Effects of laser pulse characteristics on the form of wakefield generated by the high intensity ultrashort laser pulse-plasma interaction.

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Abstract

Wakefield generation by femtosecond laser pulse in its interaction with ionized gas is described. The amplitude of the wakefield A_p is studied as a function of laser pulse and background gas parameters and is compared with results of preliminary ionized plasma A_{pi}. It is found that ionization process can increase A_p compared to A_{pi}. The effect of polarization of laser beam on the wakefield amplitude is studied and linear and circular polarizations are compared. Differences between the effects of square and Gaussian beam shapes in the intensity of A_p are investigated. The strongest enhancement A_p in comparison with A_{pi} takes place for longer laser pulse with a duration in excess of plasma wave period when the resonant conditions of pondermotive excitation of the wakefield are not matched.

Keywords: plasma wake field, plasma based accelerators, Langmuir wave, polarization direction, pondermotive force.

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Introduction:

It has been about 25 years since Tajima and Dawson proposed using laser beam to excite plasma waves. After that, due to advances in technology, especially the development of compact terawatt laser systems, there have been tremendous progresses in theory and experiments. There have been many different kinds of experiments on the use of intense laser pulse to excite large amplitude (up to 100 GeV/m) plasma wakes [(1-3)]. Generation of intense Langmuir wave in plasmas has been opened the new possibilities in elaboration of the compact ultrahigh gradient plasma based accelerators of charged particles. The energy of these wakes can be used for exciting the radiation as well [(4-7)]. As can be seen the theory of a wake plasma wave generation by intense laser pulse in plasmas was worked out intensively during the past decade and has checked out experimentally. At the present time, however, in the current and planned experiments on laser plasma acceleration it is assumed that the laser pulse will produce a plasma while propagating in a neutral gas. The theory of laser pulse propagation and plasma wakefield generation in an ionizing gas should include the adequate description of ionization kinetics in the intense laser field and the influence of the ionization processes to the plasma waves excitation in a laser produced plasma [(8)].

In the present paper we have analyzed systematically the efficiency of wakefield generation depending on the gas and laser pulse parameters. A special attention has been concentrated on the dependence of wakefield amplitude on the polarization of the laser pulse in the conditions when ionization processes play an important role in the wakefield generation. It is shown that the increase of the wakefield amplitude due to the gas ionization is more pronounced for more circularly polarized laser pulses as compared to the linear polarization of the laser. It is found also that ionization front (at the leading front of the laser pulse) provides the detectable wakefield amplitude, determined by the threshold for ionization laser intensity, independent of the relation between the laser pulse duration and plasma wave period.

The paper is organized as follows; after this short introduction, in section 2 the basic equations that describe the slowly varying amplitude of the laser field and the plasma wave electric field in the presence of tunneling optical field ionization processes is presented and on the basis of laser field energy conservation the conditions of the smallness of the laser energy losses due to gas ionization is discussed. One- dimensional (1-D) analytical solutions for the wakefield generated by a given laser pulse in an ionizing gas are obtained and analyzed. Conclusion is presented in section 3.

Discussion and results

A. Basic equations

Further the wakefield generation in terms of slowly varying amplitudes approximation is described which implies that the laser pulse duration is in excess of a few optical cycles. The validity of such approximation is also a consequence of the fact that the ionization of comparatively light atoms occurs at nonrelativistic intensities. Beside, here we restrict our consideration to 1-D approximation of the problem, which is justified if a wide laser pulse propagates over the distance smaller than its Rayleigh length, or diffraction effects are suppressed.

