

Multiple elastic scattering π^- - ${}^7\text{Li}$ in cluster model

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Abstract

Introduction: A theoretical analysis of the collision of particles with hadrons¹ (wavelength of the incident particle is much smaller than the ranges of its interaction with nucleons), is carried out in the high energy approximation². This approximation, which corresponds to a generalized form of diffraction theory, takes explicit account of n-collision processes as well as single ones. It is used to express the amplitudes for elastic scattering by hadrons in terms of the elastic-scattering amplitudes of the hadrons components and hadrons wave functions.

Aim: Elastic scattering of negative pions by nuclei in cluster model (α, τ) ³ has been studied by means of Glauber's multiple scattering theory. Differential elastic cross sections, have been calculated.

Materials and Method: Various authors have investigated pion-nucleus scattering for the purpose of studying nuclear structure and the pion-nucleon interaction inside the nucleus. The main purpose of such investigations is to understand the mean field encountered by the pion while traversing the target nucleus. This field is usually described in terms of either multiple scattering processes, which depend upon the nuclear charge distributions, or the complex optical model potential. We will restrict ourselves here to the first one where the projectile undergoes multiple scatterings during its passage through the nucleus. The independent multiple scattering theory have been used to describe the pion elastic scatterings from nuclei, is the well known Glauber's multiple scattering theory, which is under discussion here. The theory is based on high-energy Eikonal approximation, in which the interacting particles are almost frozen in their instantaneous positions during the passage of the projectile through the target and the trajectory is nearly straightforward.

Results: The resulting expressions are used to evaluate differential cross section for elastic and the contributions to these cross sections of various single and n-rescattering processes of pion-lithium in Alpha-Tritium cluster model in the energy $\sim 164\text{MeV}$ are exhibited individually. The results clearly agree and much more closely with the experimental values of the cross sections of (α, τ) .

Conclusion: We have accurately calculated scattering amplitude of pion collision with clusters of Lithium nucleuses, and result from surveying theoretical diagrams, clarifies that it is only fourth diagram ($\Sigma \pi, {}^7\text{Li}$) from diagram-1, which encompasses all once and twice repeated collision between clusters and pion, while other diagrams only illustrate collision of pion with single clusters separately.

¹. Part of elementary particles participate in high energy interactions (Baryons and mesons)

². Eikonal or Glauber approximation.

³. Alpha-Tritium cluster

Keywords: Glauber's multiple scattering theory, Green function; Profile function, Alpha-Tritium cluster, πN - amplitude, Differential elastic cross section.

Introduction

Light diffraction phenomenon in Optics: In this phenomenon we observe that wavelength of incoming particle, is much lower than the size of the collide particle and target nucleus absorbs all incident waves to the nucleus, and it can be considered as a complete dark body. Such a collision has a particular position in the physic of elastic and strong collision. Collision of particles in high energy (at least 100 MeV) with very small momentum compared with size of the nucleus, will be a complete absorbed collision and the nucleus can be considered as fully or semi dark body. ^[1]

Collision between particles in high energy therefore has light diffraction properties and through expands of diffraction phenomenon we will can survey high energy hadrons collision with heavy and semi-heavy nucleuses in high energies, and express nuclear diffraction phenomenon applying light diffraction phenomenon. In nuclear diffraction phenomena, observes collision of incident particles with target nucleus applying separate collision of that particle with each nucleon inside the nucleus, then survey properties of the collision. Total amplitude of collision is the sum of the amplitude of incident particle with each particle inside the target nucleus, which has a direct relation with the nuclear shape (which relates to nucleus structure). For the dark body in diffraction phenomenon scattering elements matrix (scattering amplitude) is defined as following:

$$f(k', k) = -\frac{m}{2\pi\hbar^2} \langle k' | \hat{t} | k \rangle, \quad (1)$$

where \hat{t} , - is converting factor from primary to secondary status.

