Study of Roughness Effect on Waves And Currents Combined Flow

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Abstract

In this paper, An analytical and experimental models will be presented , to study the effect of waves and currents interaction phenomena.In this experiments we will consider the viscosity effect and sea bed roughness .This model has been drived by using the general form of the so called (N-S) Equations of motions , and shear stress calculations which are a Combinations of theory and experiments in this field .Our major concern will be to define and express the governning dimensionless parameters in a such complicated phenomena.The results of experiments data will be good compared by Brevik of Ass[1] (1980), van rijn[3] (1993) and Jonsson[5] (1976).Finally the experimental analysis shows that, the dimensionless parameter developed in present study , have essentially reasonable agreement with the theory.

Key words: Sea Waves , Currents , Interaction , Roughness , Dimensionless Parameter

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Introduction

Interaction of waves , currents and sea bed roughness is a complicated phenomena in fluid dynamics , study and analysis of this Combined flow is very important , in true dealing with sediment transport,approach channels , platforms ,sea bed pipe line , and forces on the coastal structures.The friction at the seabed gives rise to erosion and transportation of sea bed sediments , determines the dispersion of pollutants, and influences the forces on structures on the bed . For example when waves Interface with a tidal current, the wave and current boundary layers interact in a such away that the time – mean bed shear stress is larger than that the same depth averaged current in the absence of waves .A number of theories of wave - current combined flow have been proposed in the last two and a half decode , Listed for example by Kamphius and soulsby[4] (1988), which mostly consider , wave dominated case , or a current dominated of the real sea but , in this experiment,we will study the effect of different roughness patterns,on the wave parameters such as wave height (H), wave length, wave period (T) wave direction (ϕ),and waves celerity(C),as well as the current velocity (U),To drive the nondimensional parameters of the combined flow.

Theory

To demonstrate the importance of the various parameters in waves, currents and roughness system, the general form of (N-S) Equations, have been considered as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}.\frac{\partial \rho}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}.\frac{\partial \rho}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) \qquad (2)$$

$$\frac{\partial u}{\partial x} + u\frac{\partial w}{\partial z} = 0, \qquad (3)$$

Let us discuss now the interaction between a regular linear wave train and a steady shear current with arbitrary profile in two dimensional plane(Fig:1),the surface elevation $\xi(x,t)$ is sinusoidal ,i.e:

$$\xi(x,t) = a\cos(kx - \omega_1 t) \qquad (4)$$

$$u_1(z) = \frac{u_*}{k}Ln\left(\frac{z}{z_0}\right) \qquad (5)$$

$$\omega_1 = \omega_0 + ku\cos(\phi - \phi_0) \qquad (6)$$

$$z_0 = 0/018, Ks\left(\frac{u_* .Ks}{\gamma}\right)^{0/15},$$ (7)

$$\mathrm{re}\, U = U_1(z) + u(z).\cos(Kx - \omega t)$$ (8)

$$w = w(z).e^{i(kx-ct)}, K = \frac{2\pi}{L}, C = \frac{L}{T}, \omega = K.C$$ (9)

By placing the operator $\frac{\partial}{\partial t} by - C\frac{\partial}{\partial x}$, and eliminations the pressure terms , from

equations (1),(2),and nonviscous term, and also $\left|u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right| \le \left|\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x}\right|$, then we obtain

the combined flow equations as follows .

$$\frac{d^2 w}{dz^2} - \left[k^2 - \frac{k}{w - ku_1}.\frac{d^2 u}{dz^2}\right]w = 0$$ (10)

