Chaotic particle dynamics in free-electron laser with coaxial wiggler

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Abstract

The motion of relativistic test electron in a free electron laser can be altered significantly by an ideal coaxial wiggler field and uniform axial guide field. We have investigated group I, II and III orbits and finally have found that electron motion become chaotic at sufficiently high beam density and at sufficiently high amplitude of wiggler field. The threshold value of the wiggler amplitude for the onset of chaos is estimated analytically and confirmed by computer simulation. It is shown that the electron dynamic is nonintegrable. There is evidence for chaos from numerical calculation of Poincare maps and nonzero Lyapunov exponent using different approaches of Benettin's method which are described and compared. Moreover, it is shown that the particle motion become chaotic on a time scale comparable with the beam transit time through a few wiggler periods.

Key Words:Free-electron laser,Chaos,Coaxial wiggler, Poincare map, Lyaponov exponents

Introduction

Hamiltonian chaos has been an active area of research in physics and applied sciences.The work of Kolomogrov, Arnold, and Moser (KAM) show that the phase space of integrable classical Hamiltonian systems. Subject to small perturbations, contains three types of orbits: stable periodic orbits, stable quasi periodic orbits, and chaotic orbits.

Riyopoulos and Tang have analyzed side band induced chaos in the electron motion in the field configuration consisting of an ideal helical–wiggler field, the electromagnetic signal wave field, and the side band wave field(1). Chen and Schmidt have shown that the electromagnetic signal wave can also cause chaotic electron motion in the combined helical-wiggler and axial guide field configuration(2). Then Billardon has observed evidence of chaotic behavior in the radiation field in a modulated storage rugfel. Chen and Davidson have found the electron dynamics in the self-magnetic field produced by the non-neutral electron beam in the field configuration consisting of a constant amplitude helical-wiggler magnetic field, and uniform axial magnetic field become chaotic(4,5). The motion of an electron in a linearly polarized wiggler with an axial guide field investigated by Michel-Lours *et al*(6).

In this paper, we analyzed the motion of a relativistic electron in the field configuration consisting of a coaxial wiggler, and uniform axial magnetic fields. It is shown that the motion is nonintegrable. Poincare surface of section plots and nonzero Lyapunov exponents are generated to demonstrate the nontegrability and choaticity of the motion.

The organization of this paper is as follows. In Sec. II, theoretical formulation of the problem is analyzed. In Sec. III, the results of numerical computations and some conclusions are presented.

Theoretical formulation

The motion of one electron in a free electron laser (FEL) with coaxial wiggler B_w and a guide field B_0 is considered. The self-field produced by the electron beam are neglected. The motion of the electron takes place in the coaxial wiggler. The total magnetic field inside a coaxial wiggler will be taken to be of the form

$$\mathrm{B} = B_r \hat{\mathrm{r}} + B_z \hat{\mathrm{z}}, \quad (1)$$

$$B_r = B_w F_r(r, z), \quad (2)$$

$$B_z = B_0 + B_w F_z(r, z), \quad (3)$$

where B_0 is a uniform static axial guide field, F_r and F_z are known functions of cylindrical coordinates r and z.

$$F_r = F_{r1} \sin(k_w z) + F_{r3} \sin(3k_w z), \quad (4)$$

$$F_z = F_{z1} \cos(k_w z) + F_{z3} \cos(3k_w z), \quad (5)$$

where

$$F_{rn} = G_n^{-1}\left[S_n I_1(nk_w r) + T_n K_1(nk_w r)\right], \quad (6)$$

$$F_{zn} = G_n^{-1}\left[S_n I_0(nk_w r) - T_n K_0(nk_w r)\right], \quad (7)$$

$$G_n \equiv I_0(nk_w R_{out})K_0(nk_w R_{in}) - I_0(nk_w R_{in})K_0(nk_w R_{out}), \quad (8)$$

$$S_n = \frac{2}{n\pi}\sin(n\pi/2)\left[K_0(nk_w R_{in}) + K_0(nk_w R_{out})\right], \quad (9)$$

$$T_n = \frac{2}{n\pi}\sin(n\pi/2)\left[I_0(nk_w R_{in}) + I_0(nk_w R_{out})\right], \quad (10)$$

where $n = 1,3$; R_{in} and R_{out} are the inner and outer radii of the coaxial waveguide, $k_w = 2\pi/\lambda_w$ where λ_w is the wiggler (spatial) period, and I_0, I_1 , K_0 , and K_1 are modified Bessel functions.
Vector potential associated with B is

$$\mathrm{A} = \left[\frac{B_w}{nk_w}F_{rn}\cos(nk_w z) + \frac{rB_0}{2}\right]\hat{\boldsymbol{\theta}} \quad (11)$$

where F_{rn} has radial dependency for $(n = 1,3)$ first and third harmony. The corresponding Hamiltonian is

$$H = c\left[p_r^2 + \frac{1}{r^2}\left(p_\theta + \frac{reB_0}{2c} + \sum_{n=1,3}\frac{eB_w}{nk_w c}F_{rn}Cos(nk_w z)\right)^2 + p_z^2 + m^2c^2\right]^{1/2}. \quad (12)$$

As the Hamiltonian is not an explicit function of time, H is a constant of motion, and also H is independent of θ, so it follows that $p_\theta = Const$.