In small scattering angles or in other word where the angle between k', k vectors is small, incident amplitude in high energies will be maximum, when there is not such a maximum, we can conclude that changes is momentum is small, in other word $k' \cong k$. Considering very trivial changes of momentum in high energy physic, therefore collision procedure will be explained and surveyed only in middle position (which momentum is almost equal to primary momentum), this estimate is called high energy approximation which also known as Eikonal or Glauber approximation.

Cluster Model: In nuclear physic, we have nucleus, which constitute by neutron and proton in nucleus. This model lets us describe results of several experiments about structure of nucleus and nuclear reactions. Cluster model is applied to describe other nuclear reactions and experiment, which cannot be explained by former model. In this model, there is particular condition which nucleus nucleon in the nucleus, composted a cluster much more stable than before, in this state clusters demonstrate special properties, and its nucleon will no longer be separated. Alpha and Tritium particles are one of stable formation of cluster nucleons, which are very useful to explain collision in light nucleuses such as Lithium and Beryllium with hadrons.

Surveying multi-particle and cluster systems in collision physic with low-middle and high energy is very important, so that nowadays there are other calculating models such as Mount Carlo calculating method, ^[2] Potential and EFT models, which are applied to describe and calculate differential cross section of these reactions, especially hadrons with light nucleuses such as Lithium and Beryllium. Collision between pion and six-nucleon Lithium nucleus in cluster model of Alpha particle and two nucleon were studies in previous articles

and transformation to cylindrical coordinate model and also equations about Jacobean particle coordination system transformation has been explained completely, so to have better understanding of equations of this article we suggest referring to those articles. In this article we will have look on collision procedure and cluster effects in scattering amplitude of pion from unstable seven-nucleon Lithium under Alpha-Tritium cluster. The reason for selection Lithium-pion is the proper prouder of this collision in explaining static properties: binding energy, resonance surface spectrum, electromagnetic form factor and other collision properties.

Results and discussion

Theoretical frameworks of pion-Lithium collision in cluster model

The interaction of pions with nuclei has been actively studied since 1978. The main focus of this effort to date has been to work at energies at, or below the pion production threshold, which occurs for the pion-nucleon (πN) interaction at p kinetic energies of $\sim 175 \text{ MeV}$. At these energies, the basic πN interaction is completely elastic which is under discussion here. The elastic interaction of pion with finite nuclei and with nuclear matter is of considerable theoretical interest for a number of reasons. We are dealing here with one of the few examples of multiple scattering by a nucleus of an elementary particle projectile, the main other example of which is provided by the nucleon-nucleus interaction. Even as a more repetition of the latter with a different interaction, the $\pi^- {}^7\text{Li}$ nuclear, multiple scattering would therefore be of interest. In particular, the $\pi^- {}^7\text{Li}$ nuclear interaction is however, of such a qualitatively different structure compared to the nucleon one, that it must be considered to be of interest in its own right. In addition, its connection to the basic processes is clearer and more immediate than in the case of nucleons. Moreover, the absence of poles in the low energy (πN) scattering as well as the short range of the (πN) interaction opens the possibility of establishing a very transparent connection between the elementary scattering and the πN scattering, a connection that unfortunately is not so direct in the case of nucleons. Even as a more repetition of the latter with a different interaction, the $\pi^- {}^7\text{Li}$ nuclear multiple scattering in (α, τ) cluster model would therefore be of interest.^[3,4] The main purpose of such investigations is to understand the mean field encountered by the pion while traversing the target Alpha and Tritium particles. This field is usually described in terms of either multiple scattering processes which depend upon the nuclear charge distributions or the complex optical model potential.^[5,6] We will restrict ourselves here to the first one where the projectile undergoes multiple scatterings during its passage through the nuclei ${}^7\text{Li}$ in cluster model. These calculations are carried out for the elastic collisions, $\pi^- {}^7\text{Li}$ at incident energy $\sim 164 \text{ MeV}$. The full series calculations of differential cross with multiple interactions are better in describing the scattering data than using the single scattering. We have studied the scattering effects in $\pi^- {}^7\text{Li}$ scattering. A single channel cluster model is used to calculate the wave function for the ground states of ${}^7\text{Li}$. The multiple scattering theories have been used to describe the pion elastic scatterings from nuclei ${}^7\text{Li}$ in cluster model, is the well-known Glauber's multiple scattering theory (GMST), which is under discussion here.