However , the system of equation (10) can not be solved analytically for arbitrary wave number K and frequency ω unless the current velocity varies linearly with depth, Equ(5), for special case , it is necessary to solve the equation in non dimensional from , and experimentally measured value of wave length ,velocity profile ,and roughness patterns , will help us , to determine the value of parameters .The velocity profile due to waves and currents is as follow :

$$U_{(z)} = \frac{u_{*c}}{K}.\frac{1}{u_{*c} + u_{*w}} Ln\left(\frac{z}{K_s / 30}\right),$$ (11)
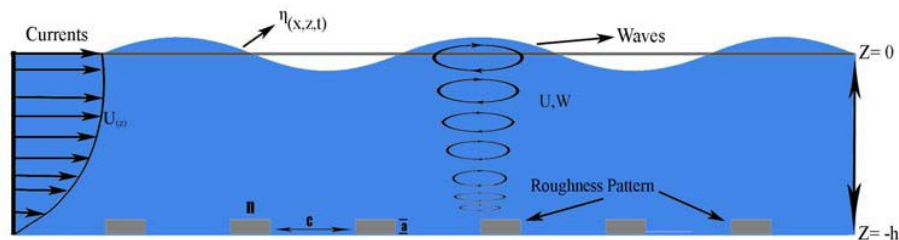

Fig (1): Definition of Waves + Currents + Roughness System

Which K_S is grain size diameter , u_{*c} Current shear velocity u_{*w} waves shear velocity , K= 0.4 Von Karman Coefficient,and $u_{(z)}$ is the real current velocity due to waves and currents combined flow.In experimental analysis u_{*w} and δ_w waves boundary layer thickness calculated by Jonsson and sleath [5](1976),

$$(12)\, \delta_w = \left(\frac{\pi v T}{4}\right)^{1/2} \qquad\qquad u_{*w} = \frac{\omega.\delta_w}{4k}$$

In our experiments we found new expression of equation (11)during the tests by different roughness patterns.

$$U_{(z)} = \frac{u_{*c}}{k} \cdot \frac{1}{u_{*c} + u_{*w}} \cdot Ln\left(\frac{z}{k_s / 30}\right) \cdot \frac{1}{\left(\frac{u_b \cdot D_{n50}}{v}\right)^{0.022}}$$ (13)

Which will describe the effect of roughness nominal diameter D_{n50}, and viscosity effect in sea bed ,U_b is the sea bed orbital velocity due to combined flow ,which is calculated by $u_b = \frac{\omega H}{2} \cdot \frac{1}{\sinh(kh)}$ (14)

In which ,waves frequency ω ,should be calculated by using equation (6).New dimensionless parameters is some how roughness reynolds number . $R_{er} = \frac{u_b \cdot D_{n50}}{v}$ (15)

Which describes the types of flow near the sea bed in shallow water regions .In this experiment,we have used different shapes of roughness elements ,which will produce turbulent flow near the bed.
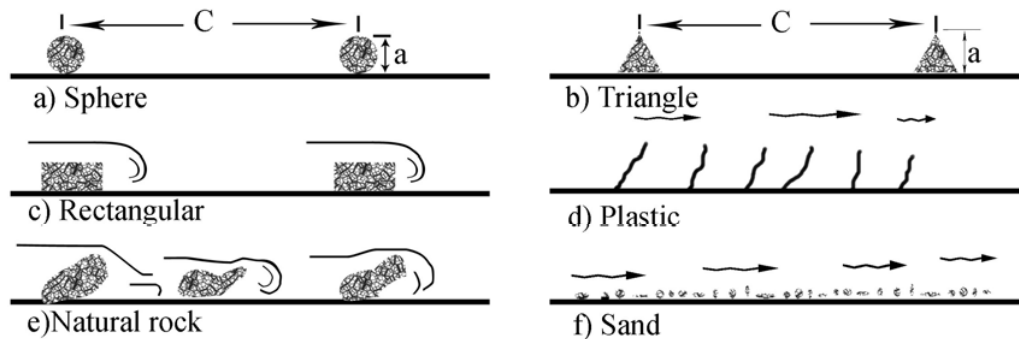

Fig:(2):Different roughness patterns

Due to different geometry of the roughness ,various roughness reynolds numbers ,can be defined ,which will present in another paper in the very near future.Another important problem,in the derivation of the(N-S) equations is to define the stream line governning equations.In the case of pure waves the stream lines equation on the viscous form are as follows.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial P}{\partial x} + \gamma \nabla^2 .u$$ (16)

