N[\alpha_c I T_c + K(T_h - T_c) + 0.5I^2R + 0.5\overline{\mu}I(T_h - T_c)] \tag{20}$$

where $\alpha_h = \alpha_{hp} - \alpha_{hn}$ and $\alpha_c = \alpha_{cp} - \alpha_{cn}$ are the Seebeck coefficients of the thermoelectric elements at hot and cold sides. $\overline{\mu} = \overline{\mu_p} - \overline{\mu_n} = 2\overline{\mu_p}$, $K = K_p + K_n$ and $R = R_p + R_n$ are the total Thomson coefficient, thermal conductance and electrical resistance of a thermoelectric element. αIT, $K\Delta T$, I^2R and $\overline{\mu}I\Delta T$ are the rates of Peltier heat, Fourier heat, Joule heat and Thomson heat, respectively.

K and R are given by:

$$K = K_p + K_n = \overline{k_p}A_p/L_p + \overline{k_n}A_n/L_n = 2\overline{k}A/L, \tag{21}$$

$$R = R_p + R_n = L_p/(\overline{\sigma_p}A_p) + L_n/(\overline{\sigma_n}A_n) = 2L/(\overline{\sigma}A). \tag{22}$$

The power output is given by:

$$P = K_H(T_H^n - T_h^n)^m - K_L(T_c^n - T_L^n)^m. \tag{23}$$

The efficiency $\eta = P/\dot{Q}_H$ is given by:

$$\eta = 1 - \frac{K_L(T_c^n - T_L^n)^m}{K_H(T_H^n - T_h^n)^m}. \tag{24}$$

The power output of a thermoelectric generator depends on the working electrical current I. For a given thermoelectric generator, there exists an optimal electrical current corresponding to the maximum power output, which has been proved by much literature. Thus, the optimization of electrical current is a basic problem of the internal optimization of a thermoelectric generator.

More external thermal conductance means better heat exchange but bulkier structures and higher cost. When the external heat transfer is fixed, the allocation of thermal conductance of heat exchangers would affect the performance of the device. Therefore, it is an important problem that how to allocate fixed total external thermal conductance among hot and cold sides of the device for maximum power output or maximum efficiency. The optimum allocation of thermal conductance for conventional power and refrigeration plants was first advanced by Bejan [16,63,64] and advanced by many researchers. To describe the allocation, a ratio of thermal conductance allocation is introduced as $f = K_H/(K_H + K_L)$. Then one has $K_H = fK_T$ and $K_L = (1 - f)K_T$.

Much work has shown that when the external heat transfer law is linear i.e. $\dot{Q} \propto \Delta T$, there will exist optimal electrical currents and optimal ratios of thermal conductance allocation corresponding to the maximum power output and maximum efficiency, respectively. This research will prove that when the external heat transfer law is nonlinear i.e. $\dot{Q} \propto (\Delta T^n)^m$ ($n \neq 1$ or $m \neq 1$), there will also exist optimal electrical currents and optimal ratios of thermal conductance allocation.

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The Euler–Lagrange function for maximum power output can be established by combining Eq. (23) with Eqs. (14) and (15) as follows:

$$\begin{aligned} L^P = {} & K_T f(T_H^n - T_h^n)^m - K_T(1-f)(T_c^n - T_L^n)^m \\ & + \lambda_P(K_T f(T_H^n - T_h^n)^m - N(\alpha_h IT_h + K(T_h - T_c) \\ & - 0.5\, I^2R - 0.5\,\overline{\mu} I(T_h - T_c))) \\ & + \phi_P(K_T(1-f)(T_c^n - T_L^n)^m - N(\alpha_c IT_c \\ & + K(T_h - T_c) + 0.5\, I^2R + 0.5\,\overline{\mu} I(T_h - T_c))), \end{aligned} \tag{25}$$

where λ_P and ϕ_P are the Lagrangian multipliers, which are constants to be determined. The optimal ratio of thermal conductance allocation, f_P, optimal electrical current, I_P, and the corresponding hot junction temperature, T_h, and cold junction temperature, T_c, should satisfy the Euler–Lagrange equations:

$$\frac{\partial L^P}{\partial T_h} = 0, \qquad \frac{\partial L^P}{\partial T_c} = 0, \qquad \frac{\partial L^P}{\partial f} = 0, \qquad \frac{\partial L^P}{\partial I} = 0. \tag{26}$$