The theory is based on high energy Eikonal approximation, in which the interacting particles are almost frozen in their instantaneous positions during the passage of the projectile through the target and the trajectory is nearly straight forward that means momentum of indicate particles proximally equivalent with momentum of scattered particles. However, because the Glauber theory is principally derived for the higher energy and the small-angle situations, the reliability of its results may be questioned in the case of low energy and large angle. The GMST has the great advantage of leading to straightforward calculations of the elastic pion-nucleui (${}^7\text{Li}$ in cluster model) scattering cross-sections from knowledge of free pion-nucleon scattering amplitude and nuclear densities.

The preliminary applications of GMST were found to have great successes in reproducing the hadron-nucleus scattering data. The confidence in this theory encouraged the extension of its application to nucleus-nucleus collisions. Correlations within the nucleus are of fundamental theoretical interest but unfortunately, they are difficult to study experimentally. In pion-nuclei ^7Li in cluster model scattering, they manifest themselves through a change of the effective pion-nucleon cross section via the so-called in-medium pion-nucleon amplitude. With regard that scattered particles resulted from collision in long distances from the center of collision are moving freely, and their relative motion energy always would remain positive and non-quantum. Therefore, considering Eikonal approximation condition in elastic collision, energy spectrum of particle collision^[4] having m as mass, in $V(\vec{r})$ potential field and using Green function in relation (1) will be as follows:

$$(\nabla^2 + k^2)G(\vec{r} - \vec{r}') = \delta(\vec{r} - \vec{r}'), \quad (2)$$

the scattering wave function has the form ($\varphi_0 = e^{ikz}$ - Incident wave function, $f(\theta, \varphi)$ - Scattered wave function):

$$\Psi(\vec{r}) = \varphi_0 + f(\theta, \varphi) \frac{e^{ikr}}{r}, \quad (3)$$

$$f(\theta, \varphi) = -\frac{m}{2\pi\eta^2} \int d^3\vec{r}' \cdot \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}') \phi(\vec{r}'), \quad (4)$$

we used cylindring coordinate, gave $\eta = c = 1$ and $(\phi // \vec{k})$ by substitution by substitution to Eq.(3), then we can write:

$$\Psi(\vec{r}) = e^{ikz} - \frac{im}{k} \iint d\vec{r}' \cdot V(\vec{r}, \vec{r}'), \quad (5)$$

in this theory, the elastic scattering amplitude between a pion and nuclei of ^7Li is:

$$f(\vec{k}', \vec{k}) = -\frac{m}{2\pi\eta^2} \int d\vec{r} \frac{e^{ikr - imr}}{k} \cdot e^{-ik'\vec{r}} \cdot V(\vec{r}) \int d\vec{r}' V(\vec{r}, \vec{r}'), \quad (6)$$

where k - is scattering momentum and k' - is the incident momentum of pion. Inserting transferred momentum $q = k - k'$ in equation (6):

$$\begin{aligned} f(\vec{q}) &= \frac{-m}{2\pi} \int d\vec{r} d\vec{r}' \cdot e^{-iq\vec{r}} V(\vec{r}, \vec{r}') \cdot e^{\frac{-im}{k} \int d\vec{r}'' V(\vec{r}, \vec{r}'')} = \\ &= \frac{-m}{2\pi} \int d\vec{r} \cdot e^{-iq\vec{r}} V(\vec{r}, \vec{r}') \cdot e^{\frac{-im}{k} \int d\vec{r}'' V(\vec{r}, \vec{r}'')}, \end{aligned} \quad (7)$$

now use

$$\Psi(\vec{r}) = R e^{ikr}, R = e^{\frac{-im}{k} \int d\vec{r}'' V(\vec{r}, \vec{r}'')},$$

where R is function,

inserting $2ik \frac{\partial R}{\partial z} = 2m.VR$ into eq. above, we obtain:

$$f(\vec{q}) = \frac{-ik}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \left[1 - \frac{im}{k} \int V(\vec{\rho}, \vec{z}') d\vec{z}' \right], \quad (8)$$

and we determinate:

$$f(\vec{q}) = \frac{ik}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \left[1 - e^{-\frac{m}{k} \int d\vec{z}' V(\vec{\rho}, \vec{z}')} \right]. \quad (9)$$