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial P}{\partial z} + \gamma \nabla^2 .w - g$$ (17)

$$u = -\frac{\partial \Psi}{\partial z}, w = \frac{\partial \Psi}{\partial x}$$ (18)

By eliminating pressure gradients parameters ,we will have ,

$$(19) \frac{\partial}{\partial t}\left(\nabla^2 \Psi\right) = \gamma.\left(\nabla^4 \Psi\right)$$

To find the stream line functions for combined flow we have used the equation (10), and substitute $u = -\dfrac{\partial \Psi}{\partial z}, w = -\dfrac{\partial \Psi}{\partial x}$ as follows:

$$\frac{k}{\omega - ku_1}.\frac{\partial^4 \Psi}{\partial x.\partial z^3} + \frac{\partial^3 \Psi}{\partial x.\partial z^2} - \left[k^2 - \frac{u_*.z_0}{\omega - ku_1}\right].\frac{\partial \Psi}{\partial x} = 0 \qquad (20)$$

$$If \omega = KC, K = \frac{2\pi}{L}, C = \frac{L}{T}, \omega^2 \cong gK.\tanh(kh)$$

then ,we will have

$$\frac{\partial^4 \Psi}{\partial x.\partial z^3} + (C - U).\frac{\partial^3 \Psi}{\partial x.\partial z^2} - \left[k^2(C - U_1) - \frac{u_*.z_0}{k}\right].\frac{\partial \Psi}{\partial x} = 0 \qquad (21)$$

In equation (21),we will see the stream lines are in the 4th order of Ψ ,which can not be solved analytically ,but by injection of some dyes the path lines can be measured experimentally in the combined flow.

Applied Formulas

Near the sea bed the effect of inertia and neglected with those of friction can de defined on the form quadratic friction law.

$$(22) \tau_0 = \rho.C_d.U^2$$

This formula is only for pure currents+roughness ,in which c_d is the drag coefficient

$$(23) C_d \cong \left[\frac{K}{0.71 + Ln\left(z_0 \big/ h\right)}\right]$$

$$(24) z_0 \cong \frac{k_s}{30} + \frac{v}{9u_*} \text{ In which}$$

But ,when waves are present ,the shear stress can not be calculated by using the equation (23),In this experiment we have received a new formula after Bijker[1] (1967).To calculate the shear stress due to waves of currents.(Morovvati[8],2000)

$$\tau_0 = \frac{\rho.g}{\sqrt{2.\pi.c}}.v.\sqrt{h}.\sqrt{f_w}.U \qquad (25)$$

In which C in the chezy coefficient in coastal area, and can be calculated by this formula

$$(26) C = 18\log\frac{12h}{D_{n50} + D_{n90}}$$

Which is presented by the writer for coastal shallow water ,as a results of anormous tests over different roughness patterns.Another friction term which is considered in combined flow,is the eddy viscosity terms which appears in the equation of motion in the form of

$$A_X.\frac{\partial^2 u}{\partial x^2}, A_X.\frac{\partial^2 u}{\partial z^2}, A_X.\frac{\partial^2 w}{\partial z^2}, and A_X.\frac{\partial^2 w}{\partial x^2}, Which A_X and Az$$

Are $A_X \cong U.L$ And $A_z = \dfrac{UH^2}{L}$ In dimentionless .forms By a dimensional Analysis of all the governing parameters ,which are in this functions are as follows:

$$(27)\ f\left(H,T,L,h,u,w,a_b,u_b,U,u_*,z_0,\nu,\rho,\mu,k_s,D_{n50},D_{n90},C,n,A_x,A_z,\Delta\rho,g,....\right)=0$$