Eq. (26) can be calculated as:

$$\begin{aligned} & (1+\lambda_P)C_h + \lambda_P N\,(\alpha_h I + K - 0.5\overline{\mu} I) \\ & \quad - \phi_P N\,(K + 0.5\,\overline{\mu} I) = 0, \end{aligned} \tag{27}$$

$$\begin{aligned} & (1+\phi_P)C_c + \lambda_P N\,(-K + 0.5\,\overline{\mu} I) \\ & \quad - \phi_P N\,(\alpha_c I - K - 0.5\,\overline{\mu} I) = 0, \end{aligned} \tag{28}$$

$$(1+\lambda_P)\,\left(T_H^n - T_h^n\right)^m + (1+\phi_P)\,\left(T_c^n - T_L^n\right)^m = 0, \tag{29}$$

$$\begin{aligned} & \lambda_P\,\left[T_h\,\alpha_h - IR - 0.5\,\overline{\mu}\,(T_h - T_c)\right] \\ & \quad - \phi_P\,\left[T_c\,\alpha_c + IR + 0.5\,\overline{\mu}\,(T_h - T_c)\right] = 0, \end{aligned} \tag{30}$$

where:

$$C_h = K_T fmnT_h^{n-1}\left(T_H^n - T_h^n\right)^{m-1}, \tag{31}$$

$$C_c = K_T\,(1-f)\,mnT_c^{n-1}\left(T_c^n - T_L^n\right)^{m-1}. \tag{32}$$

The Euler–Lagrange function for maximum efficiency can be established by combining Eq. (24) with Eqs. (14) and (15) as follows:

$$\begin{aligned} L^\eta = {} & 1 - (1-f)f^{-1}\left(T_c^n - T_L^n\right)^m \left(T_H^n - T_h^n\right)^{-m} \\ & + \lambda_\eta(K_T f(T_H^n - T_h^n)^m - N(\alpha_h IT_h + K(T_h - T_c) \\ & - 0.5\, I^2R - 0.5\overline{\mu} I(T_h - T_c))) \\ & + \phi_\eta(K_T(1-f)(T_c^n - T_L^n)^m - N(\alpha_c IT_c \\ & + K(T_h - T_c) + 0.5I^2R + 0.5\,\overline{\mu} I(T_h - T_c))), \end{aligned} \tag{33}$$

where λ_η and ϕ_η are the Lagrangian multipliers, which are constants to be determined. The optimal ratio of thermal conductance allocation, f_η, optimal electrical current, I_η, and the corresponding hot junction temperature, T_h, and cold junction temperature, T_c, should satisfy the Euler–Lagrange equations:

$$\frac{\partial L^\eta}{\partial T_h} = 0, \qquad \frac{\partial L^\eta}{\partial T_c} = 0, \qquad \frac{\partial L^\eta}{\partial f} = 0, \qquad \frac{\partial L^\eta}{\partial I} = 0. \tag{34}$$

Eq. (34) can be calculated as:

$$\begin{aligned} & (1-f)f^{-1}D_c mnT_h^{n-1}D_h^{-1}\left(T_H^n - T_h^n\right)^{-1} \\ & \quad - \lambda_\eta(-K_T fmnT_h^{n-1}D_h\left(T_H^n - T_h^n\right)^{-1} \\ & \quad - N(\alpha_h I + K - 0.5\overline{\mu} I)) - \phi_\eta N(K + 0.5\overline{\mu} I) = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} & (1-f)f^{-1}mnT_c^{n-1}D_cD_h^{-1}\left(T_c^n - T_L^n\right)^{-1} \\ & \quad + \lambda_\eta N(-K + 0.5\overline{\mu} I) - \phi_\eta(N(\alpha_c I - K - 0.5\overline{\mu} I) \\ & \quad - K_T(1-f)mnT_c^{n-1}D_c(T_c^n - T_L^n)^{-1}) = 0, \end{aligned} \tag{36}$$

