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$\Re$-torsion free Acts Over Monoids

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Abstract

In this paper firt of all we introduce a generalization of torsion freeness of acts over monoids, called $\Re$-torsion freeness. Then in section 1 of results we give some general properties and in sections 2, 3 and 4 we give a characterization of monoids for which this property of their right Rees factor, cyclic and acts in general implies some other properties, respectively.

Keywords: $\Re$-torsion free; Rees factor act; cyclic act

Introduction

Throughout this paper *S* will denote a monoid with identity element 1. We refer the reader to [11] and [12] for basic definitions and terminology relating to semigroups and acts over monoids and to [1], [13] and [14] for definitions and results on flatness which are used here.

A monoid *S* is called *left (right) collapsible* if for any $s, s' \in S$ there exists $z \in S$ such that $zs = zs'$ ($sz = s'z$). A submonoid *P* of *S* is called *weakly left collapsible* if for any $s, s' \in P$, $z \in S$, $sz = s'z$ implies the existence of $u \in P$ such that $us = us'$. It is obvious that every left collapsible submonoid is weakly left collapsible, but not the converse. A monoid *S* is called *right (left) reversible*, if for any $s, s' \in S$, there exist $u, v \in S$ such that $us = vs'$ ($su = s'v$). A submonoid *P* of *S* is called *weakly right reversible*, if for any $s, s' \in P$, $z \in S$, $sz = s'z$ implies the existence of $u, v \in P$ such that $us = vs'$. A right ideal K_S of a monoid *S* is called *left stabilizing*, if for any $k \in K_S$, there exists $l \in K_S$ such that $lk = k$. K_S is called *left annihilating*, if for any $t \in S$, $x, y \in S \setminus K_S$, $xt, yt \in K_S$ implies that $xt = yt$. K_S is called *strongly left annihilating*, if for all $s, t \in S \setminus K_S$ and for all homomorphisms $f :_S (St \cup Ss) \to_S S$ $f(s), f(t) \in K_S$ implies that $f(s) = f(t)$. K_S is called *completely left annihilating*, if for all $x, y, z, t, t' \in S$,

$$[(xt \neq yt') \wedge (tz = t'z)] \Rightarrow [(xt \notin K_S) \vee (yt' \notin K_S) \vee (x \in K_S) \vee (y \in K_S)]$$

K_S is called P_E*-left annihilating*, if for all $x, y, t, t' \in S$,

$$(xt \neq yt') \Rightarrow [(xt \notin K_S) \vee (yt' \notin K_S) \vee (x \in K_S) \vee (y \in K_S) \vee (\exists u, v \in S, e, f \in E(S), et = t, ft' = t', ut = vt', xe \neq ue \Rightarrow xe, ue \in K_S, yf \neq vf \Rightarrow yf, vf \in K_S)]$$

K_S is called *E-left annihilating*, if for all $x, y, t \in S$,

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$$(xt \neq yt) \Rightarrow [(xt \notin K_S) \vee (yt \notin K_S) \vee$$
$$(x \in K_S) \vee (y \in K_S) \vee$$
$$(\exists u,v \in S, e,f \in E(S), et = t = ft$$
$$, ut = vt, xe \neq ue \Rightarrow xe, ue \in K_S,$$
$$yf \neq vf \Rightarrow yf, vf \in K_S)]$$

A nonempty set A is called a *right S-act*, usually denoted A_S , if S acts on A unitarily from the right; that is, there exists a mapping $A \times S \to A$, $(a,s) \mapsto as$, satisfying the conditions $(as)t = a(st)$ and $a1 = a$, for all $a \in A$ and all $s,t \in S$. Left S-acts ${}_S A$ are defined dually. If A_S be an act, then we define Green's equivalence relation $\Re$ on A_S by the following rule:

$$(a,b) \in \Re \Leftrightarrow aS = bS$$

for all $a,b \in A$.