For numerical calculation, dimensionless variables are introduced:
$\hat{p}_i = p_i/mc$, $\hat{z} = k_w z$, $\hat{r} = k_w r$, $\hat{\Omega}_c = \Omega_c / ck_w$ with $\Omega_c = eB_0/m$, $a_w = eB_w/mck_w$, $\gamma = H/mc^2$, $\tau = ck_w t$.

In the new variables, one obtains

$$\gamma = \hat{H} = \left[\hat{p}_r^2 + \hat{p}_z^2 + \frac{1}{\hat{r}^2}\left(\hat{p}_\theta + a_w F_{r1}\cos(\hat{z}) + \frac{a_w}{3}\cos(3\hat{z}) + \frac{\hat{r}\hat{\Omega}_c}{2}\right)^2 + 1\right]^{1/2} \quad (13)$$

We tried to find a canonical transformation, for finding two other constants, but having failed in finding all constants of motion. We have plotted the trajectory of an electron. The motion looks chaotic for some initial conditions. Chaos, in fact confirmed by performing Poincare section and calculating nonzero Lyapunov exponents. For this purpose, the following normalized equations of motion derived from Eq. (13), have been solved numerically

$$\dot{r} = \frac{\partial H}{\partial p_r}, \quad \dot{\theta} = \frac{\partial H}{\partial p_\theta}, \quad \dot{z} = \frac{\partial H}{\partial p_z},$$
$$\dot{p}_r = -\frac{\partial H}{\partial r}, \quad \dot{p}_\theta = 0, \quad \dot{p}_z = -\frac{\partial H}{\partial z} \qquad (14)$$

The plane (r, p_r) with $z = 0 \bmod 2\pi$ is chosen to be the Poincare surface of section. The numerical method for solving differential equations is a fourth order Runge-Kutta.

The existence of chaotic trajectories is confirmed by calculating nonzero Lyapunov exponents by two approaches. The first consists in considering two nearby trajectories with an initial tangential vector of norm d_0. The distance d_n between those trajectories is calculated numerically, and as soon as d_n / d_0 is greater than a quantity between 2 and 3, we renormalize d_n to d_0. The Lyapunov exponent corresponding to the Poincare map is given by

$$\sigma = \lim_{\substack{t_n \to \infty \\ d_0 \to 0}} \frac{1}{t_n} \sum_{1}^{n} \log\left(\frac{d_n}{d_0}\right) \qquad (15)$$

Numerical results and conclusions

A numerical computation is conduced to investigate the properties of the equilibrium orbits of electrons inside a coaxial wiggler. Wiggler wavelength $2\pi / \lambda_w$ and lab-frame electron density n_0 were taken to be $3\ cm$ and $10^{12} cm^{-3}$, respectively. The wiggler magnetic field B_w were taken to be $3745\ G$ which corresponds to the relativistic wiggler frequency $\Omega_w / ck_w = 0.442$. Electron-beam energy $(\gamma_0 - 1)m_0 c^2$ was taken to be $700\ keV$, corresponding to a Lorentz factor $\gamma_0 = 2.37$ and the axial magnetic field B_0 was varied from 0 to $25.3\ kG$ corresponding to a variation from 0 to 3 in the normalized relativistic cyclotron frequency Ω_0 / ck_w associated with B_0. The inner and outer radii of the coaxial wiggler were assumed to be $R_{in} = 1.5\ cm$ and $R_{out} = 3\ cm$, respectively[7].

Figure 1 shows the variation of σ (the largest Lyapunov exponent) with the normalized time τ for three classes of solutions, group I orbits for which $0 < \Omega_0 / ck_w < v_\parallel / c$, group II orbits with $v_\parallel / c < \Omega_0 / ck_w < 3v_\parallel / c$, and group III orbits with $\Omega_0 / ck_w > 3v_\parallel / c$. The existence of group III orbits is due to the presence of the third spatial harmonics of the wiggler field, which also produced the second magnetoresonance at $\Omega_0 / ck_w \approx 3v_\parallel / c$. Growth of group III orbits is larger than group I and group II orbits. As the figure shows the Lyapunov exponent are larger than zero and all group become chaotic. In this figure two near conditions in space-phase are:

$r_1 = r_2 = 2\pi$,

$\theta_1 = \theta_2 = 0$,

$z_1 = z_2 = \pi / 2$,

$p_{r1} = p_{r2} = 0$,
$p_{z1} = p_{z2} = 0.3$,
$p_{\theta 1} = \left(H_0^2 - p_{z1}^2 - p_{r1}^2 - 1\right)r^2$,
$p_{\theta 2} = \left(0.0001\times H_0^2 - p_{z2}^2 - p_{r2}^2 - 1\right)r^2$,

where the initial distance is d_0. After time t_n, the perturbation grows and distance between two trajectories will be d_n. We used Eq. (15) for finding the Lyapunov exponents.