$$f^{-2}D_cD_h^{-1} + \lambda_\eta K_T D_h + \phi_\eta K_T D_c = 0, \tag{37}$$

$$\begin{aligned} & -\lambda_\eta N\,(\alpha_h\, T_h - IR - 0.5\,\overline{\mu}\,(T_h - T_c)) \\ & \quad + \phi_\eta N\,(\alpha_c\, T_c + IR + 0.5\,\overline{\mu}\,(T_h - T_c)) = 0, \end{aligned} \tag{38}$$

where:

$$D_h = \left(T_H^n - T_h^n\right)^m, \tag{39}$$

$$D_c = \left(T_c^n - T_L^n\right)^m. \tag{40}$$

It is important to note that not only the external heat transfer, but also the temperature dependence of thermoelectric properties is considered in this paper. It is not the full temperature dependence which could only be determined numerically, but it comes indirectly into play due to averages. These averages are constants and the equations are solved by adapting constant material properties. For given heat source and heat sink temperatures, T_H and T_L, the junction temperatures of thermoelectric element, T_h and T_c, are unknown, thus the thermoelectric properties, α_h, α_c, $\overline{k}$, $\overline{\sigma}$ and $\overline{\mu}$, are unknown. Alteration method is adopted herein to determine the junction temperatures. For given initial values of T_h and T_c ($T_h = T_H$, $T_c = T_L$, for example), α_h, α_c, $\overline{k}$, $\overline{\sigma}$ and $\overline{\mu}$ can be calculated by the fitting equation of the thermoelectric material (Eqs. (41)–(43) hereinafter). Then T_h and T_c can be calculated. The alteration process will be repeated until required precision is obtained.

3. Numerical example

A practical thermoelectric generator in engineering is adopted herein to analyze the effects of heat transfer laws on the maximum power output and maximum efficiency. In the numerical analysis and optimization, the number and size of the thermoelectric elements are set as $N = 127$, $A = 1 \times 1$ mm^2, and $L = 2$ mm [65]. The heat reservoir temperatures are set as $T_H = 450$ K and $T_L = 300$ K. The physical properties of the commercially available material (Bi_2Te_3) by Melcor are shown as follows [65]:

$$\alpha_p = (22\,224.0 + 930.6T - 0.9905T^2)10^{-9}\ \mathrm{VK^{-1}}, \tag{41}$$

$$\rho = (5112.0 + 163.4T + 0.6279T^2)10^{-10}\ \Omega\mathrm{m}, \tag{42}$$

$$k = (62\,605.0 - 277.7T + 0.4131T^2)10^{-4}\ \mathrm{Wm^{-1}\,K^{-1}}, \tag{43}$$

where α_p, ρ and k are the Seebeck coefficient, electrical resistivity and thermal conductivity. The Thomson coefficient is given by the second Kelvin relationship [1–5]

$$\mu = T\frac{d\alpha}{dT}. \tag{44}$$

Figures 2–4 show the effect of heat transfer law on temperature difference ($\Delta T = T_h - T_c$) between the hot and cold junctions, power output P and efficiency η versus working electrical current I, respectively. Figure 5 shows the effect of heat transfer law on power output P versus efficiency η. In the calculations, the ratio of thermal conductance allocation is set as $f = 0.5$. It can be seen that the heat transfer law does affect the performance of the irreversible thermoelectric generator. According to the order of temperature difference, they are ranked as follows: Special complex heat transfer law, Dulong–Petit heat transfer law, radiative heat transfer law, linear phenomenological heat transfer law, and Newtonian heat transfer. The effects of heat transfer law on the characteristic of power output and efficiency versus working electrical current

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Table 1: Optimization results of the optimal variables and the corresponding optimal performance with different heat transfer law.