After the analysis , the dimensionless numbers of combined flow have been driven in the forms of froude numbers and reynolds numbers:

$$R_{eb} = \frac{u_b.a_b}{\nu} \qquad\qquad\qquad (28)$$

$$R_{eA} = \frac{u_b.L_\Delta}{\nu} \qquad\qquad\qquad (29)$$

$$R_{ed} = \frac{u_b.D_{n50}}{\nu} \qquad\qquad\qquad (30)$$

$$R_{e_{cw}} = \frac{(c-u)L}{\nu} \qquad\qquad\qquad (31)$$

Another very important dimensionless parameters are π_1,π_2 and π_3 which are defined as follows , that describe the relations between viscosity ,nominal roughness diameter ,current velocity U and waves parameters:

$$(29)\qquad\qquad \pi_1 = \frac{\omega H^2}{\nu}$$

$$(30)\qquad\qquad \pi_2 = \frac{\omega H^2.\nu}{D_{n50}^2.(c-u)^2}$$

$$(31)\qquad\qquad \pi_3 = \frac{XH^2}{D_{n50}.\nu}$$

π_1 Express the waves parameters and viscousity effect relation , by increasing the viscosity of fluid , the wave height decrease . π_2 express the relations between wave height, roughness elements and current velocity U ,in the following direction , π_3 is a very special number of this experiment ,which satisfies wave height decrement ,due to current velocity in creasing , and also it explaines a logical relations between the waves + currents and roughness complicated phenomena.

EXPERIMENT AND DATA ANALYSIS

Experiments were carried out in the wave – current flume as shown in (Fig :3),and also use the results some special bed roughness data analysis From (Morovvati[8],1995) , the flume is 30m long , 1m high ,and 1m wide ,and also some experiment in aspecial tilting flume (L=10m , h = 45cm ,w = 30 cm, m=15/1000), It is equipped with a piston –type random wave generator at one end and a wave absorbing beach at the other end .The beach has a slope of about 1 /2.2 and also a wave absorber by a net of 1/5 in meshes. Currents can be generated by pumping the water through the pipe under – neath the flume

as shown in (Fig:3) , many experiments have been done by waves and currents in the following and opposite directions.Measurements of wave height ,wave period ,and current velocity were performed from a carriage moving over rails above the flume , water level variation were measured by three resistance probe and velocities by propellar and pito-tube currents meteres in different depth.
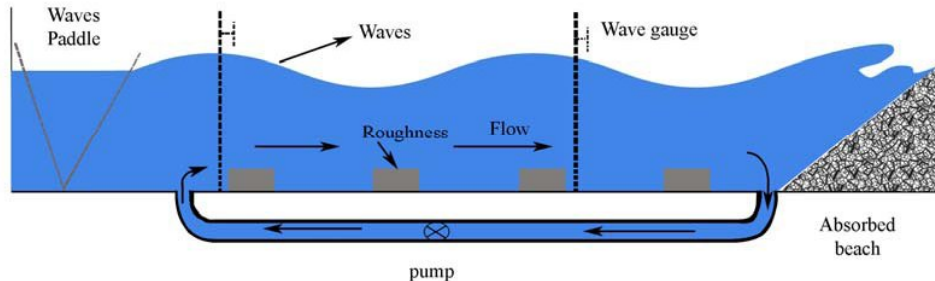

Fig:3: Waves + Currents + Roughness

In order to have beter understanding of the complex system, we have done many tests in a vavest currents over different raughness patterns. Domnent parameters are (H, T, h, C, n, L), and the fallowing experimental arangements have been done:

- Water depth   20 <h < 40 (cm)
- Wave height   7 < H < 15 (cm)
- Wave period   0.7<T<1.4(cm)
- Current velocity   10<U<120(cm/sec)
- Roughness elements (Ripples , Sand , Gravels, 5*3*2 cm concrete block)
- Roughness patterns (regular, irregular , 4spots,5 spots)….
-dye injection of crystal of ($k_2 MnO_4$) ,of organics of ($k_2 Cr O_4$)and ($k_2 Cr O_7$) and fluorescen $C_2OH_{12}O_5$ which gives good fluorescien to the water, eaimm sulfide which is dissolve in cold water, and has very good fluorescein by $\rho \cong 1030$, for path line observation .The test program and basic hydrodynamic data are given in tables. Detailed of waves ,currents ,and roughness the presented by the writer[3] (1990)

EXPERIMENTAL RESULTS

The results of each tests gives the wave height before and after the roughness area (H1 and H2), the roughness pattern characteristics (C, Lr, Δ , a , Ks) , ratio of wae height changes, waves excursion (ao) , and important parameter as the ration (C/ao), the wave length for T = 10.4 sec wve measured. All tests results can be found in tables 1-4.

