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A STUDY OF GAMMA RAY EXPOSURE BUILDUP FACTORS IN STRATIFIED SHIELDS FOR POINT ISOTROPIC SOURCES, INCLUDING THE EFFECTS OF INCOHERENT AND COHERENT SCATTERING

M.H. Alamatsaz[*] and A. Shirani

Department of Physics, Isfahan University of Technology, Isfahan, Islamic Republic of Iran

Abstract

The effects of including incoherent (bound-electron) and coherent (Rayleigh) scattering in exposure buildup factor calculations for point isotropic gamma ray sources, penetrating a two-layer water-lead shield have been investigated in the gamma ray energy (E_γ) range of 40 keV to 3 MeV. Incoherent scattering decreases the values of these factors in both layers up to $E_\gamma \sim 200$ keV and the effect is more significant in the water layer. Coherent scattering increases the factor values and is more effective in the lead layer. This effect vanishes in the water layer at $E_\gamma \sim 200$ keV but is present in the lead layer up to $E_\gamma \sim 1,000$ keV. The combined effects of both incoherent and coherent scattering have also been investigated. The combined effect increases the exposure buildup factor values. These factors are presented for different thicknesses of water and lead layers.

Keywords: Monte Carlo method; Gamma ray buildup factor; Nuclear radiation shielding

1. Introduction

In gamma ray transport through a medium, the response of a detector to photons at a distance r from a monoenergetic point source is given by

$$R^0(r) = BR_0\, e^{-\mu r} \quad (1)$$

where R_0 is the detector response in the absence of the medium (*i.e.*, a vacuum between source and detector), μ is the linear attenuation coefficient for photons of the source energy and B is the buildup factor which accounts for photons which are scattered by the shield into the detector [1]. The buildup factor can be obtained, in principle by experiment; but since the attenuation coefficients and the scattering cross sections are known with reasonable accuracy, buildup factors are customarily obtained either by the solution of the photon transport equation or by Monte Carlo method. Buildup factors, which are important data in nuclear radiation shielding and absorbed dose calculations, have been widely studied by various research groups. Details concerning these studies are given in the review paper of buildup factors by Harima [2].

In this work, a Monte-Carlo program was designed and written to calculate the exposure buildup factors for

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[*] *E-mail: alamatsa@cc.iut.ac.ir*

a gamma ray point source located at the center of a stratified spherical shield of water and lead, and the effects of incoherent and coherent scattering, that have not been considered in previous works of this kind [3], were investigated. Formulas concerning incoherent and coherent scattering are given in [4-6], where these phenomena have been considered for single-material shields of various types.

2. Method of Calculation

Simulation of transport of gamma rays through matter, by Monte Carlo method is fully described in [7,8]. In the following we give a brief description of the Monte-Carlo simulation program which was written to calculate the exposure buildup factors. In the program a gamma ray point source of definite energy E_γ is assumed to be located at the center of a stratified spherical shield consisting of a specified thickness of water followed by a known thickness of lead. The initial direction of a source photon is selected by means of internally generated random numbers to simulate an isotropic source. Photons generated in this way travel in the shield and interact with the shielding media according to various types of interactions and their corresponding cross sections. By this method the history of each photon (from the time it is given birth by the source until it is absorbed or leaves the system under consideration) is determined. If E_i is the energy of the ith photon that has passed the surface of a sphere with radius r, and θ_i is the angle between the direction of this photon and the normal to the surface at the crossing point, exposure buildup factor for photons of energy E_γ at a distance r from the source is calculated as follows:

$$B(r) = \frac{\sum_{i=1}^{N'} E_i \left(\frac{\mu_{en}}{\rho}\right)_{iair} \left(\frac{1}{|\cos\theta_i|}\right)}{E_\gamma N_0 e^{-\mu_1 R_1} . e^{-\mu_2 (r-R_1)} . \left(\frac{\mu_{en}}{\rho}\right)_{air}} \quad \text{for } r > R_1$$