Transfer law	f_P	I_P	$P_{\max}$ (W)	η_P (10^{-2})	f_η	I_η	$\eta_{\max}$	P_η (W)
Newtonian ($n = 1$, $m = 1$)	0.5268	0.6014	2.9122	0.0656	0.5287	0.5111	0.0670	2.8465
Linear phenomenological ($n = -1$, $m = 1$)	0.6201	0.5996	2.8982	0.0655	0.6221	0.5099	0.0669	2.8333
Radiative ($n = 4$, $m = 1$)	0.3873	0.5967	2.8769	0.0653	0.3478	0.5061	0.0667	2.8092
Dulong–Petit ($n = 1$, $m = 1.25$)	0.5277	0.5730	2.6827	0.0639	0.5313	0.4886	0.0652	2.6246
Special complex ($n = 4$, $m = 1.25$)	0.3837	0.5651	2.6222	0.0634	0.3857	0.4818	0.0647	2.5653

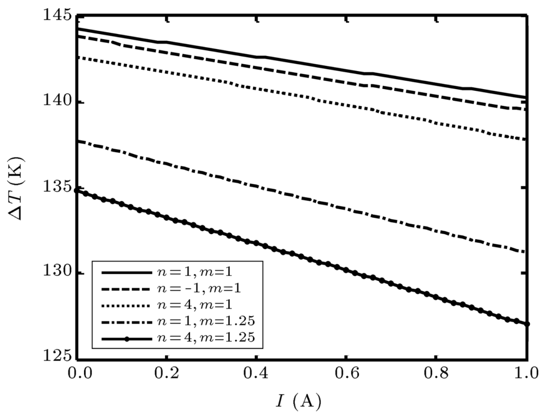

Figure 2: Effect of heat transfer law on temperature difference versus working electrical current.

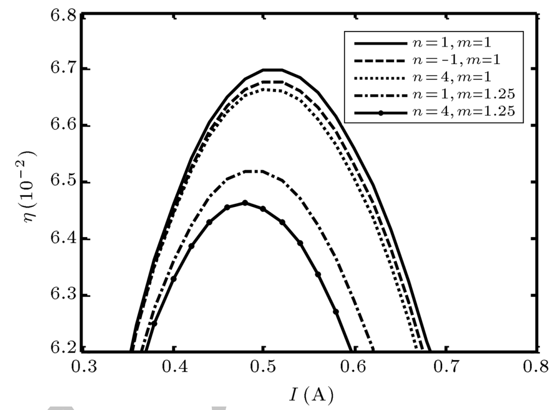

Figure 4: Effect of heat transfer law on efficiency versus working electrical current.

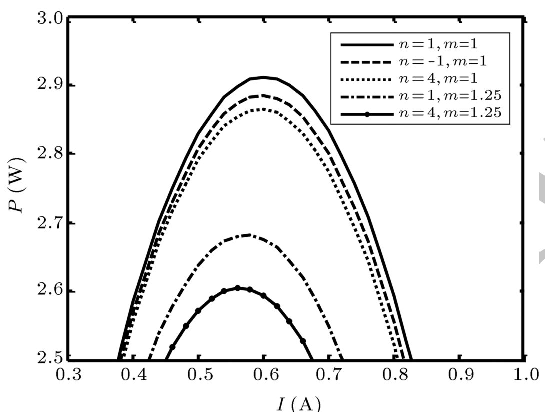

Figure 3: Effect of heat transfer law on power output versus working electrical current.