For transversal laser field $\mathbf{E_L}$ the only first harmonic of laser frequency ω_0 is taken into account:

$$E_L = \frac{1}{2}(E_x e_x + E_y e_y)\exp[-i\omega_0(t - z/c)] + c.c. \qquad (1)$$

Longitudinal plasma electric field $\mathbf{E_z}$ is described up to the second order with respect to the zero and second harmonics of ω_0:

$$E_z(z,t) = E_p(z,t) + \frac{1}{2}(E_{p2}(z,t)\exp[-2i\omega_0(t - z/c)] + c.c.) \qquad (2)$$

The source of free electrons Γ (total ionization rate) consists of ionization rates of ions with any ionization degree k from 0 (neutral) to z-1 (H-like atomic rest) (8):

$$\Gamma = \sum_{k=0}^{z-1}\Gamma_k \qquad (3)$$

It is also expanded into the laser frequency harmonics series:

$$\Gamma(z,t) = \Gamma_0(z,t) + \frac{1}{2}(\Gamma_2(z,t)\exp[-2i\omega_0(t - z/c)] + c.c. + ... \qquad (4)$$

Harmonics of Γ originate from the strong nonlinear dependence of optic field tunneling ionization probability of any ion on the laser field $\mathbf{E_L}$.

With the help of hydrodynamic equations obtained in (8) from kinetic description of plasma electrons in the presence of optical field ionization processes we derive the equation for slowly varying amplitude of transversal laser electric field:

$$2\omega_0\frac{\partial a}{\partial\tau} - 2ic\frac{\partial^2 a}{\partial\xi\partial\tau} + i\left(\omega_0^2\frac{n_e}{n_c} - \omega_p^2\frac{|a|^2}{4}\right)a = -2\omega_0\frac{a}{|a|^2}\sum_{k=0}^{z-1}\frac{\Gamma_0^k}{n_c}\frac{U_k}{mc^2} + \frac{\Gamma_2\omega_0}{4n_c}a^* \qquad (5)$$

and for slowly varying plasma wave electric field:

$$\frac{\partial^2 a_p}{\partial^2\xi} + k_p^2 a_p = \frac{k_p^2}{4}\left(\frac{c}{\Omega_p}\frac{\partial|a|^2}{\partial\xi} - 2|a|^2\frac{\Gamma_0}{n_0\Omega_p} + 2\,\mathrm{Re}\left[\left(a^*\right)^2\frac{\Gamma_2}{4n_0\Omega_p}\right]\right) \qquad (6)$$

in which $a=(E_xe_x+E_ye_y)/m\omega_. c$, $a_p=eE_z/m\Omega_p c$, $\omega_p^2=4\pi e^2 n_0/m$,

$$n_0 = (-\int_{\infty}^{\xi}\Gamma_0(\xi)d\xi)/c\,,$$

$$\Omega_p^2 = (4\pi e^2\int_{-\infty}^{+\infty}\Gamma_0(\xi)d\xi/mc\,,$$

$k_p=\omega_p/c$, $n_c=m\omega_0^2/4\pi e^2$ is the electron critical density, m(e) is the electron mass (charge), ξ=z-ct and τ=t$ are coordinates in the comoving frame.

The electron density perturbation n_e-n_0 driven by the laser pulse can be expressed (in 1-d approximation which is valid for laser spot size, and so far transverse scales of the wake, in excess of plasma wavelength $2\pi/k_p$) through the longitudinal field of the wake as follows: $(n_e - n_0)/n_e = (\Omega_p c/\omega_0^2)(\partial a_p/\partial\xi)$. It might be well to point out that in the present paper equation 6 is derived on the bases of kinetic description of the gas ionization. This equation completed by equation 5 for the laser pulse envelope a and equations for n_0. Γ_0 and Γ_2 describes the wakefield generation in the presence of optical field ionization self consistently, in contrast to phenomenological approaches (9-10).

From Eq. 5 one can derived the following energy conservation relation

$$\frac{\partial}{\partial\tau}\int_{-\infty}^{\infty} d\xi \int d^2\mathbf{r}_\perp |\mathbf{a}|^2 = -\frac{c}{2}\int_{-\infty}^{\infty} d\xi \int d^2\mathbf{r}_\perp \frac{n_e-n_0}{n_e}\frac{\partial|\mathbf{a}|^2}{\partial\xi} +$$

$$\frac{c}{2}\int_{-\infty}^{\infty} d\xi \int d^2\mathbf{r}_\perp \left\{ |\mathbf{a}|^2 \frac{\partial}{\partial\xi}\left(\frac{n_0}{n_c}\right) + \mathbf{Re}\left(\frac{\Gamma_2}{2cn_c}(\mathbf{a}^*)^2\right)\right\} - 2\sum_{k=0}^{z-1}\frac{U_k}{mc^2 n_c}\int_{-\infty}^{\infty} d\xi \int d^2\mathbf{r}_\perp \Gamma_0^{<k>} \qquad (7)$$