A right S-act A satisfies Condition (P), if for all $a,a' \in A$, $s,s' \in S$, $as = a's'$ implies that there exist $b \in A$, $u,v \in S$ such that $a = bu$, $a' = bv$ and $us = vs'$. A monoid S is called *right PCP*, if all principal right ideals of S satisfy Condition (P). A right S-act A satisfies Condition (P'), if for all $a,a' \in A$, $s,s',z \in S$, $as = a's'$, $sz = s'z$ imply that there exist $b \in A$, $u,v \in S$ such that $a = bu$, $a' = bv$ and $us = vs'$. A right S-act A satisfies Condition (P_E), if for all $a,a' \in A$, $s,s' \in S$, $as = a's'$ implies that there exist $b \in A$, $u,v,e^2 = e, f^2 = f \in S$ such that $ae = bue$, $a'f = bvf$, $es = s$, $fs' = s'$ and $us = vs'$. It is obvious that Condition (P) implies Condition (P_E), but not the converse, for this see [2]. A satisfies Condition (E), if for all $a \in A$, $s,s' \in S$, $as = as'$ implies that there exist $b \in A$, $u \in S$ such that $a = bu$ and $us = us'$. A satisfies Condition (EP), if for all $a \in A$, $s,s' \in S$, $as = as'$ implies that there exist $b \in A$, $u,v \in S$ such that $a = bu = bv$ and $us = vs'$. A satisfies Condition (E'), if for all $a \in A$, $s,s',z \in S$, $as = as'$, $sz = s'z$ imply that there exist $b \in A$, $u \in S$ such that $a = bu$ and $us = us'$. A satisfies Condition ($E'P$), if for all $a \in A$, $s,s',z \in S$, $as = as'$, $sz = s'z$ imply that there exist $b \in A$, $u,v \in S$ such that $a = bu = bv$ and $us = vs'$. It is obvious that Condition (E) $\Rightarrow$ Condition (EP) $\Rightarrow$ Condition ($E'P$) and Condition (E) $\Rightarrow$ Condition (E') $\Rightarrow$ Condition ($E'P$). In [3] and [4] we gave a characterization of monoids by Conditions (EP) and ($E'P$) of their acts. A right S-act A satisfies Condition (PWP), if for all $a,a' \in A$, $s \in S$, $as = a's$ implies that there exist $b \in A$ and $u,v \in S$ such that $a = bu$, $a' = bv$ and $us = vs$. A right S-act A satisfies Condition (PWP_E), if for all $a,a' \in A$, $s \in S$, $as = a's$ implies that there exist $b \in A$ and $u,v,e^2 = e, f^2 = f \in S$ such that $ae = bue$, $a'f = bvf$, $es = fs = s$ and $us = vs$. In [7] we gave a characterization of monoids by Condition (PWP_E) of their acts. A is called regular, if all cyclic subacts of A are projective. A is called *faithful*, if for $s,t \in S$ the equality $as = at$ for all $a \in A$ implies $s = t$. A is called *strongly faithful*, if for $s,t \in S$ the equality $as = at$ for some $a \in A$ implies that $s = t$. A is called *P-regular*, if all cyclic subacts of A satisfy Condition (P). In [9] we gave a characterization of monoids by P-regularity of their acts. A is called *strongly* (P) *-cyclic* if for any $a \in A$ there exists $z \in S$ such that $\ker \lambda_z = \ker \lambda_a$ and zS satisfies Condition (P). In [8] we gave a characterization of monoids by strong (P)-cyclic of their acts.

Let S be a monoid and I be a proper right ideal of S. Let x, y and z denote elements not belonging to S. If $A = ((S \setminus I) \times \{x, y\}) \bigcup (I \times \{z\})$ and S acts on A from the right as follows:

$$(u,x)s = \begin{cases} (us,x), \text{ if } us \notin I \\ (us,z), \text{ if } us \in I \end{cases}$$
$$(u,y)s = \begin{cases} (us,y), \text{ if } us \notin I \\ (us,z), \text{ if } us \in I \end{cases}$$
$$(u,z)s = (us,z),$$

then the right S-act A is called *amalgam* of S by I and is denoted by $S \coprod^I S$.