In describing scattering amplitude and properties of the collision system, profile function plays an important role and incident wave will have phase δ_i as result of each collision with nucleon and finally scattered wave (out coming) will have scattering phase equal to $\delta(\vec{\rho})$, which can be stated as the result of sum of scattering phases with single particles $\delta_i = (\vec{\rho}, \vec{\rho}_i)$, $\vec{\rho}$ is the position vector of nucleons. Surface vectors $\vec{\rho}, \vec{\rho}_i$ are equal to image of radius vector of incident particles \vec{r} and scattered particle \vec{r}' in a plate perpendicular to primary momentum of \vec{k} of incident particle. Accordingly, the modified optical phase-shift function $\delta(\vec{\rho})$, can be written sum phase-shift function for $\pi\alpha$ - ${}^7\text{Li}$ scattering, which is equal to:

$$2\delta(\vec{\rho}) = -\frac{m}{k} \int d\vec{z}' V(\vec{\rho}, \vec{z}'). \quad (10)$$

Applying functions (9, 10) and performing some sort of replacements, we will have the following relations:

$$f(\vec{q}) = \frac{ik}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \left[1 - e^{2\delta(\vec{\rho})} \right] \quad (11)$$

or

$$\omega(\vec{\rho} - \vec{\rho}_i) = \frac{1}{2\pi k} \int d\vec{q} \cdot e^{i\vec{q}(\vec{\rho} - \vec{\rho}_i)} f(\vec{q}), \quad (12)$$

where $\omega(\vec{\rho}) = 1 - e^{2\delta(\vec{\rho})}$ is total profile function.

Profile function of whole of the nucleus is always equal to total of profile function of particles, and this is known as the first theoretical approximation of nucleus diffraction. [7]

We define the scattering wave function of the ground state ${}^7\text{Li}$ in $\pi\alpha$ - ${}^7\text{Li}$ elastic scattering as follow:

$$f(\vec{q}) = \frac{ik}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \omega(\vec{\rho}). \quad (13)$$

The survey of cluster models used at the same time for the description of the properties of the light, medium and heavy nuclei was carried out. It was shown that (α, τ) cluster model describes different characteristics of light nuclei. On the basis of the multiple diffraction scattering theory and the (α, τ) cluster model with dispersion the approach for

description of the observables for elastic scattering of particles and light nuclei by light (α, τ) cluster nuclei was considered.

The calculated on the basis of this approach differential cross sections and polarization observables for elastic scattering of particles and light nuclei by light (α, τ) cluster nuclei were presented. These results testified that (α, τ) cluster model allows us to explain different experimental data for the scattering of intermediate energy particles by nuclei. Lithium nuclei in (α, τ) cluster model look like: ^[8,9]

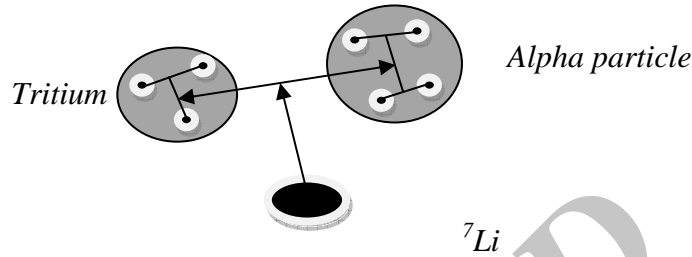


Fig.1. Lithium nuclei in Alpha-Tritium (α, τ) cluster model.

