Potential of grains in two dimensional hexagonal dusty plasma crystals

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Abstract

The massive, negatively charged dust grains are considered as discrete particles, while the electrons and ions assume to be distributed by Boltzmannean with the same temperatures. Assuming the grains to be conductors and charge of grain depend on the surface potential of the grain. The Poisson equation for small potentials takes then the form of the Helmholtz equation. The spatial distribution of the potential in the lattice includes the effect of whole system of dust particles. Such a self consistent description gives the dispersion relation for the dust lattice waves. It is shown that for the existence of ideal lattice the dusty plasma parameters must satisfy the definite relation. The calculations are carried out for two dimensional hexagonal lattice putting the cyclic boundary condition on dust grains. New relation for potential is found and there is a comparison with Yukawa system.

Key word: dusty plasma, hexagonal lattice, Yukawa potential.

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Introduction

Lattices with a reduced dimensionality are an interesting class of soft condenses matter. These lattices consist of particles which arrange themselves in a crystalline structure in the presence of external and inter particles forces. Typical examples of two-dimensional (2D) systems are colloidal suspensions(1), electrons in liquid helium(2), and Longmuir monolayer(3). A number of interesting physical processes have been studied in these lattices ,e.g., solid liquid phase transition, phonon propagation, and sublimation. Typical examples of 1D systems are quantum wires(4). Another way of preparing a lattice with reduced dimensionality is to use a plasma crystal, in which micro-size charged particles interact with each other via a Yokawa or screened-coulomb potential.

Several investigation(5-8) have dealt with the lattice oscillations in the dust plasma crystal. In Ref. (8) most consistent construction of the dust-lattice wave (DLW) theory has been presented. In the mentioned paper the interaction only between nearest dust-grain is taken into account. Furthermore, the charge of dust-grain is assumed being fixed. Under this assumption the main Eq. (8) of Ref. (8) has obtained the form of an inhomogeneous equation, that restrict the application of methods, known from the solid physics, for the description of oscillatory phenomenon in dust lattice.

In this paper we investigate potential of grains in hexagonal lattices on the basis of Kroning-Penny model (9). The charge of dust grain is connected with its surface potential, which depends on the potential in plasma (10). We assume grains being spherical conductors with the radius a_{nm}, thus the grain charge Q_{nm} is proportional to its potential. Such a dependence of grain charge on its potential charge the type of main equation, obtained from the Poisson equation. For small value of potential it becomes the form of the Helmholtz equation for potential. The calculations are carried out for the two dimensional lattice putting the cyclic boundary condition on the chain of dust grains. The spatial distribution of potential along the lattice chain is found.

Dusty plasma model

A two dimensional dusty plasma in which extremely massive, negatively charged dust grains are considered as discrete particles, while the electrons and ions assume to be distributed by Bultzmannean with the same temperatures, $T_e = T_i = T$.

$$n_e = n_0 \exp[e\varphi / k_B T], \tag{1}$$

$$n_i = n_0 \exp[-e\varphi / k_B T], \tag{2}$$

where n_0 is the number density of electrons and ions at $\varphi = 0$, and e is the magnitude of the electron charge. The potential should be satisfies Poisson's equation

$$\nabla^2 \phi - 4\pi e\left[n_e(\phi) - n_i(\phi)\right] = -\sum_n q_n(r,\theta,\phi), \tag{3}$$

where on the right-hand side of eq. (3), summation over the dust grains is carried out. The charge density, corresponding to the single spherical grains of radius a, can be represented as $q_n(\mathbf{r},\phi) = \frac{Q_n}{4\pi a_n{}^2} \int dS(\mathbf{n})\delta(\mathbf{r} - (\mathbf{r_n} + a_n\mathbf{n}))$, where S_n is the surface area of the grain, $\mathbf{r}_n$ the radius-vector of the grain's center and $\mathbf{n}$ is unit vector, perpendicular to the surface element. And Q_n denotes the total charge of the grain. Assuming the grains to be conductors, we have $Q_{nm} = a_{nm}\phi_{nm}$, where ϕ_{nm} is the potential on grain's surface. When the grain's size is smaller than the grains separation distance, $a_{nm} << d$, the grains position (x_n, y_m) can be described by $\delta -$ function, as: $q(r,\phi) = \sum Q_n \delta(r - r_n) = \sum a_n \phi(r) \delta(r - r_n)$. Below we consider the two –dimensional case only. This approximation looks rather artificial, however its results can give some notion for the more general, three-dimensional case. The equation (3) can be simplify to