and

$$B(r) = \frac{\sum_{i=1}^{N'} E_i \left(\frac{\mu_{en}}{\rho}\right)_{iair} \left(\frac{1}{|\cos\theta_i|}\right)}{E_\gamma N_0 e^{-\mu_1 r} \left(\frac{\mu_{en}}{\rho}\right)_{air}} \quad \text{for } r < R_1 \qquad (2)$$

where R_1 is the thickness of the first medium (water), N_0 is total number of the source photons generated with energy E_γ, N′ is the number of photons that has passed the surface of the sphere with radius r, μ_1 and μ_2 are total linear attenuation coefficients of the first and second media respectively, $(\mu_{en} / \rho)_{air}$ and $(\mu_{en} / \rho)_{iair}$ are mass energy absorption coefficients of air for photons of energy E_γ and E_i respectively. In the above relation, $\theta = \pi/2$ can only occur when a photon passes tangent to a given sphere of radius r. This case is not considered as a hit to that sphere and therefore no singularity can ever happen.

The type of interactions considered in the program are photoelectric, either Compton scattering (Klein-Nishina cross section applied to free electrons) or incoherent scattering (which is Compton scattering considering binding effects of electrons), coherent (Rayleigh) scattering and pair production. Fluorescence radiation for lead is treated as in [9]. Only K-shell X-rays are considered and fluorescence production is assumed to be zero in the region less than K edge energy of 0.088 MeV. The K-shell contribution to photoelectric absorption is assumed to be 0.788 and the X-ray fluorescence is represented by the average transition energy of 76.539 keV. Isotropic emission is assumed for the produced X-rays. Bremsstrahlung radiation is not included in the program, because this radiation makes reasonable contribution at gamma ray energies above 4 MeV and we intended to investigate the effects of incoherent and coherent scattering which are significant at low energies (below 3 MeV).

Using the above mentioned program, gamma ray exposure buildup factors were calculated for a two-layer shield consisting of 5 mean free paths (mfp) water followed by 8 mfp lead in the gamma ray energy range between 40 keV and 3 MeV. At each energy, calculations were made for penetration depth up to 10 mfp and the effects of incoherent and coherent scattering were investigated, the results of which are given and discussed in the next section. In order to reduce statistical errors on buildup factors, the number of generated events (N_0) was taken to be 5,000,000. With this option, maximum relative standard deviation which occurs at 10 mfp ($\mu r = 10$) is about 7 percent.

3. Results and Discussion

3.1. Comparison with Previous Works

Using the point Monte Carlo code EGS4 [10], Harima and Hirayama [3] have calculated exposure buildup factors for a two-layer shield of 5 mfp water followed by 8 mfp lead, for gamma ray energies between 0.1 and 10 MeV. In their calculations, they have used free electron cross sections for Compton scattering, and the DLC-15 [11] cross sections for photoelectric and pair production interactions. To test the function of our Monte Carlo program, we ran it under conditions similar to those of [3] (that is, with both coherent and incoherent scattering excluded) and calculated exposure buildup factors. The results of these calculations, at some gamma ray energies are compared with the results of [3] in Figure 1. It is seen that the two results agree very well and our program can therefore be used for further investigations. In addition, in special cases where the shield was considered to be water or lead alone, our two-layer program, with both coherent and incoherent scattering included, resulted in exposure buildup factors consistent with those of [4,6,12].

3.2. Photon Cross-Sections

The cross-sections used in obtaining the results presented in the following sections were taken from the Internet [13]. To see the effects of these cross sections on the exposure buildup factors, we calculated exposure buildup factors for the two-layer water-lead shield using the DLC-15 and the Internet cross sections separately. The results obtained, in the gamma ray energy range between 40 keV to 10 MeV, were similar within statistical errors and can conclude that the differences due to cross sections are not significant.

3.3. Effects of Incoherent Scattering

In most buildup factor calculations, binding energy of electrons in their atoms are ignored and photons are assumed to be scattered by free electrons. In addition coherent (or Rayleigh) scattering events are also ignored. The binding energy of electrons can be considered in the calculations by multiplying the Klein-Nishina free electron distribution by a scattering function S (x,z) which is a function of the momentum transfer (x) and the atomic number (z) of the absorbing material [14]. In this case the scattering is called incoherent scattering.