According to cluster model, pion is scattering with Alpha and Tritium that means it is not scatter with all nuclei in ^7Li . The quantum numbers of Lithium nuclei in the ground state are given as follow:

$$\begin{aligned}
 p : (1S_{1/2})^2(1P_{3/2})^1 &\rightarrow J_p = \frac{3}{2}, \quad L_p = 1, \quad S_p = 1/2, \\
 n : (1S_{1/2})^2(1P_{3/2})^2 &\rightarrow J_n = \frac{3}{2}, \quad L_n = 1, \quad S_n = 1/2, \\
 (J^\pi, T) &= \left(\frac{3}{2}^-, \frac{1}{2}\right),
 \end{aligned} \tag{14}$$

where

J_n, J_p -Total orbit momentum of proton and neutron;

S_n, S_p -Proton and neutron spin;

L_n, L_p -Orbit momentum of proton and neutron;

π -Parity of nucleus;

J -Total momentum of nucleus;

T -Isotopic spin.

In calculation of final amplitude of pion-Lithium scattering in cluster model, we should move from single particle coordination system of nucleons to Jacobean coordination system (*Jacobian Coordinate* is used to represent, see Appendix. 1). After performing a series of replacements and applying dimensional calculation, we will find new coordination of clusters and relative coordination of particle motion and coordination of center of total mass of the nucleus and replacing new coordination, pion scattering amplitude of Alpha particle, Tritium and each couple particle will be equal to following equations (For all details see ^[9,10]):

$$\Psi_{Li} = (LM_L SM_S | JM) \phi_\alpha \phi_\tau \phi_{\alpha\tau} \chi_{S, M_S}, \quad (15)$$

$$f(\vec{q}) = f_1(\Omega_\alpha) + f_2(\Omega_\tau) - f_3(\Omega_{\alpha\tau}), \quad (16)$$

where

$$\Omega_\alpha = \omega(\vec{p} - \vec{p}_\alpha) = \frac{1}{2i\pi k} \oint d\vec{q} \cdot e^{-i\vec{q}(\vec{p} - \vec{p}_\alpha)} f(\vec{q}), \quad (17)$$

$$\Omega_\tau = \omega(\vec{p} - \vec{p}_\tau) = \frac{1}{2i\pi k} \oint d\vec{q} \cdot e^{-i\vec{q}(\vec{p} - \vec{p}_\tau)} f(\vec{q}), \quad (18)$$

$$\Omega_{\alpha\tau} = \Omega_\alpha \Omega_\tau = \omega(\vec{p} - \vec{p}_{\alpha\tau}) = \frac{1}{2i\pi k} \oint d\vec{q} \cdot e^{-i\vec{q}(\vec{p} - \vec{p}_{\alpha\tau})} f(\vec{q}), \quad (19)$$

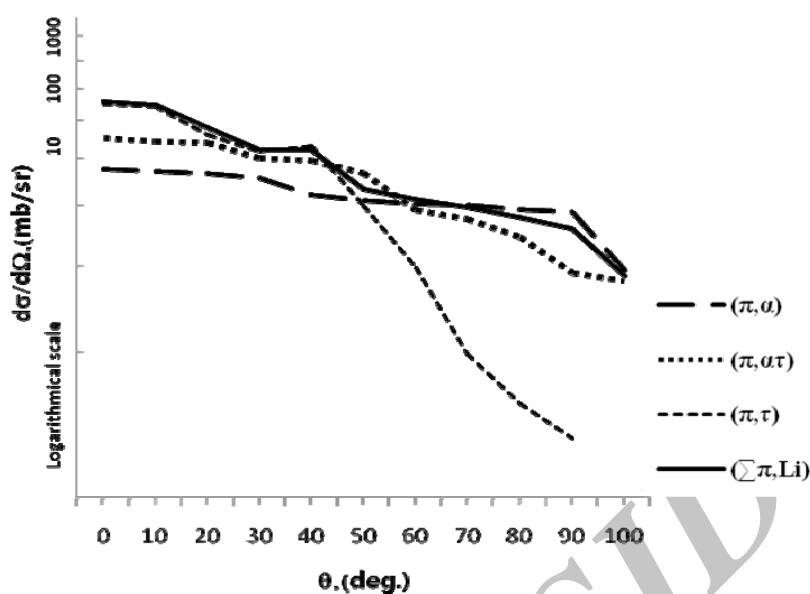
here χ_{S, M_S} - spins function of nucleon;

$\phi_\alpha, \phi_\tau, \phi_{\alpha\tau}$ -Wave function of each cluster and have function of relative movement of both clusters; $f_1(\Omega_\alpha), f_2(\Omega_\tau), f_3(\Omega_{\alpha\tau})$ -scattering amplitude of pion with clusters and multiple scattering.