$$\nabla^2 \Phi - k_D^2 \sinh(\Phi) = -\sum_n a_n \delta(\mathbf{r} - \mathbf{r}_n) \Phi(\mathbf{r}) , \quad (4)$$

where $\Phi = e\phi / k_B T$ (5)

and k_D is the inverse of Debye length λ_D,

$$k_D = 1/\lambda_D = (8\pi e^2 n_0 / k_B T)^{1/2} \quad (6)$$

For equal charge and size of grains, and in the linear limit $(\Phi << 1)$ we obtain

$$\nabla^2 \Phi - k_D^2 \Phi = -a\Phi_0 \sum_n \delta(\mathbf{r} - \mathbf{r}_n) \quad (7)$$

Equation (7) is often used in the solid state physics for the description of the lattice waves in the one dimensional approximation. This approximation looks rather artificial, although its results can give some notion for the more general three dimensional case.

In two dimensional hexagonal lattice (figure 1), we will obtain potential from boundary value problem.

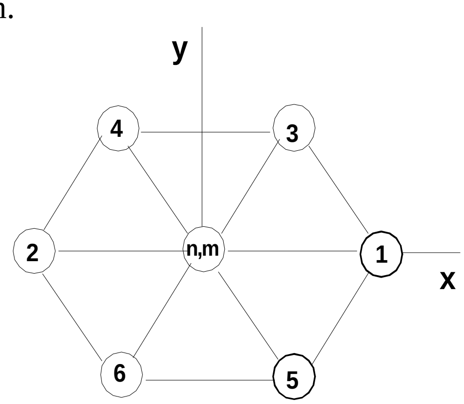

Figure 1: hexagonal lattice.

$$\Phi(r,\theta) = A_0 I_0 (k_D r) + B_0 K_0 (k_D r) + \sum_{m=1} (A_m I_m (k_D r) + B_m K_m (k_D r)) \cos(m\theta)$$
$$+ \sum_{m=1} (C_m I_m (k_D r) + D_m K_m (k_D r)) \sin(m\theta) \quad (8)$$

where I_m, K_m are modified Bessel functions, respectively. Due to condition $\Phi(\theta) = \Phi(-\theta)$, all coefficients C_m and D_m should be set equal to zero. The other boundary conditions are:

$$r = a \Rightarrow \Phi(r,\theta) = \Phi_0 \quad (9)$$

$$\Phi(\theta) = \Phi(\theta + \frac{l\pi}{3}), \quad l = 1,2,3,4,5 \quad (10)$$

$$r = d, \theta = 0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3 \Rightarrow \Phi = \Phi_0 \quad (11)$$

$$\Phi \big|_{r=d/2, \theta = l\pi/3} = \Phi \big|_{r=\sqrt{3}d/2, \theta = l\pi/6}, \quad l = 1,2,3,4,5 \quad (12)$$

On the surface of grain, the potential is Φ_0 and independent of angle θ, so close to grain only $m = 0$, with $A_0 = 0$ and $B_0 = \Phi_0 / K_0(k_D a)$ satisfies all conditions. Conditions (10), (11) and (12) come from symmetry[11]. One can obtain coefficients (A_m, B_m) by using all conditions.

By using of boundary conditions, and determination of all coefficients, the complete form of the potential function of grains to be obtained. So have plotted the potential function versus r, for a definite value of θ, and Yukawa potential simultaneously for comparison, in figure 2.