Results

1. General properties

Definition 1.1. An act A_S is called $\Re$ *-torsion free* if for any $a,b \in A$ and $c \in S$, c right cancellabe, $ac = bc$ and $a \Re b$ imply that $a = b$.

We use the abbreviation $\Re$TF for $\Re$-torsion freeness. It is clear that torsion freeness implies $\Re$-torsion freeness, but not the converse, see the following example.

Example 1.1. Let $S=(\mathbf{N},\cdot)$, and consider the amalgam $A_S=\mathbf{N}\coprod^{\mathbf{N}\setminus\{1\}}\mathbf{N}$. Then $(1,x)\neq(1,y)$, but $(1,x)2=(1,y)2$. Hence A_S is not torsion free. It can easily be seen that A_S is $\Re$ -torsion free.

Proposition 1.1. *Let S be a monoid. Then:*

(1) *The one-element act* Θ_S *is* $\Re$ *-torsion free.*

(2) S_S *is* $\Re$ *-torsion free.*

(3) *If an act is* $\Re$ *-torsion free, then all its subacts are* $\Re$ *-torsion free.*

(4) A_i, $i\in I$, *are* $\Re$ *-torsion free if and only if* $A_S=\coprod_{i\in I}A_i$ *is* $\Re$ *-torsion free.*

(5) *If* A_i, $i\in I$, *are* $\Re$ *-torsion free right S-acts, then* $A_S=\prod_{i\in I}A_i$ *is* $\Re$ *-torsion free.*

Proof. It is clear from definitions.□

Proposition 1.2. *Let S be a monoid. Then:*

(1) *All right S-acts satisfying Condition (EP) are* $\Re$ *-torsion free.*

(2) *All right S-acts satisfying Condition (E) are* $\Re$ *-torsion free.*

Proof. (1). Suppose the right *S*-act A_S satisfies Condition (*EP*) and let $ac=a'c$, aRa', for $a,a'\in A_S$ and right cancellable $c\in S$. Since aRa', there exist $s,t\in S$ such that $a=a's$ and $a'=at$. Since A_S satisfies Condition (*EP*), the equality $ac=atc$ implies that there exist $b\in A_S$ and $u,v\in S$ such that $a=bu=bv$ and $uc=vtc$. Then the right cancellability of c implies $u=vt$, and so $a'=at=bvt=bu=a$, as required.

(2). Since Condition (*E*) $\Rightarrow$ Condition (*EP*), it is obvious. □

Proposition 1.3. *Let S be a monoid. Then:*

(1) *All P-regular right S-acts are* $\Re$ *-torsion free.*

(2) *All strongly (P)-cyclic right S-acts are* $\Re$ *-torsion free.*

(3) *All regular right S-acts are* $\Re$ *-torsion free.*

(4) *All strongly faithful right S-acts are* $\Re$ *-torsion free.*

Proof. (1). It follows from [9, Theorem 2.2] and using the same argument as in the proof of (1) of Proposition 1.2.

Since strong faithfulness $\Rightarrow$ regularity $\Rightarrow$ strong (*P*)-cyclic $\Rightarrow$ *P*-regularity, (2), (3) and (4) are obvious. □

Notice that it is not yet known if the faithfulness implies $\Re$ -torsion freeness.

