

Discrete modes in two dimension dusty plasma crystals

B. Farokhi*

Physics Department, Arak Branch, Islamic Azad University, Arak, Iran

Abstract

Introduction: Recently, interplay between nonlinear dynamics and geometry has received particular attention in physics of intrinsic localized modes. Apart from its conceptual and theoretical interest, this has real importance in science and technology.

Aim: I aim at focusing on the nonlinear aspects of the transverse (vertical) dust-lattice mode on the two dimensional (hexagonal) lattices. I am interested in investigating the conditions for the occurrence of discrete multisite lattice excitations in a nonlinear Klein-Gordon-like chain, which is characterized by an inverse dispersion law.

Material and Method: The occurrence of discrete vibrational modes in 2D hexagonal dusty plasma lattices is investigated. The system is described by a Klein – Gordon hexagonal lattice. Discrete equation of motion was found and it change to discrete nonlinear Schrodinger type equation.

Results: The discrete dispersion relation was found. It is demonstrated that highly localized structures involving vertical oscillations of charged dust grains may exist in a 2D hexagonal dust lattice.

Conclusion: The results show, the difference in structure from the usual nonlinear Klein– Gordon equation used to describe oscillator chains is: the phonons (here) are stable only in the presence of the electric field force.

Keywords: Dusty plasma, Debye crystals, Hexagonal, Discrete modes

Introduction

Recently, interplay between nonlinear dynamics and geometry has received particular attention in physics of intrinsic localized modes. Apart from its conceptual and theoretical interest, this has real importance in science and technology. A landmark is the prove of the existence of spatially localized, time – periodic oscillations in discrete nonlinear systems, by MacKay and Aubry.^[1] They are now termed discrete breathers (DB), nonlinear localized excitations, or intrinsic localized modes, and over since there are many experimental data and theoretical results published about these important new phenomenne in physics. They appear in lattices of oscillators interacting through nonlinear forces. A type of system is called Klein-Gordon lattices. They are Hamiltonian systems consisting of one degree of freedom anharmonics Hamiltonian oscillators coupled weakly in a lattice, that is, the frequency of oscillation of each oscillator varies non- trivially with the amplitude. Most studies on breathers have been done utilizing this type of models, with application to molecular crystals, duty plasma crystals, biomolecules, such as DNA, solids, etc.

It is known from solid state physics that periodic lattices may sustain, a variety of localized excitations, due to a mutual balance between the intrinsic nonlinearity of the

medium and the mode dispersion. Such structures, typically investigated in a continuum approximation, include nontopological solitons (pulses), kinks, and localized modulated envelope structures.

Nonlinear mechanism related to dust lattice modes is admittedly still in a preliminary stage. Small amplitude localized longitudinal excitations were considered in Refs, [2-3] also the amplitude modulation of longitudinal [4,5] and transverse [6-7] dust lattice waves was recently considered. Those studies were recently extended to more realistic, hexagonal crystalline geometries. [8] All of these studies have relied on a quasicontinuum description of dust lattice dynamics. The discrete character of dust – lattice oscillations has not been studied, apart from some recent investigations which were restricted to single – mode transverse dust breathers [9-11] The one-dimensional quintic nonlinear Schrodinger equation was investigated by K. I. Maruno *et. al.* [13] They were checked the integrability of the symmetric mapping by singularity confinement criteria and growth properties. Resently, The two dimensional dusty plasma lattice was investigated by Hamiltonian and perturbation method. [14] They found odd and even localized modes in hexagonal lattice.

In this paper, I aim at focusing on the nonlinear aspects of the transverse (vertical) dust-lattice mode on the two dimensional (hexagonal) lattices. I am interested in investigating the conditions for the occurrence of discrete multisite lattice excitations in a nonlinear Klein-Gordon-like chain, which is characterized by an inverse dispersion law.