Exposure buildup factors for the two-layer shield of 5 mfp water plus 8 mfp lead were calculated, with and without binding effects of electrons, at various gamma ray energies and up to 10 mfp at each energy. The results obtained showed that the binding effects, which decrease buildup factor values, are only significant at low energy values and large values of μr. Figure 2 compares exposure buildup factors obtained with and without binding effects at gamma ray energy of 40 keV, where incoherent scattering showed maximum effect (note that, in the figure, a given mean free path does not correspond to the same shielding thickness for the two curves, because of the different values of μ's). It is seen that the difference between the two curves increases with μr in water medium (below 5 mfp) and has a maximum value of about 14% at 5 mfp. The differences in the lead layer (above 5 mfp) are smaller and at μr = 10 are within statistical errors. In Figure 3, $(B-B_i)/B$ is plotted against gamma ray energy at 5 and 10 mfp, where B and B_i denote exposure buildup factors without and with electron binding effects respectively. It is seen that the effect at 5 mfp decreases with gamma ray energy and vanishes at about 200 keV. At 10 mfp (in lead layer) the differences are not significant and are within statistical errors at all energies.

3.4. Effects of Coherent Scattering

When a photon is scattered by an atom as a whole and is, therefore, scattered without any significant change in its energy, the scattering is called coherent scattering. Coherent scattering can be considered in the calculations by applying the appropriate distribution in which form factors are included [4,14].

Exposure buildup factors were calculated with and without considering coherent scattering at various gamma ray energies. The results obtained showed that coherent scattering increases buildup factor values at all energies and the effect in both layers is more significant at low energy values and large values of μr. Exposure buildup factors with and without coherent scattering are compared in Figure 4 at 40 keV gamma ray energy. It is seen that the differences between the two curves are significant at all μr values and the effect increases with μr, so that the difference at 5 mfp (that is at the end of water layer) is about 50% and at 10 mfp (in lead layer) is about 80%. In Figure 5, $(B_c-B)/B$, is plotted against gamma ray energy at 5 and 10 mfp. In this figure, B and B_c denote exposure buildup factors without and with coherent scattering, respectively. It is seen that at 5 mean free paths, coherent scattering effect decreases with gamma ray energy and vanishes at about 200 keV, while at 10 mfp (in lead layer) the effect is somehow different and decreases above 80 keV and is considerable up to about 1,000 keV. The reason for the sudden fall at 100 keV in Figure 5(b), is that at this energy, the K-shell effect in lead is maximum, and smaller effects such as incoherent and coherent scattering do not show up strongly.

3.5. Combined Effects of Incoherent and Coherent Scattering

It is, usually, the combined effects of incoherent and coherent scattering which are important in buildup