2. Characterization by $\Re$ *-torsion freeness of right Rees factor acts*

In this section we characterize monoids by $\Re$ -torsion freeness of right Rees factor acts. We recall that if K_S is a right ideal of *S*, the Rees congruence ρ_K is defined by $(a,b)\in\rho_K$ if $a,b\in K$ or $a=b$ and the resulting factor act is called the Rees factor act and is denoted by S/K_S. We say an ideal K_S of *S* satisfies Condition (*), if $xc,yc\in K_S$ and $x\Re y$, $x,y\in S\setminus K_S$, $c\in S$ right cancellable, imply $x=y$.

Lemma 2.1. *Let S be a monoid and* K_S *be a right ideal of S. Then:*

(1) $[x]_{\rho_K}\Re[y]_{\rho_K}$ *implies either* $x,y\in S\setminus K_S$ *or* $x,y\in K_S$, *for all* $x,y\in S$.

(2) $x\Re y$ *implies* $[x]_{\rho_K}\Re[y]_{\rho_K}$, *for all* $x,y\in S$.

(3) $x\Re y$ *if and only if* $[x]_{\rho_K}\Re[y]_{\rho_K}$, *for all* $x,y\in S\setminus K_S$.

Proof. (1). If $[x]_{\rho_K}\Re[y]_{\rho_K}$, then there exist $s,t\in S$ such that $[x]_{\rho_K}=[y]_{\rho_K}s=[ys]_{\rho_K}$ and $[y]_{\rho_K}=[x]_{\rho_K}t=[xt]_{\rho_K}$. Thus either $x=ys$ or $x,ys\in K_S$ and either $y=xt$ or $y,xt\in K_S$. If $x\notin K_S$, then $x=ys$, and so $y\notin K_S$. If $x\in K_S$, then $y\in K_S$, since $y=xt$ or $y,xt\in K_S$. Thus $x\in K_S$ if and only if $y\in K_S$.

(2). It is obvious.

(3). Let $x,y\in S\setminus K_S$. If $x\Re y$, then $[x]_{\rho_K}\Re[y]_{\rho_K}$. If $[x]_{\rho_K}\Re[y]_{\rho_K}$, then there exist $s,t\in S$ such that either $x=ys$ or $x,ys\in K_S$ and either $y = xt$ or $y, xt \in K_S$. Since $x, y \in S \setminus K_S$, we have $x = ys$ and $y = xt$, by (1), and so $x\Re y$. □

Theorem 2.1. *Let S be a monoid and K_S be a right ideal of S. Then the right Rees factor S-act S/K_S is $\Re$-torsion free if and only if K_S satisfies Condition (*).*

Proof. Necessity. Suppose the right Rees factor *S*-act S/K_S is $\Re$-torsion free, and let $xc, yc \in K_S$, $x\Re y$, for $x, y \in S \setminus K_S$, $c \in S$ right cancellable. Then $[x]_{\rho_K} c = [y]_{\rho_K} c$ and $[x]_{\rho_K} \Re [y]_{\rho_K}$, by (2) of Lemma 2.1. Hence, $[x]_{\rho_K} = [y]_{\rho_K}$, and so $x = y$ or $x, y \in K_S$. But $x, y \in S \setminus K_S$, and so $x = y$, as required.

Sufficiency. Suppose $[x]_{\rho_K} c = [y]_{\rho_K} c$ and $[x]_{\rho_K} \Re [y]_{\rho_K}$, for $x, y \in S$, $c \in S$ right cancellable. Then $xc = yc$ or $xc, yc \in K_S$. If $xc = yc$, then $x = y$, and so $[x]_{\rho_K} = [y]_{\rho_K}$, as required. Thus we suppose $xc, yc \in K_S$. Since $[x]_{\rho_K} \Re [y]_{\rho_K}$, either $x, y \in K_S$ or $x, y \in S \setminus K_S$, by (1) of Lemma 2.1. If $x, y \in K_S$, then $[x]_{\rho_K} = [y]_{\rho_K}$, as required. If $x, y \in S \setminus K_S$, then $x\Re y$, by (3) of Lemma 2.1. Thus by the assumption $x = y$, and so $[x]_{\rho_K} = [y]_{\rho_K}$, as required. □