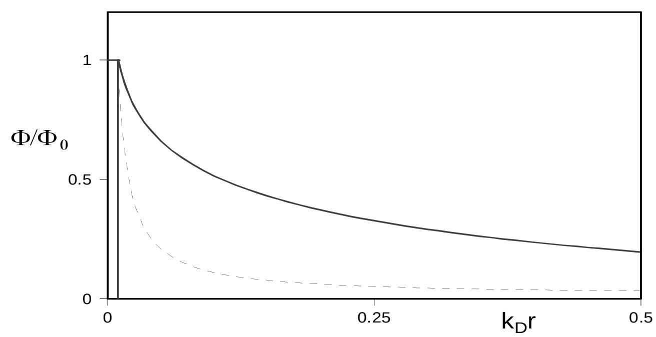

Figure 2: Comparison of grain's potential (solid line) with Yukawa potential (dashed line).

Figure (2) shows $\Phi(r, \theta = 0)$ as a function of normalized $k_D r$ up to $m = 12$. The potential is compared with Yukawa potential form.
The potential function of grains have plotted in two dimensional coordinate as a function of r, θ in figure 3.

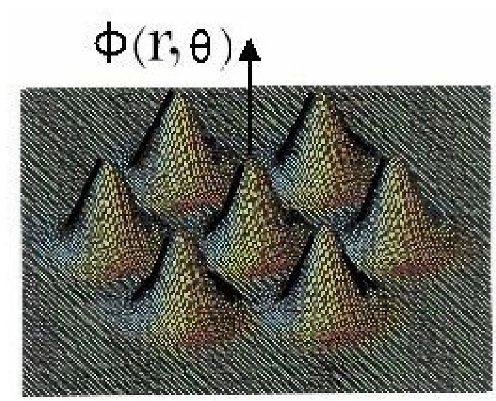

Figure 3: The potential of grains as a function of r, θ.

Dispersion relation for DLW

Let us assume the dust particles to execute small oscillations around their equilibrium position. The grain are assumed to have the same potential (and charge) and a uniform separation distance. We consider the case, when the dust particles maintain their equilibrium potential (and charge) during oscillations. It means we assume the oscillation frequency to be larger than the characteristic frequency of dust particles charging. By using the potential (8), we can write the expression of total electrostatic energy of interaction for the dust grain's system, which yields:

$$U = \sum_n Q_n \phi_n(\vec{r}_n)$$
$$= \frac{Q_o^2}{16\pi\varepsilon_o K_O(k_D a)} \sum_n \{K_O(k_D r_n) + B_6 \times \left[K_6(k_D r_n) - \frac{K_6(k_D a)}{I_6(k_D a)} I_6(k_D r_n)\right] \quad (13)$$
$$+ B_{12} \times \left[K_{12}(k_D r_n) - \frac{K_{12}(k_D a)}{I_{12}(k_D a)} I_{12}(k_D r_n)\right]\}$$

Waves can propagate along an arbitrary direction, denoted by angle θ, which represents the angle between the wave vector **k** and a primitive translation vector (along the x axis). But for simplicity we assume that the longitudinal waves propagate in the x direction.

The grains execute small oscillations $x_n = x_{0n} + \xi_n$, around their equilibrium position.

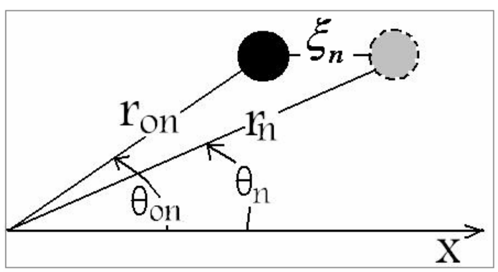

Figure 4. the relation between $r_n, r_{on}, \xi_n, \theta_n, \theta_{on}$

Figure 4 shows the relation between fluctuations of radial and tangential coordinates of displacement versus ξ_n . As usually, the wave train solution $\xi_n = A_n \exp(i(knd - \omega t))$ is used. Under the condition $k_D \xi_n << 1$, we obtain the longitudinal dispersion relation .