where integration is carried out over longitudinal coordinate ξ as well as cross section transversal to the laser pulse propagation direction r. The first term in Eq. 7 describes the laser pulse energy losses due to plasma wave generation, the second term originates from the losses to residual electron energy (11) and the third one is responsible for ionization energy losses which are proportional to the respective potentials (12). In addition, the second term in the right hand side of Eq. 5 changes the polarization of the laser pulse if the latter one was not linearly polarized at the entrance of the gas. Relative influence of this term on the laser polarization is of the same order as the relative changes in the laser pulse energy originated from the residual electron energy losses. It should be noted that Eq. 7 is valid also in the general case of three dimensional laser pulse propagation.

As it can be seen the nonadiabatic laser pulse energy losses to residual electron energy and ionization are determined mostly by the gas density, laser pulse path in the gas and ionization potentials. For example 10 cm laser pulse path in hydrogen with atomic density $N_{at}=10^{18}$ cm^{-3} at Gaussian laser pulse intensity of $I_0=5\times 10^{17}$ W/cm^2, wavelength λ=800 nm and pulse duration of 100 fs leads to about 1% laser pulse energy depletion because of the mentioned losses.

Here the energy losses to wakefield generation of comparatively low electron densities and nonrelativistic laser intensities are neglected. Neglecting these losses enable us to use stationary laser pulse propagation approximation. We represent electric field in elliptically polarized laser pulse in the following form:

$$\mathbf{E}_l = \frac{1}{2}\frac{E_l(\xi)}{\left(1+\eta^2\right)^{1/2}}\left(e_x + i\eta\,\mathbf{e}_y\right)\mathbf{exp}\left[i\omega_0\,\xi/c + i\phi\right] \qquad (8)$$

where η is the degree of ellipticity and η=0 (=1) corresponds to the linear (circular) polarization of the laser field. By a pertinent choice of a phase ϕ we make an amplitude E_l a real function of ξ which relates with laser irradiation intensity as $I_l=(c/8\pi)\, E_l^2$.

It should be noted that the source Γ in general case of an arbitrary ellipticity η contains an infinite number of laser frequency harmonics, which decrease with their order because of above mentioned strong nonlinear dependence of Γ on a modulus of the oscillating laser electric field. The only case without any harmonics occurs at η

(circular polarization). Ionization rate is a function of ellipticity η, however Eq. 6 contains the zero and second harmonics of Γ only as a result of expansion up to the second order of $al=eE_l/m\omega_0 c$. Because of strong localization of laser pulse temporal profile, ionization can be treated as a threshold process with corresponding threshold intensity I_{th}. η strongly influences the relation between harmonics Γ_0 and Γ_2 of ionization rate. Further it is convenient to express Γ_2 through the value $\mu=\Gamma_2/(2\Gamma_0)$.

B. Solution

General solution to Eq. 6 behind the laser pulse has a form of harmonics oscillations:

$$a_p\left(\xi\right)=A_p\,\mathbf{cos}\left(k_p\,\xi+\psi\right) \tag{9}$$

where $k_p=\Omega_p/c$ is the wave vector and ψ is the phase of the generated plasma wave. Further with the help of Eq. 6 we are going to obtain simple analytic formulas to analyze an influence of laser field ellipticity and pulse duration on the wakefield amplitude A_p. The simple analytical solution to this equation can be easy obtained, if one takes into consideration the fact, that the ionization rate Γ is strongly localized in time with duration τ_{ion} which is much less than both of the laser pulse duration τ_{imp} and the plasma wave period $2\pi/\Omega_p$. In view of this fact we approximate the profile of an electron density $n_0(\xi)$ in the form