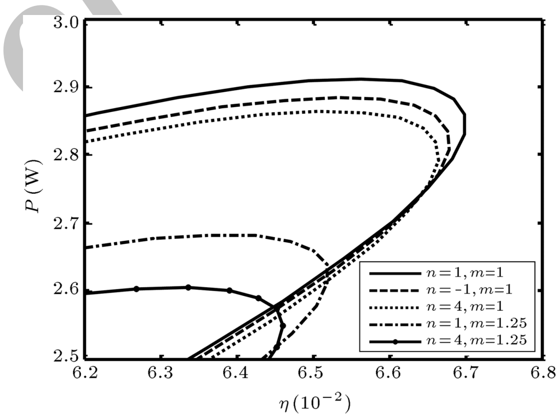

Figure 5: Effect of heat transfer law on power output versus efficiency.

curves are analogous with the effect of heat transfer law on the characteristic of temperature difference versus working electrical current. Among the several heat transfer laws, the effect of Dulong–Petit heat transfer and special complex heat transfer law are different from other heat transfer laws strikingly.

The optimization results of the optimal variables and the corresponding optimal performance with different heat transfer laws are listed in Table 1. Figure 6 shows the optimal working electrical current I_P and optimal ratio f_P of thermal conductance allocation corresponding to the maximum power output $P_{\max}$ with different heat transfer laws. Figure 7 shows the maximum power output $P_{\max}$ and the corresponding efficiency η_P with different heat transfer laws. From the table and figures, the changing features of optimal variables and optimal performance can be concluded. According to the order of the optimal ratio f_P of thermal conductance allocation, they are ranked as follows: special complex heat transfer law, radiative heat transfer law, Newtonian heat transfer law, Dulong–Petit heat transfer, and linear phenomenological heat transfer law. According to the order of the optimal electrical current I_P, they are ranked as follows: special complex heat transfer law, Dulong–Petit heat transfer, radiative heat transfer law, linear phenomenological heat transfer law and Newtonian heat transfer law. According to the order of the maximum power output $P_{\max}$, they are ranked as follows: special complex heat transfer law, Dulong–Petit heat transfer, radiative heat transfer law, linear phenomenological heat transfer law and Newtonian heat transfer law. The order of η_P is same to the order of $P_{\max}$.

Figure 8 shows the optimal working electrical current I_η and optimal ratio f_η of thermal conductance allocation corresponding to the maximum efficiency $\eta_{\max}$ with different heat transfer laws. Figure 9 shows the maximum efficiency $\eta_{\max}$ and the corresponding power output P_η with different heat transfer laws. According to the order of the optimal ratio of thermal conductance allocation f_η, they are ranked as follows:

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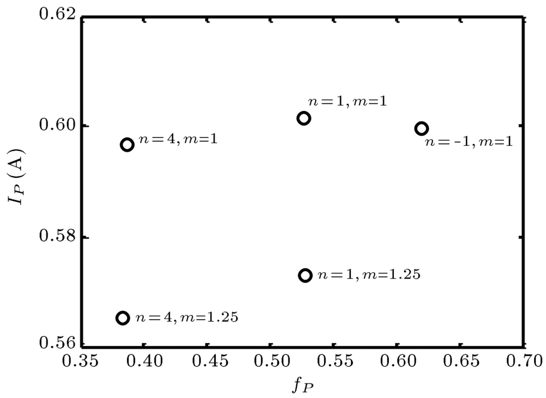

Figure 6: Optimal working electrical current and optimal ratio of thermal conductance allocation.

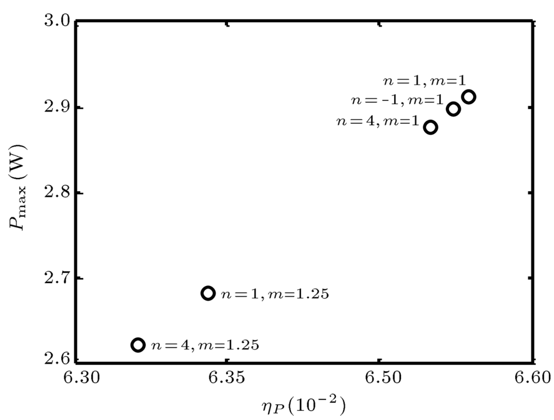

Figure 7: Maximum power output and the corresponding efficiency with different heat transfer law.