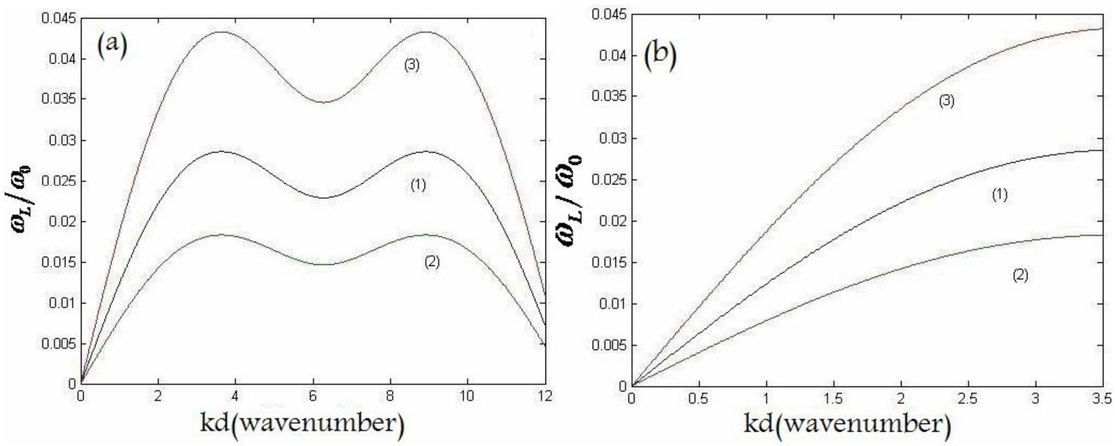

Figure5: The normalized longitudinal frequency ω_L / ω_O is depicted as a function of the normalized wave number kd , for wave propagation in the x-direction. The lattice parameter, d / λ_D here is 2.5 .(a) : (1)-For the Yukawa-type interaction, (2)-for a dressed potential energy, (3)-for proposed potential energy, eq(13). (b) : Here kd have smaller values.

Figure (5) shows the normalized frequency as a function of the normalized wave number kd for longitudinal wave. The frequency of wave on the basis of our potential is larger than the frequency which is found on the basis of Yukawa potential or dressed potential (reported in Ref. 12). This dispersion relation is the experimental result.

Conclusion

In this paper, we have calculated the potential function of grains in hexagonal dusty plasma crystal, as a function of r,θ in linear case. The comparison of our calculation and the Yukawa potential is given in figure 3. Also we have investigated the frequency dependence on the wave number (dispersion law) in x-direction. The normalized frequency in our case is larger than the normalized frequency which is found on the basis of Yukawa potential or dressed potential. The transversal dispersion relation will be the next work.

References

1- Kaganer, V.N., et al., *Rev. Mod. Phys*., **71**, 779 (1999).
2- Peeters, F.M., and X, Wu., *Phys. Rev*. A., **35**, 3109 (1987).
3- Grimes, C.C., and Adames, G., *Phys. Rev. Lett*., **42**, 705 (1979).
4- Yacoby, A., et al., *Phys. Rev. Lett*., **77**, 4612 (1996).
5- Melandso, F., *Phys. Plasmas*., **3**, 3890 (1996).
6- Homann, A., Metzer, A., Peters, S., and Piel, A., *Phys. Rev. E*., **56**, 7138 (1999).
7- Morfill, G., Ivlev, A.V., and Jokipii, J.R., *Phys. Rev. Lett*., **83**, 971 (1999).
8- Farokhi, B., Shukla, P.K., Tsinsadze, N.L., Tskhakaya, D.D., *Phys. Lett. A*., 318 (1999).
9- Farokhi, B., Shukla, P.K., Tsinsadze, N.L., Tskhakaya, D.D., *Phys. Plasmas.*, **7**, 874 (2000).
10- Shukla, P.K., Mamun, A.A., *New Journal of Phy*., **5** (2003).
11- Farokhi, B., *Proceeding of Second IAEA Technical meeting in Plasma*, *Trieste*, *Italy*, *2-4 March* (2005).
12- Farokhi, B., Kourakis, I., and Shukla, P, K., *Phys. Lett. A*., 355 (2006).