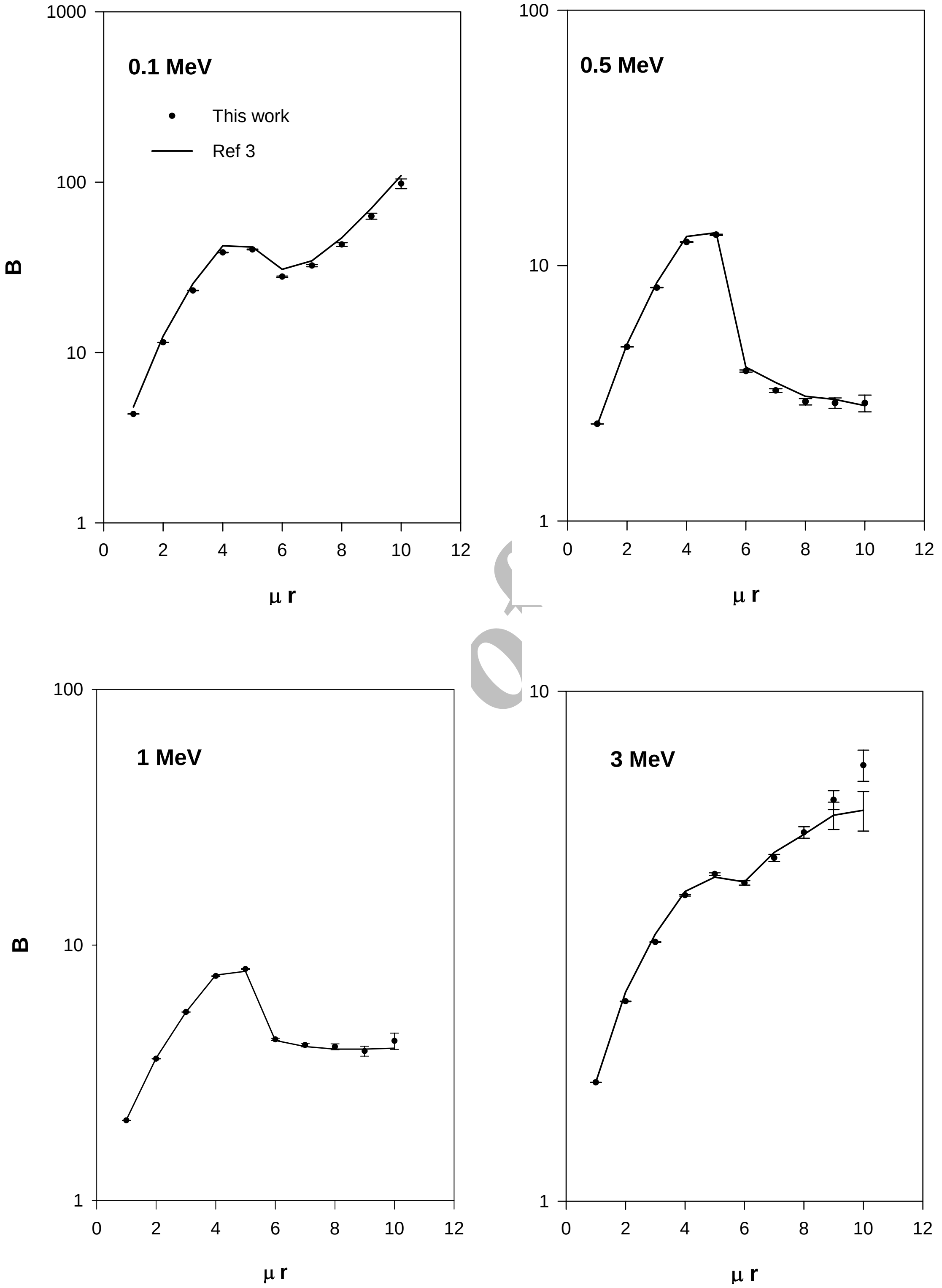


Figure 1. Comparison of exposure buildup factors with those of [3] at some gamma ray energies.