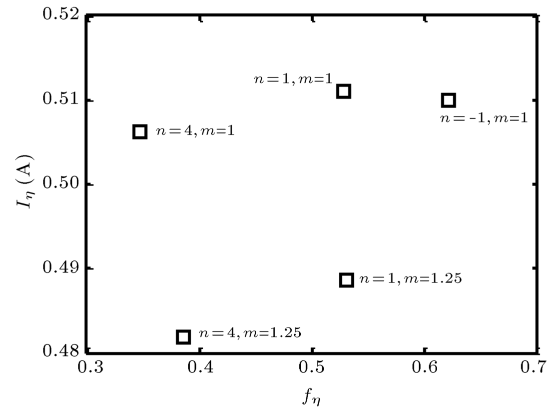

Figure 8: Optimal working electrical current and optimal ratio of thermal conductance allocation corresponding to the maximum efficiency with different heat transfer law.

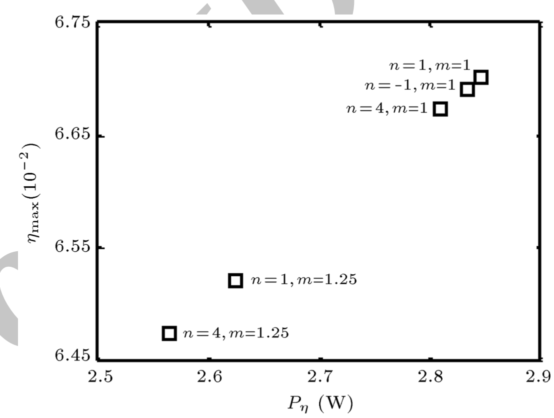

Figure 9: Maximum efficiency and the corresponding power output with different heat transfer law.

radiative heat transfer law, special complex heat transfer law, Newtonian heat transfer law, Dulong–Petit heat transfer, and linear phenomenological heat transfer law. According to the order of the optimal electrical current I_η, they are ranked as follows: special complex heat transfer law, Dulong–Petit heat transfer, radiative heat transfer law, linear phenomenological heat transfer law and Newtonian heat transfer law. According to the order of the maximum efficiency $\eta_{\max}$, they are ranked as follows: special complex heat transfer law, Dulong–Petit heat transfer, radiative heat transfer law, linear phenomenological heat transfer law and Newtonian heat transfer law. The order of P_η is the same as the order of $\eta_{\max}$.

Since each heat transfer process has a corresponding mechanism, one could not simply make a comparison just considering the effect of temperature (the power of temperature or temperature difference). For example, the K_H and K_L in the paper represent heat transfer coefficient. To convective heat transfer process, K_H and K_L represent convective heat transfer coefficient; to radiation process, K_H and K_L represent a coefficient (involves Stefan–Boltzmann constant) whose value is much smaller than the former. In the analysis mentioned above, the effect of heat transfer coefficient has not been taken into account. Some results only hold for the numerical values adopted herein.

4. Conclusions

Because the incomprehensiveness of conventional model of irreversible thermoelectric generator, an advanced model of irreversible thermoelectric generator with generalized heat transfer law $\dot{Q} \propto \Delta(T^n)^m$ is established. A class of heat transfer laws is included and multi-irreversibilities are considered in the model. Applying the model to a practical example in engineering, it is found that the external heat transfer law does affect the characteristics of the thermoelectric device. It is proved that there are optimal working electrical currents and optimal ratio of thermal conductance allocations corresponding to the maximum power output and maximum efficiency when the heat transfer law is nonlinear. The changing features of the maximum power, maximum efficiency and the corresponding design variables are analyzed in detail.

As the heat exchangers for thermoelectric device are various, the heat transfer laws are various and different from each other. One can determine the exponents n and m in $\dot{Q} \propto \Delta(T^n)^m$ by experiment or from empirical formula. Combining the heat transfer law and the results obtained herein can offer principles for the power and efficiency optimizations of practical thermoelectric generators at various external heat transfer conditions. The model and optimization method may be applied to the analysis and design of practical thermoelectric generators.

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