By using the results of four typical roughness patterns (C/a = 3.33 , 6.66 , 13.33 , 26.66) , on Fig 5, it can be seen that, wave height dcrements in the case of semi close arrangement (C/a = 6.66) is higher than the ofther case.

In the case of waves + currents (0.1<uc<1m/s), and sea bed roughness (Ripple bed, Sand bed, Gravel bed) which can be shown in fig: 5 , 6 , and 7 , the effect of waves on velocity profiles has been analyzed. It can seen that the presence of waves, appears in the form of wave eddy viscosity friction , and affect on the velacity distribution, the superposition of wave motion on a uniform current helps of the turbulence boundary layer

condition which can be seen on (Fig: 5 , 6 , 7), which are good comparable by the other experiments

Table 1: Pure Waves + Roughness (C/a = 3.33, h = 35 cm, T = 0.9 sec)

H_1	H_2	ΔH	$\Delta H/H_1$	a_o	C/a_o
7.10	5.85	1.25	0.16	2.37	4.22
9.30	7.53	1.77	0.19	3.10	3.25
10.38	8.50	1.88	0.18	3.95	2.90
12.96	10.76	2.20	0.17	4.32	2.31

Table 2: Pure Waves + Roughness (C/a = 6.66)

H_1	H_2	ΔH	$\Delta H/H_1$	a_o	C/a_o
7.10	5.79	1.31	0.184	2.37	8.44
9.30	7.46	1.84	0.197	3.10	6.50
10.38	8.41	1.97	0.489	3.45	5.80
12.96	10.70	2.26	0.174	4.32	4.62

Table 3: Velocity profile due to waves & currents (ripple bed)

h(cm)	Z	Z/h	$U/\bar{u}_c$	$U/\bar{u}_{cw}$	N
40	0	0	0.01	0.02	0.01
40	10	0.25	0.9	0.98	0.01
40	20	1.10	1.07	0.01	40
40	30	0.75	0.95	1	0.01
40	40	1	0.80	0.75	0.01

Table 4: Velocity profile due to waves & currents   (sand bed)

h(cm)	Z	Z/h	$U/\bar{u}_c$	$U/\bar{u}_{cw}$	N
40	≈ 0	≈ 0	0.008	0.01	0.02
40	10	0.25	1.10	1.25	0.02
40	20	0.50	1.30	1.22	0.02
40	30	0.75	1.07	1.0	0.02
40	40	1	0.90	0.80	0.02

Table 5:Velocity profile due to waves & currents (gravel bed)

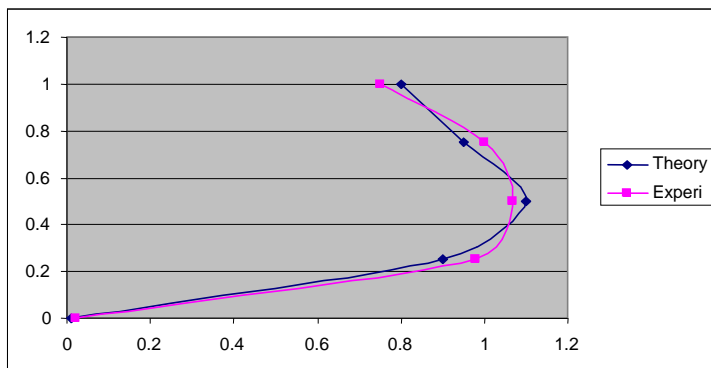| h(cm)	Z	Z/h	$U/\bar{u}_c$	$U/\bar{u}_{cw}$	N
40	0	0	0.0	0.09	0.03
40	10	0.25	1.22	1.30	0.03
40	20	0.50	1.37	1.26	0.03
40	30	0.75	1.20	1.10	0.03
40	40	1.0	1.07	0.90	0.03