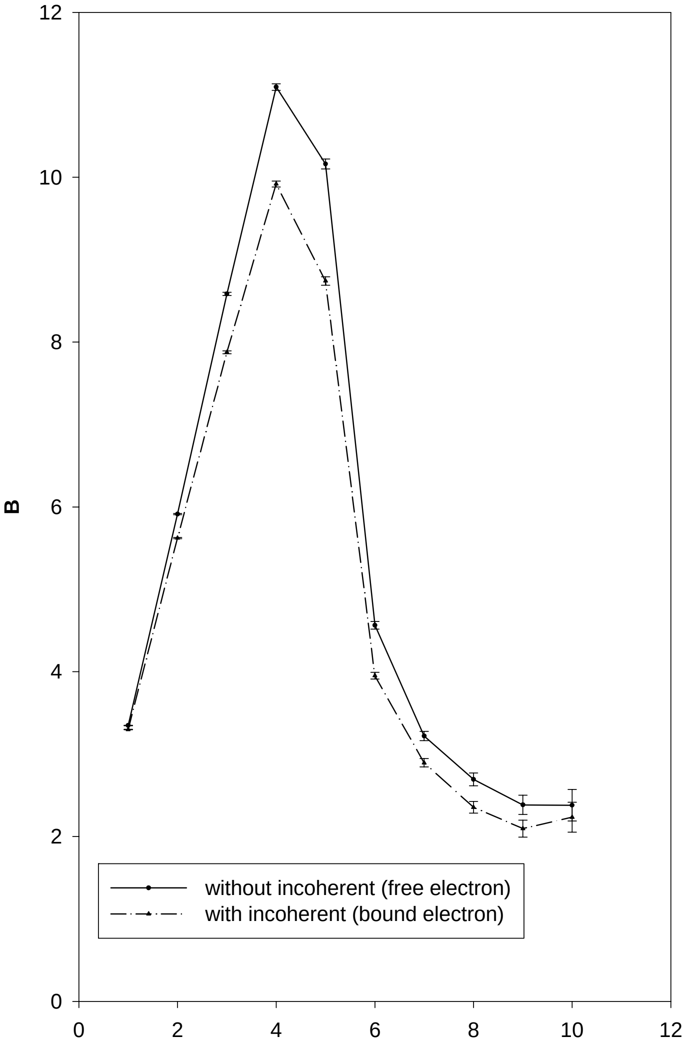
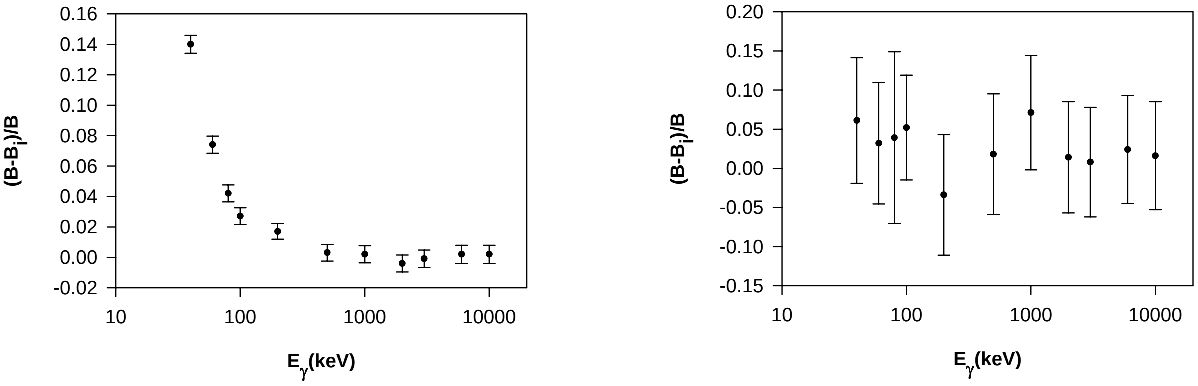


Figure 2. Exposure buildup factors without and with incoherent scattering for E_γ= 40 keV.



(a) 5 mfp

(b) 10 mfp

Figure 3. Effects of incoherent scattering at (a) 5 and (b) 10 mean free paths.The error bars are fractional errors on exposure buildup factors without electron binding effects (*i.e.*, DB/B).