Remark 2.1. If K_S is a left annihilating right ideal of a monoid *S*, then K_S satisfies Condition (*), but not the converse, otherwise, by Theorem 2.1, [12, III, 10.11] and that principal weak flatness implies $\Re$-torsion freeness, all left stabilizing right ideals are left annihilating, and so by [14, Theorem 10], all principally weakly flat right Rees factor *S*-acts satisfy Condition (*PWP*), which is not true. By [6, Lemma 3.4], all P_E-left annihilating right ideals are left stabilizing, thus every P_E-left annihilating right ideal satisfies Condition (*), but not the converse, otherwise, all torsion free right Rees factor *S*-acts are principally weakly flat, which is not true.

The following example shows that there are monoids *S* and right Rees factor *S*-acts which are not $\Re$-torsion free.

Example 2.1. Let $S = \mathbf{Z}[i] = \{a + bi \mid a, b \in \mathbf{Z}\}$. Then *S* with multiplication is a commutative and cancellative monoid. If $K_S = (1+i)S = \{a + bi \mid a, b \in \mathbf{Z}, 2 \mid a + b\}$, then $5, -5 \in S \setminus K_S$, $-5 \times 2 = -10 \in K_S$, $5 \times 2 = 10 \in K_S$, and $5\Re -5$, but $5 \neq -5$, thus the right Rees factor *S*-act S/K_S is not $\Re$-torsion free, by Theorem 2.1.

As we saw in Example 1.1, the following example shows also that for Rees factor acts, $\Re$-torsion freeness does not imply torsion freeness.

Example 2.2. Let $S = (\mathbf{N}, \cdot)$. If $K_S = 2S$, then S/K_S is not torsion free, but it is $\Re$-torsion free. Thus for Rees factor acts $\Re$-torsion freeness does not imply torsion freeness and all properties which imply torsion freeness.

Now, it is natural to ask for monoids over which $\Re$-torsion freeness of Rees factor acts implies torsion freeness and all properties which imply torsion freeness.

Theorem 2.2. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free right Rees factor S-acts are torsion free.*

(2) *If a proper right ideal K_S of S satisfies Condition (*), then K_S satisfies the following condition:*

$xc \in K_S$, $x, c \in S$, c right cancellable, implies $x \in K_S$.

Proof. It follows from Theorem 2.1, and [12, III, 8.10]. □

Theorem 2.3. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free right Rees factor S-acts are principally weakly flat.*

(2) *If a proper right ideal K_S of S satisfies Condition (*), then K_S is left stabilizing.*

Proof. It follows from Theorem 2.1, and [12, III, 10.11]. □

Theorem 2.4. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free right Rees factor S-acts*

satisfy Condition (PWP).

(2) *If a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is left stabilizing and left annihilating.*

Proof. It follows from Theorem 2.1, and [14, Theorem 10]. □

Theorem 2.5. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (* PWP_E *).*

(2) *If a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is left stabilizing and E-left annihilating.*

Proof. It follows from Theorem 2.1, and [7, Theorem 4.2]. □

Theorem 2.6. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are flat.*

(2) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are weakly flat.*

(3) *S is right reversible and if a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is left stabilizing.*

Proof. It follows from Theorem 2.1, [12, III, 12.2], and [12, III, 12.17]. □

Theorem 2.7. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (WP).*

(2) *S is right reversible and if a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is left stabilizing and strongly left annihilating.*

Proof. It follows from Theorem 2.1, [14, Theorem 17], and [14, Corollary 18]. □

Theorem 2.8. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (P).*

(2) *S is right reversible and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S|=1$.