Figure 2 shows the typical phase-space structure for group I orbits. We have chosen surface $p_r = 0$ as a Poncare map section.

Figure 3 and 4 show the typical phase-space structure for group II and group III orbits, respectively.

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Fig. 3. Nonintegrable surface of section plot in the (z, p_z) plane at $p_r = 0$ for group II orbits with parameters $a_w = 2.11\pi/3,\ \hat{\Omega}_c = 3.55,$ and $\gamma = 2.37$.

Fig. 4. Nonintegrable surface of section plot in the (z, p_z) plane at $p_r = 0$ for group III orbits with parameters $a_w = 2.11\pi/3,\ \hat{\Omega}_c = 6.63,$ and $\gamma = 2.37$.

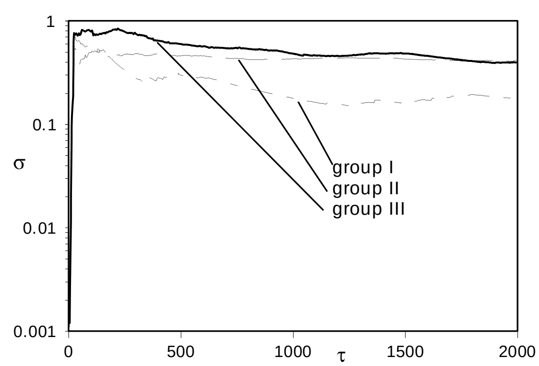
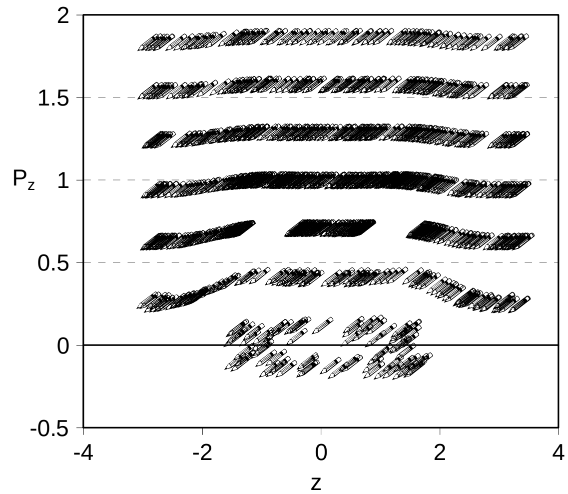


Figure 1



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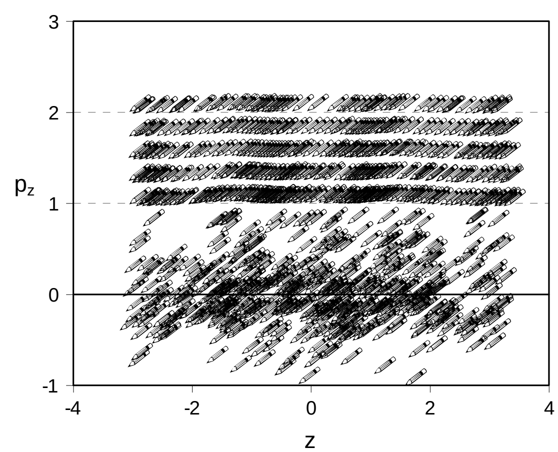

Figure 3

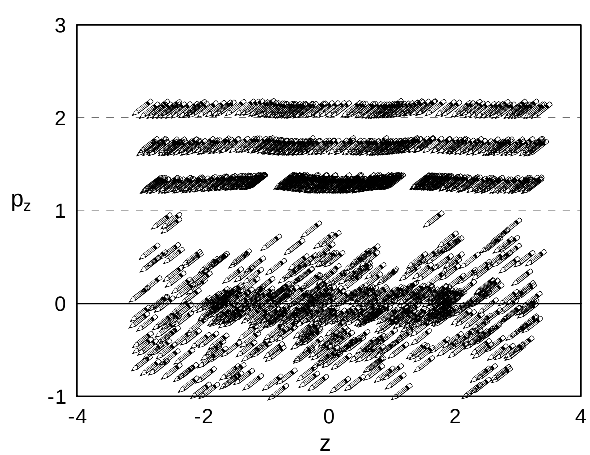

Figure 4

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