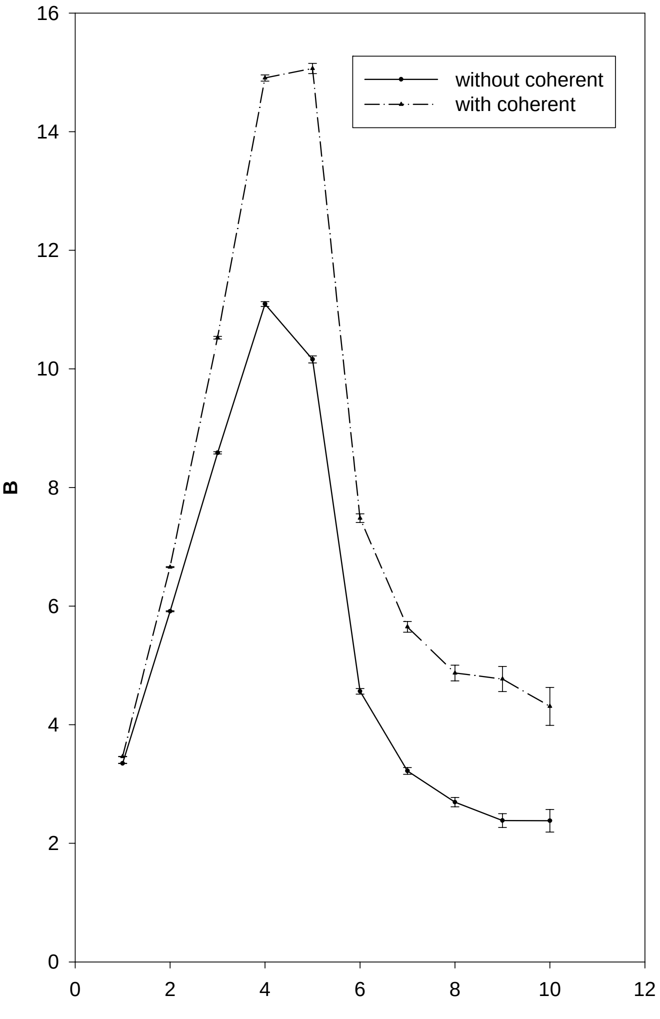
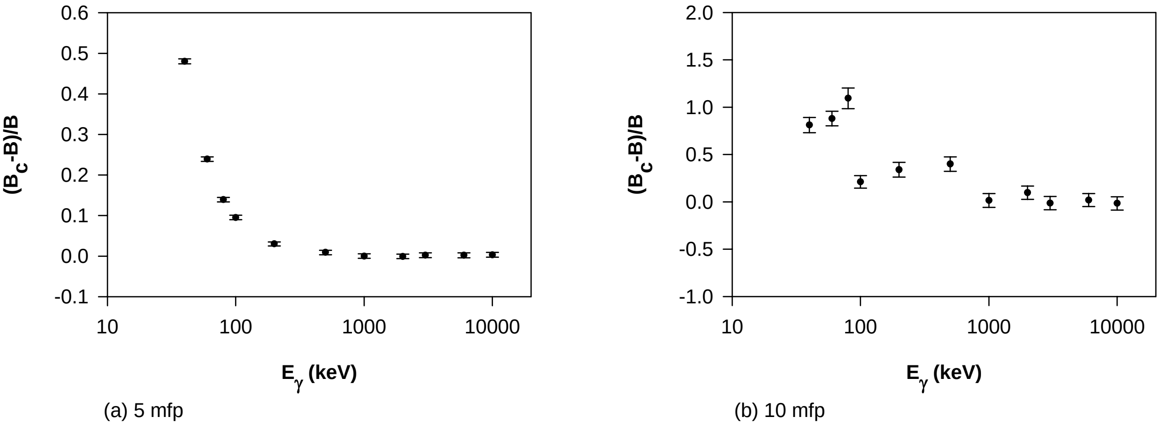


Figure 4. Exposure buildup factors without and with coherent scattering for E_γ= 40 keV.



(a) 5 mfp

(b) 10 mfp

Figure 5. Effects of coherent scattering at (a) 5 and (b) 10 mean free paths. The error bars are fractional errors on exposure buildup factors without coherent scattering (*i.e.*, DB/B).