Proof. It follows from Theorem 2.1, [12, III, 13.7], and [12, III, 13.9]. □

Theorem 2.9. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (* P_E *).*

(2) *S is right reversible and if a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is* P_E *-left annihilating.*

Proof. It follows from Theorem 2.1, and [6, Theorem 3.5]. □

Theorem 2.10. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (P').*

(2) *S is weakly right reversible and if a proper right ideal* K_S *of S satisfies Condition (*), then* K_S *is left stabilizing and completely left annihilating.*

Proof. It follows from Theorem 2.1, and [10, Theorem 4.3]. □

Theorem 2.11. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts satisfy Condition (E).*

(2) *S is left collapsible and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S|=1$.

Proof. It follows from Theorem 2.1, [12, III, 14.3], and [12, III, 14.10]. □

Theorem 2.12. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are projective.*

(2) *S contains a left zero and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S|=1$.

Proof. It follows from Theorem 2.1, [12, III, 17.2], and [12, III, 17.15]. □

Theorem 2.13. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are free.*

(2) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are projective generators.*

(3) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are generators.*

(4) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are faithful.*

(5) *All* $\mathfrak{R}$ *-torsion free right Rees factor S-acts are*

strongly faithful.

(6) $S = \{1\}$.

Proof. Implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (4). It follows from [12, III, 18.1].

Since $\Theta_S \cong S / S_S$ is an $\Re$-torsion free cyclic right Rees factor *S*-act, and Θ_S is faithful (strongly faithful) if and only if $S = \{1\}$, implications (4) $\Rightarrow$ (6) and (5) $\Rightarrow$ (6) are obvious.

(6) $\Rightarrow$ (1), (5). If $S = \{1\}$, then all right *S*-acts are free (strongly faithful). □

Theorem 2.14. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free right Rees factor S-acts are P-regular.*

(2) *S is right reversible, if S contains a left zero, then S is right PCP, and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S| = 1$.

Proof. It follows from Theorem 2.1, and [9, Theorem 3.1]. □

Theorem 2.15. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free right Rees factor S-acts are strongly (P)-cyclic.*

(2) *S is right PCP, contains a left zero and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S| = 1$.

Proof. It follows from Theorem 2.1, and [8, Theorem 3.1]. □

Theorem 2.16. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free right Rees factor S-acts are are regular.*

(2) *S is right PP, contains a left zero and if a proper right ideal* K_S *of S satisfies Condition (*), then* $|K_S| = 1$.

Proof. It follows from Theorem 2.1, [12, III, 19.4], and [12, III, 19.6] . □

3. Characterization by $\Re$ ***-torsion freeness of cyclic right acts***

In this section we characterize monoids by $\Re$-torsion freeness of cyclic right acts.

Let *S* be a monoid, $s, t \in S$ and C_r be the set of all right cancellable elements of *S*. Set

$$F_1 = \{(x, y) \in S \times S \mid \exists c \in C_r, (xc, yc) \in \rho(s,t), [x]_{\rho(s,t)} \Re [y]_{\rho(s,t)}\},$$

$$F_{i+1} = \{(x, y) \in S \times S \mid \exists c \in C_r, (xc, yc) \in \rho(F_i), [x]_{\rho(F_i)} \Re [y]_{\rho(F_i)}\}$$

for $i \in \mathbf{N}$. It can easily be seen that F_i is reflexive and symmetric, for every $i \in \mathbf{N}$. Also,

$$\rho(s,t) \subseteq F_1 \subseteq \rho(F_1) \subseteq F_2 \subseteq \rho(F_2) \subseteq .....\rho(F_i) \subseteq F_{i+1} \subseteq .....$$

It is clear that $\rho_{\Re TF}(s,t) = \bigcup_{i \in \mathbf{N}} \rho(F_i)$ is a right congruence on *S* containing (*s*,*t*).