Results and Discussion

Analytical model

We take into account the nearest neighbors interactions only, i.e., each particle (m, n) interact with the three other pairs of particles $(m \pm 1, n)$, $(m \pm 1/2, n - \sqrt{3}/2)$ and $(m \pm 1/2, n + \sqrt{3}/2)$. The physical situation considered is a two-dimensional hexagonal crystal (assumed infinite, for simplicity) consisting of negative dust grains, which are located at equidistant sites a . If the particles are not at their equilibrium positions, we may define the six length variables l_1, l_2, l_3, l_4, l_5 , and l_6 , which represent the distances from the particle (m, n) to the nearest particles, respectively

$$l_1 = \sqrt{a^2 + (\Delta z_1)^2}, \quad (1-a)$$

$$l_2 = \sqrt{a^2 + (\Delta z_2)^2}, \quad (1-b)$$

$$l_3 = \sqrt{(a/2)^2 + (\sqrt{3}a/2)^2 + (\Delta z_3)^2}, \quad (1-c)$$

$$l_4 = \sqrt{(a/2)^2 + (\sqrt{3}a/2)^2 + (\Delta z_4)^2}, \quad (1-d)$$

$$l_5 = \sqrt{(a/2)^2 + (\sqrt{3}a/2)^2 + (\Delta z_5)^2}, \quad (1-e)$$

$$l_6 = \sqrt{(a/2)^2 + (\sqrt{3}a/2)^2 + (\Delta z_6)^2}, \quad (1-f)$$

where z_{ij} denote the displacements of the respective particles from their equilibrium positions in z direction, and

$$\begin{aligned} \Delta z_1 &= z_{m+1,n} - z_{m,n}, & \Delta z_2 &= z_{m-1,n} - z_{m,n}, & \Delta z_3 &= z_{m+1/2,n+\sqrt{3}/2} - z_{m,n}, \\ \Delta z_4 &= z_{m-1/2,n+\sqrt{3}/2} - z_{m,n}, & \Delta z_5 &= z_{m+1/2,n-\sqrt{3}/2} - z_{m,n}, & \Delta z_6 &= z_{m-1/2,n-\sqrt{3}/2} - z_{m,n} \end{aligned} \quad (2)$$

The equation of motion in z - direction is

$$\frac{d^2 z_{m,n}}{dt^2} + \nu \frac{dz_{m,n}}{dt} = \frac{\nu}{M} (F_e - Mg + F_z), \quad (3)$$

where the electrostatic binary interaction force in z – direction $F_z = -\partial U(r) / \partial z$ exerted on two grains situated at a distance r is derived from a Debye – Huckel potential function $U_D(r) = q^\vee \exp(-r / \lambda_D) / \epsilon \pi \epsilon_0 r$. Lattice discreteness, on the other hand is manifested via the weak interacting potential energy among neighboring grains, as compared to the energy “stored” in an isolated (single –site) vibration, so

$$F_z / M \cong -\frac{1}{M} \frac{\partial U}{\partial r} \left(\sum_{i=1}^6 \frac{\Delta z_i}{l_i} \right) \\ = \Omega^2 [6z_{m,n} - z_{m+1,n} - z_{m-1,n} - z_{m+1/2,n+\sqrt{3}/2} - z_{m+1/2,n-\sqrt{3}/2} - z_{m-1/2,n+\sqrt{3}/2} - z_{m-1/2,n-\sqrt{3}/2}] \quad (4)$$

where $\Omega^2 = q^2(1 + \kappa) \exp(-\kappa) / 4\pi\epsilon_0 M a^3$ and $\kappa = a / \lambda_D$. The damping coefficient ν accounts for dissipation due to collisions between dust grains and neutral atoms and here setting $\nu \rightarrow 0$, for simplicity; the dissipative case, will be addressed in a more detailed future work.

We shall assume a variation of the electric field intensity \mathbf{E} , as well as the grain charge q (which may vary due to charging processes) near the equilibrium position $z_0 = 0$. Thus we may develop

$$E(z) \approx E_0 + E'_0 z + \frac{1}{2} E''_0 z^2 + \dots \quad (5)$$

$$q(z) \approx q_0 + q'_0 z + \frac{1}{2} q''_0 z^2 + \dots \quad (6)$$

where the prime denotes differentiation with respect to z and subscript “0” denotes evaluation at $z = z_0$. Accordingly, the electric force $F_e - Mg = q(z)E(z) - Mg$ is expressed as

$$F_e(z) - Mg \approx -Mg + q_0 E_0 + (q_0 E'_0 + q'_0 E_0) z \\ + 0.5(q_0 E''_0 + 2q'_0 E'_0 + q''_0 E_0) z^2 + \dots \\ \approx \gamma_1 z + \gamma_2 z^2 + \gamma_3 z^3 + \dots \quad (7)$$