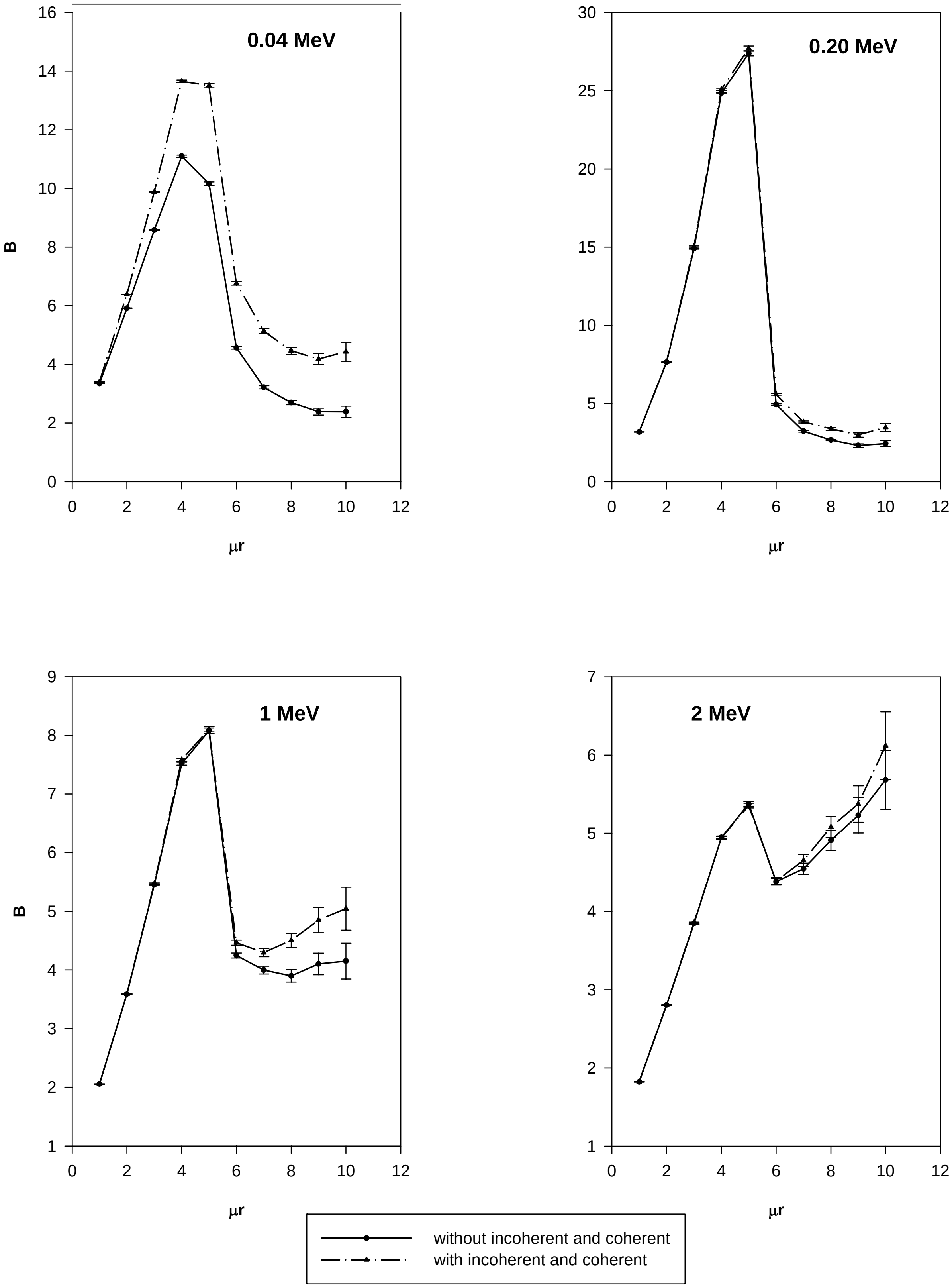


Figure 6. Exposure buildup factors without and with incoherent and coherent scattering for four different energies.

Table 1. Simultaneous effects of incoherent and coherent scattering on the exposure buildup factors at 5 and 10 mean free paths

E_γ (keV)	(B_{ic}-B) / B at 5 mfp	(B_{ic}-B) / B at 10 mfp
40	0.330	0.861
60	0.154	0.772
80	0.096	0.806
100	0.064	0.160
200	0.012	0.419
500	0.003	0.400
1000	0.003	0.215
2000	0.004	0.077
3000	0.006	0.006

B_{ic} and B denote exposure buildup factors with and without incoherent and coherent scattering, respectively.