First part of relation (16), express independent amplitude collision of incident particle with single Alpha cluster, second part express collision with single Tritium cluster and third express double collision with Alpha and Tritium clusters. From of this equation, differential cross section for $\pi^- {}^7Li$ scattering determinate as follow:

$$\frac{d\sigma}{d\Omega} = |f(\theta, \varphi)|^2, \quad (20)$$

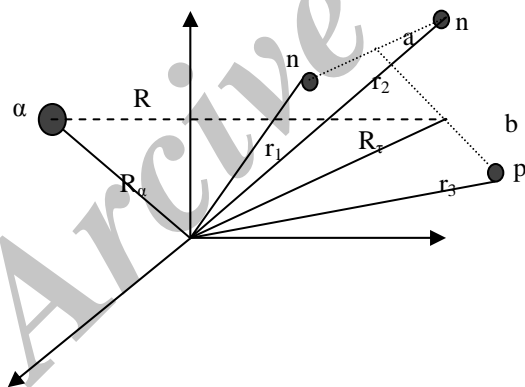
with results of equation (20), we can be easily obtained differential cross section of clusters for pion-Lithium in (α, τ) cluster model. Results for 7Li are displayed in Graphic 1.



Graph.1. Elastic differential cross sections of pion-Lithium in Alpha-Tritium (α, τ) cluster model.

Appendix.1

The coordinates of the center of mass in Jacobean coordinate (full description in [9,10]):



Coordinate of nucleons:

$$Q = P_1 - P_2, \quad P = P_1 + \frac{P_2}{2} - P_3, \quad R_\tau = \frac{P_1 + P_2 + P_3}{3}, \quad R_\alpha = \frac{P_4 + P_5 + P_6 + P_7}{4},$$

$$f_1 = R_\tau + \frac{b}{3} + \frac{a}{2}, \quad f_2 = R_\tau + \frac{b}{3} - \frac{a}{2}, \quad f_3 = R_\tau - 2\frac{b}{3},$$

$$R = R_{\alpha} - R_{\tau} - \text{coordinate of relative movement.}$$

$$\rho_{R_7} = \frac{3R_\tau + 4R_\alpha}{7} - \text{coordinate of the center of mass.}$$

Conclusion

Nowadays cluster formation theory for light and average nucleus is fully proved and satiable among other methods to study particle physic. Clustering nucleus relates to important subject to stabilize nucleus, knowing the method to such end, leads us to more accurate and clear cluster type. Choosing cluster model of Alpha and Tritium particle for unstable nucleus of ${}^7\text{Li}$ in π^- - ${}^7\text{Li}$ interaction, we will have more stable nucleus and applying Glouber Theory in high energies. We have accurately calculated scattering amplitude of pion collision with clusters of Lithium nucleuses, and result from surveying theoretical diagrams, clarifies that it is only fourth diagram ($\Sigma\pi, {}^7\text{Li}$) from diagram-1, which encompasses all once and twice repeated collision between clusters and pion, while other diagrams only illustrate collision of pion with single clusters separately. Results again clarify that in surveying of hadrons collision process with light or semi-heavy nucleus in the high energy approximation, scattering is happening in all nucleons and nuclear diffraction process of the particle under this condition is complete and concentrated and this the best state which different properties of the nucleus and collision can be described and explained. It's also notable that the result of this method in determining, differential cross section of collision are acceptably consistent with the experiment resulted from other theories and models such as Mount Carlo and Shell model.

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