The zeroth-order term of electric force balances gravity at z_0 , viz., $q_0 E_0 - Mg = 0$, while the first order $\gamma_1 = -M\omega_g^2$ is the effective width of the potential well; the value of the gap frequency ω_g may either be evaluated from *ab initio* calculations or determined experimentally. For instance, the frequency ω_g is typically of order of $\omega_g / 2\pi = 20\text{Hz}$ and $\gamma_2 = -\gamma_1 / 2$ $\gamma_3 = 0.07\gamma_1$. Now, the equation (3) become

$$\ddot{z}_{m,n} = -\omega_g^2 z_{m,n} - K_1 z_{m,n}^2 - K_2 z_{m,n}^3 \\ + \Omega^2 [6z_{m,n} - z_{m+1,n} - z_{m-1,n} - z_{m+1/2,n+\sqrt{3}/2} - z_{m+1/2,n-\sqrt{3}/2} - z_{m-1/2,n+\sqrt{3}/2} - z_{m-1/2,n-\sqrt{3}/2}] \quad (8)$$

where $K_1 = -\gamma_2/M$, and $K_2 = -\gamma_3/M$. Due to the weak interaction between neighbors, the particles can vibrate quasi independently so, the nonlinear terms between neighbors in Eq. (8) are omitted.

Waves can propagate along an arbitrary direction, which is here denoted by an angle θ , representing the angle between the wavevector k and a primitive translation vector (along the x axis), i.e. $k_x = k \cos \theta$ and $k_y = k \sin \theta$. Retaining only linear contribution in the form of "phonons" of the type

$$z_{mn} = u_0 \exp[-i\omega t + ik a(m \cos \theta + n \sin \theta)] + c.c. ,$$

we obtain an inverse-optic-mode like dispersion relation from Eq. (8),

$$\omega^2 = \omega_g^2 - 4\Omega^2 \left\{ \sin^2 \left[\frac{ka}{2} \cos \theta \right] + \sin^2 \left[\frac{ka}{2} \cos \left(\frac{\pi}{3} - \theta \right) \right] + \sin^2 \left[\frac{ka}{2} \cos \left(\frac{\pi}{3} + \theta \right) \right] \right\}, \quad (9)$$

In the special cases $\theta = 0$ or $\theta = \pi/2$ we obtain,

$$\theta = 0: \quad \omega^2 = \omega_g^2 - 4\Omega^2 \left\{ \sin^2 \left(\frac{ka}{2} \right) + 2 \sin^2 \left(\frac{ka}{4} \right) \right\}, \quad (10a)$$

$$\theta = \pi/2: \quad \omega^2 = \omega_g^2 - 8\Omega^2 \sin^2 \left(\frac{\sqrt{3}ka}{4} \right). \quad (10b)$$

The dispersion relation obtained here provides the frequency-wavenumber dependence for TDLW propagation at any direction inside the x - y plane. This expression is identical to the expression obtained by Vladimirov *et. al.* [11] The dispersion relation is an inverse optic-like dispersion, so the frequency at zero wavenumber (infinite wavelength) is finite, and the slope of the curve $\omega = \omega(k)$ is negative for small k . This information is true for all angles θ , as obvious from figures 2a and 3.

I consider the (first harmonic) amplitude of the dust grain oscillations to be small, say of order ε ($0 < \varepsilon \ll 1$; ε is a small parameter) and keep full account of the lattice discreteness, by assuming that the dust grain oscillations may be characterized by a strong variation from one site to another. According to these considerations, and following the treatment suggested in Ref. 12, one may adopt the ansatz $z_{mn} \approx \varepsilon [u_1^{(1)} e^{-i\omega_g t} + c.c.] + \varepsilon^2 [u_2^{(0)} + u_2^{(2)} e^{-2i\omega_g t} + c.c.] + \dots$, in combination with the "book-keeping" scaling assumption: $\omega_g, K_1, K_2 \sim 1$ and $d/dt, \omega_0^2 \sim \varepsilon^2$. The physical meaning of these assumptions is clear: the coupling between dust grains) measured by $\omega_0 \sim \varepsilon \omega_g$) is expected to affect the grain dynamics via the generation, to order ε^2 , of 2nd and 0th order harmonics.