Table 2. Exposure buildup factor values at 10 mfp distance from the source for different thicknesses (in mfp) of water and lead at some gamma ray energies

Water-lead (mfp)	40 keV	200 keV	1 MeV	2 MeV	3 MeV
0 – 13	1.920 ± 0.112	2.176 ± 0.123	4.873 ± 0.250	5.700 ± 0.286	5.664 ± 0.283
1 – 12	5.647 ± 0.290	3.846 ± 0.201	4.682 ± 0.241	5.541 ± 0.279	5.658 ± 0.283
2 – 11	4.900 ± 0.256	3.656 ± 0.191	4.577 ± 0.236	5.573 ± 0.281	5.800 ± 0.290
3 – 10	4.018 ± 0.214	3.349 ± 0.176	4.525 ± 0.234	5.660 ± 0.285	5.669 ± 0.284
4 – 9	3.973 ± 0.211	3.211 ± 0.170	4.800 ± 0.247	5.757 ± 0.289	5.840 ± 0.292
5 – 8	4.277 ± 0.225	3.185 ± 0.170	4.838 ± 0.248	5.985 ± 0.300	5.965 ± 0.297
6 – 7	5.219 ± 0.270	3.567 ± 0.188	5.068 ± 0.259	5.890 ± 0.295	6.098 ± 0.304
7 – 6	6.660 ± 0.338	4.191 ± 0.217	5.429 ± 0.275	6.162 ± 0.308	6.131 ± 0.305
8 – 5	9.446 ± 0.471	5.916 ± 0.297	6.082 ± 0.306	6.456 ± 0.322	6.249 ± 0.311
9 – 4	15.215 ± 0.744	11.975 ± 0.579	7.726 ± 0.382	7.044 ± 0.349	6.349 ± 0.315
10 – 3	39.732 ± 1.906	120.854 ± 5.689	19.142 ± 0.908	10.591 ± 0.508	7.619 ± 0.369

factor calculations. Our investigations showed that these two effects together increase buildup factor values in both layers (water and lead). Exposure buildup factors calculated with and without these two effects, for the two-layer shield under consideration, are shown in Figure 6 at some gamma ray energies. It is seen that the effect in both media is maximum at 40 keV and it vanishes in water medium (below 5 mfp) at about 200 keV, while in lead medium (above 5 mfp) it is quite considerable up to about 1000 keV. In Table 1, the quantity (B_{ic}-B)/B is shown at 5 and 10 mean free paths for gamma ray energies up to 3000 keV. In this table B_{ic} and B denote exposure buildup factors with and without simultaneous incoherent and coherent scattering effects, respectively. The relative standard deviation of B (that is $\Delta B/B$) at 5 mfp is about 0.005 and at 10 mfp is about 0.07 (these errors have been obtained with 5,000,000 histories). It is seen that the effect at 5 mfp is about 30% at 40 keV and it goes below statistical errors above 200 keV. At 10 mfp the effect is about 90% at 40 keV and it is comparable with statistical errors at about 2000 keV.

3.6. Exposure Buildup Factor Values

With all types of gamma ray interactions (including incoherent and coherent scattering) considered in our program, exposure buildup factors were calculated at 10 mfp distance from the source for different thicknesses (in mfp) of water and lead layers. These factors which are given in Table 2 for some gamma ray energies, can be used in calculations of gamma ray transmission through such shields.

Conclusion

Exposure buildup factors were calculated for a two-layer shield of water and lead, and the effects of incoherent and coherent scattering on these factors were investigated. The results obtained showed that the effects of incoherent scattering in water layer are considerable (maximum 15%) for gamma rays of energies up to about 200 keV and are less significant in the lead layer. The effects of coherent scattering (which is proportional to square of the atomic number of the absorbing material) are more significant in the lead layer and increase exposure buildup factor values by a maximum amount of about 90% at gamma ray energies below 100 keV. This effect in both materials decreases with gamma ray energy and it vanishes in water at about 200 keV, while in lead, it is present up to gamma ray energies of about 1000 keV. These results agree with the results obtained in [6] and [12] for single material shield.

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