$$n_0(\xi)=N_{at}\sum_{k=1}^{z_{\max}}\theta(\xi_k-\xi)\,,$$

where ξ_k is the coordinate of the k_{th} ionization front, and z_{max} is the integer number equal to the largest possible number of ionized electron levels at given laser radiation parameters. In the general case of multielectron atoms (when z_{max} is large), the solution has a comparatively complicated form; therefore we reproduce it for hydrogen (z_{max}=1) only:

$$a_p=-\frac{1}{2}a_l^2\left(\xi_1\right)G_\Gamma\left(\xi_1\right)\mathbf{sin}\left[k_p\left(\xi_k-\xi\right)\right]+\frac{1}{4}\int_{\xi_1}^{\xi}a_l^2\left(\xi\right)\mathbf{cos}\left[k_p\left(\xi_l-\xi\right)\right]k_p\,d\xi+\frac{B}{4}a_l^2\left(\xi_1\right)\,\mathbf{sin}\left[k_p\,\left(\xi_1-\xi\right)\right] \tag{10}$$

in this equation

$$G_\Gamma\left(\xi\right)\equiv 1-\frac{1-\eta^2}{1+\eta^2}\,\frac{\mu\left(\xi\right)}{2}$$

and $a_l=eE_l/m\omega_0 c$ is the point of a maximum of ionization rate Γ_0 and $G_\Gamma(\xi)$ is the "ionization" source of the wakefield; the second and the third terms in a right hand side of Eq.10 represent the "pondermotive" source of the wake field. Coefficient B=0 if the laser pulse propagates through the preionized plasma (with $\xi_l=\infty$ and $a_l(\xi_l)=0$);

B=1 if the gas is ionized by the laser pulse with smooth forward front and its spatial scale L_f considerably exceeds the ionization front width $L_{ion}=c\tau_{ion}$.

Eq.10 is convenient for analytical exploration of the influence of ionization processes at the forward front of a laser pulse on the wakefield generation in the ionized gas. Let's first consider the approximation of nearly rectangular pulse with envelop $E_l(\xi)=E_0$ for $|\xi|<L_{imp}/2$ assuming its duration L_{imp} to be much larger the leading front ($L_{imp}>> L_f$; but $L_f>> L_{ion}$). For the rectangular pulse propagating through the homogeneous preionized plasma the second term in the Eq.10 at $\xi<-L_{imp}/2$ yields a well known result $a_p=-(1/2)a_0^2\cos(k_p\xi)\sin(k_pL_{imp}\}/2)$, where $a_0=eE_0/(m\omega_0c)$. In this case the wakefield amplitude is maximum at $k_pL_{imp}=\pi(1+2n)$, where n is integer, and is equal to $a_0^2/2$. In the case when a rectangular laser pulse ionizes gas during its propagation, Eq.10 gives the following value of the wake field at $\xi<-L_{imp}/2$:

$$a_p(\xi)=-\frac{a_0^2}{2}u\,\mathbf{cos}\left(k_p\xi+\psi\right) \tag{11}$$

Where $u=[u_1^2-u_1\cos(k_pL_{imp})+1/4]^{1/2}$, $u_1=(a_1/a_0)^2(G_\Gamma-1/2)+1/2$ and $\psi=\cos^{-1}[(u_1+1/2)\sin(k_pL_{imp}/2)/c]$ in which $a_1=a_l(\xi=\xi_1)$. According to Eq.11 the wakefield amplitude A_p is maximum under the same condition $k_pL_{imp}=\pi(1+2n)$ and equal to:

$$A_{p,\mathbf{max}}=\left(\frac{a_0^2}{2}\right)\left[1+\left(\frac{a_1}{a_0}\right)^2\left(G_\Gamma-\frac{1}{2}\right)\right] \tag{12}$$

The phase $\psi=0$ in this case.