Inserting the above ansatz into Eq. (8) and keeping terms up to order ε^3 , one obtains a discrete nonlinear Schrodinger type of the form

$$i \frac{dA_{mn}}{dt} + P(A_{m+1,n} + A_{m-1,n} + A_{m+1/2,n+\sqrt{3}/2} + A_{m+1/2,n-\sqrt{3}/2} + A_{m-1/2,n+\sqrt{3}/2} + A_{m-1/2,n-\sqrt{3}/2} - 6A_{mn}) + Q|A_{mn}|^2 A_{mn} = 0 \quad (11)$$

where $A_{mn} = u_1^{(1)}$, along with the harmonic amplitude relations

$$u_2^{(0)} = -\frac{2K_1}{\omega_g^2} |A_{mn}|^2, \quad u_2^{(2)} = \frac{K_1}{3\omega_g^2} A_{mn}^2, \quad (12)$$

and the definitions

$$P = -\frac{\omega_0^2}{2\omega_g} < 0, \quad Q = \frac{1}{2\omega_g} \left(\frac{10K_1^2}{3\omega_g^2} - 3K_2 \right). \quad (13)$$

Conclusions

Note that $P < 0$, and the sign of Q depends on the sheath characteristics and cannot be prescribed. Note the value of K_1 , K_2 (and Q) depend on experimental conditions and are not constant.

The envelope equation (11) yields a plane wave solution

$A_{mn} = A_0 \exp[-i(m\tilde{k}a \cos \theta + n\tilde{k}a \sin \theta - \tilde{\omega}t)] + c.c.$, where the envelope frequency $\tilde{\omega}$ obeys the dispersion relation

$$\tilde{\omega}(\tilde{k}) = -4P \left\{ \sin^2 \left[\frac{\tilde{k}a}{2} \cos \theta \right] + \sin^2 \left[\frac{\tilde{k}a}{2} \cos \left(\frac{\pi}{3} - \theta \right) \right] + \sin^2 \left[\frac{\tilde{k}a}{2} \cos \left(\frac{\pi}{3} + \theta \right) \right] \right\} + Q|A_0|^2. \quad (14)$$

The instability analysis (cf. Ref. 6) show that, instability (viz., $Im \omega \neq 0$) occurs if $PQ \cos \tilde{k}a > 0$ (i.e., essentially $Q \cos \tilde{k}a < 0$ here) and (at the same time) the amplitude A_0 exceeds $\sqrt{2P/Q}$. The difference in structure from the usual nonlinear Klein – Gordon equation used to describe oscillator chains is the phonons (here) are stable only in the presence of the electric field force.

References:

1. MacKey, R.S., and Aubry, S., *Nonlinearity* **7**, 1623 (1994).
2. Melandso, F., *Phys. Plasmas*, **3**, 3890 (1996).
3. Kourakis, I., Shukla, P.K., *Eur. Phys. J. D*, **29**, 247 (2004).
4. Amin, M.R., Morfill, G.E., and Shukla, P.K., *Phys. Plasmas*, **5**, 2578 (1998).
5. Kourakis, I., Shukla, P.K., *Phys. Plasmas*, **11**, 1384 (2004).
6. Kourakis, I., Shukla, P.K., *Phys. Plasmas*, **11**, 2322 (2004).
7. Kourakis, I., Shukla, P.K., *Phys. Plasmas*, **11**, 3665 (2004).
8. Farokhi, B., Kourakis, I., Shukla, P.K., *Phys. Plasmas*, **13**, 122304 (2006).
9. Kourakis, I., Shukla, P.K., *Phys. Plasmas*, **12**, 014502 (2005).
10. Koukouloyannis, V., Kourakis, I., *Phys. Review E*, **76**, 016402 (2007).
11. Vladimirov, S.V., Yaroshenko, V.V., and Morfill, G.E., *Phys. Plasmas*, **13**, 030703 (2006).
12. Kivshar, Yu., *Phys. Lett. A*, **173**, 172 (1993).
13. Maruno, K.I., Ohta, Y., Joshi, N., *Phys. Lett. A*, **311**, 214 (2003).
14. Koukouloyannis, V., Kourakis, I., *Phys. Review E*, **80**, 026402 (2009).