Fig. 5: Mean Current velocity Profiles as modified by waves and ripple bed
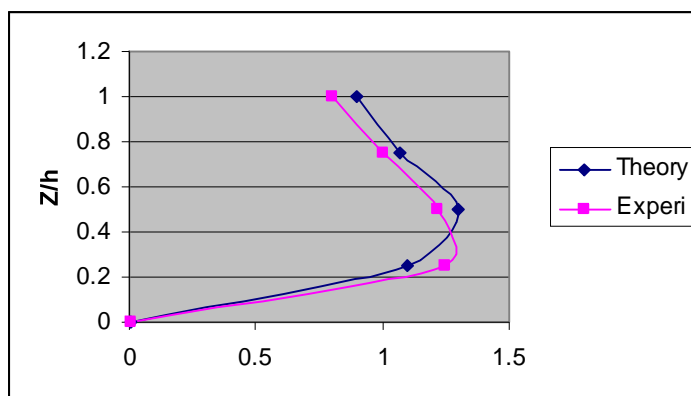

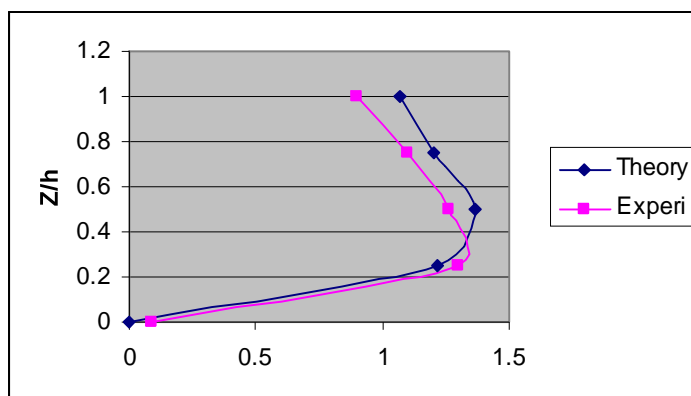**Fig. 6: Mean current velocity profiles as modified by waves and sand bed**.

Fig. 7: Mean current velocity profiles as modified by waves and gravel bed

CONCLUSIONS

From the analysis with the experiment data, the folliwng conclusions can be made:

1. Waves & currents in the following or opposite directions, affect each other , the carrents modifies on waves parameters, such as wae height wave period, wav length, wave direction and waves geometry.

2. The presence of roguhness has a little effect on waves than on a current, but, in a waves + curent + roughness system, the effect of waves on current are umch sever than the sea bed roughness.

3. With different arrangement of roughness elements, we will have different roughness coefficient factor, and the distance between rigid bodies is one of the main important factors. The conclusion that the rougliness elements become smaller, is not accurate. The maximum resistance occures in some medium spacing arrangement, (In our case $\frac{c}{a} \cong 0.66$).

4. The most noticeable effect as regards to our wave hight decay measurements is the reduction of the waveheight due to current influence to waves parameters. In other words in a waes + currrents + roghness element system, te presence of the current plays a dominat role in the change of wave parameters thean the roughness elements.

5. Shear stress due to waves and currents (τ_{cw}) is much largerthan in the case of pure current (τ_c), and pure waves (τ_w) for the calcution of longshore sediment transport, τ_{cw} should be considered instend of τ_c or τ_w.

6. The current Eddy viscosity ($\in_C$) is different from the wave $\in$ddy viscodity ($\in_w$), and Waves + Current eddy viscosity $\in_{CW}$ is much larger than $\in_c$ and $\in_w$, and plays a dominant role in the damping of waves parameters.

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