It follows from Eq.10 for $\mu(\xi)$ that G_Γ increases with η: G_Γ ~0.6 for linear polarization ($\eta=0$); $G_\Gamma=1$ for circular polarization ($\eta=1$) of the laser pulse. Besides, the highest magnitude of the relative value A_p/a_0^2 will be achieved at laser pulse peak intensity I_0 close to ionization threshold intensity I_{th}. At such conditions $a_1\sim a_0$. In this case Eq. 12 leads to $(A_p/a_0^2)_{max}=1/4+G_\Gamma/2$ which is ~0.55 for linear and ~0.75 for circular polarization. Thus, the maximum relative wakefield amplitude exceeds $(A_p/a_02)_{max}$ for preliminary totally ionized plasma by 10% for linear polarization and by 50% for circular polarization of laser radiation at $I_0=I_{th}$. The minimum relative wake field amplitude is attained at $k_pL_{imp}=2\pi n$, n=1,2,... and according to Eq.11 equals to $(A_p/a_0^2)_{min}=(a_1^2/a_0^2)(G_\Gamma\}/2+1/4)-1/2$. This value is not equal to zero in contrast to the case of preionized plasma, when $A_{p,min}=0$. Notice also, that according to Eq. 12, the contribution of ponderovotive force into the wakefield generation in the case $a_1\sim a_0$ is a half of that in the case of preionized gas.

At this point lets consider the Gaussian laser pulse with envelop $a_l(\xi)=a_0\exp[-(\xi^2/\sigma_\xi)^2)]$ in which $\sigma_\xi=L_{imp}/(2\ln2)^{1/2}$ and L_{imp} is the full width at half of maximum intensity. In this case Eq.10 can be transformed to

$$a_p(\xi)=-\frac{a_0^2}{2}e^{-2(\xi_1/\sigma_\xi)^2}\left(G_\Gamma-\frac{1}{2}\right)\mathbf{sin}\left[k_p(\xi_1-\xi)\right]-$$

$$\left\{\frac{a_0^2}{8}G_p\,e^{ik_p\xi}\left[erf\left(\frac{-\sqrt{2}\xi}{\sigma_\xi}-i\frac{k_p\sigma_\xi}{2\sqrt{2}}\right)+erf\left(\frac{\sqrt{2}\xi_1}{\sigma_\xi}+i\frac{k_p\sigma_\xi}{2\sqrt{2}}\right)\right]\right\} \qquad (13)$$

In this equation

$$G_p \equiv \frac{1}{2}\sqrt{\frac{\pi}{2}}\,k_p\,\sigma_\xi\,e^{-k_p^2\sigma_\xi^2/8}$$

Let's consider a limiting case of the obtained formula. If $I_0 >> I_{th}$, then the ionization occurs at the laser pulse leading front far from its peak intensity. In this case the laser pulse width is about the plasma wave length (so near-resonant region), so as $k_p\sigma_\xi \sim 2$ and the inequality $k_p\sigma_\xi < 4\xi_1/\sigma_\xi$ holds. Setting $\xi_1 \rightarrow \infty$ for this case and omitting first two terms in Eq.13, we obtain the same wakefield, as in the case of preliminary completely ionized plasma:

$$a_p(\xi) = -\frac{a_0^2}{4}G_p\left[\mathbf{cos}\left(k_p\xi\right)+\frac{1}{2}e^{ik_p\xi}\,erf\left(\frac{-\sqrt{2}\xi}{\sigma_\xi}-i\frac{k_p\sigma_\xi}{2\sqrt{2}}\right)\right] \qquad (14)$$

which amplitude A_p at $\xi << -\sigma_\xi$ is equal to $A_p = 0.5a_0^2G_p$. This amplitude reaches it's maximum at $k_p\sigma_\xi = 2$.

Conclusion

Wakefield generation by femtosecond laser pulse is described in the frame of slowly varying amplitudes approximation. Amplitude of the wakefield A_p is studied as a function of laser pulse and background gas parameters. It is found that ionization processes increase A_p in comparison with A_{pi} at relatively high laser peak intensities. Besides the enhancement of A_p due to gas ionization is more pronounced for circularly polarized laser pulses than for linear polarization of the laser radiation. This difference originates from the partial compensation of "ionization" source by higher harmonics of electron production rate for linear polarization, that reflects the fact that electron density increase due to ionization in the linear polarized laser pulse takes place mainly at the moments when the laser field is maximum and so the velocity of free electron oscillations is closer to zero (in contrast to the circular polarization when ionization rate does not depend on the phase of the laser pulse).

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