Theorem 3.1. *Let S be a monoid and* $s, t \in S$. *Then* $\rho_{\Re TF}(s,t)$ *is the smallest right congruence containing (s,t), where* $S / \rho_{\Re TF}(s,t)$ *is* $\Re$*-torsion free.*

Proof. If $[x]_{\rho_{\Re TF}(s,t)} c = [y]_{\rho_{\Re TF}(s,t)} c$ and $[x]_{\rho_{\Re TF}(s,t)} \Re [y]_{\rho_{\Re TF}(s,t)}$, for $x, y \in S$ and $c \in C_r$, then there exist $l_1, l_2 \in S$ such that $(x, yl_1), (y, xl_2) \in \rho_{\Re TF}(s,t)$. Thus there exist $i, j, k \in \mathbf{N}$ such that $(xc, yc) \in \rho\ (F_i)$, $(x, yl_1) \in \rho(F_j)$ and $(y, xl_2) \in \rho(F_k)$.

If $h = \max\{i, j, k\}$, then $(xc, yc), (x, yl_1), (y, xl_2) \in \rho(F_h)$, and so $(xc, yc) \in \rho(F_h)$ and $[x]_{\rho(F_h)} \Re [y]_{\rho(F_h)}$. By definition, $(x, y) \in F_{h+1}$, and so $(x, y) \in \rho(F_{h+1}) \subseteq \rho_{\Re TF}(s,t)$.

Thus $[x]_{\rho_{\Re TF}(s,t)} = [y]_{\rho_{\Re TF}(s,t)}$, as required. Let τ be a right congruence on *S* containing (*s*,*t*), where S / τ is $\Re$-torsion free. We show that $\rho_{\Re TF}(s,t) \subseteq \tau$. Since $(s,t) \in \tau$, we have $\rho(s,t) \subseteq \tau$. If $(x, y) \in F_1$, then there exists $c \in C_r$ such that $(xc, yc) \in \rho(s,t)$ and $[x]_{\rho\ (s,t)} \Re [y]_{\rho(s,t)}$, and so $(xc, yc) \in \tau$ and $[x]_\tau \Re [y]_\tau$. Since S / τ is $\Re$-

torsion free, $(x,y)\in\tau$. Thus $F_1\subseteq\tau$, and so $\rho(F_1)\subseteq\tau$. Suppose then that $\rho(F_i)\subseteq\tau$, $i\in\mathbf{N}$. If $(x,y)\in F_{i+1}$, then there exists $c\in C_r$ such that $(xc,yc)\in\rho(F_i)$ and $[x]_{\rho(F_i)}\Re[y]_{\rho(F_i)}$. Since $\rho(F_i)\subseteq\tau$ and S/τ is $\Re$-torsion free, $(x,y)\in\tau$. Hence $F_{i+1}\subseteq\tau$, and so $\rho(F_{i+1})\subseteq\tau$. Thus $\rho(F_i)\subseteq\tau$, for all $i\in\mathbf{N}$, and so $\rho_{\Re TF}(s,t)\subseteq\tau$. □

Theorem 3.2. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (P).*

(2) *For any $t,t'\in S$, there exist $u,v\in S$ such that $ut=vt'$ and $(u,1),(v,1)\in\rho_{\Re TF}(t,t')$.*

(3) *For any $s,t,t'\in S$, there exist $u,v\in S$ such that $ut=vt'$ and $(u,s),(v,s)\in\rho_{\Re TF}(st,st')$.*

Proof. (1) $\Rightarrow$ (2). The cyclic right *S*-act $S/\rho_{\Re TF}(t,t')$ is $\Re$-torsion free, and so it satisfies Condition (*P*). Thus by [12, III, 13.4], there exist $u,v\in S$ such that $ut=vt'$ and $(u,1),(v,1)\in\rho_{\Re TF}(t,t')$.

(2) $\Rightarrow$ (3). Suppose $s,t,t'\in S$. Then there exist $u',v'\in S$ such that $u'st=v'st'$ and $(u',1),(v',1)\in\rho_{\Re TF}(st,st')$. If $u:=u's$ and $v:=v's$, then $ut=vt'$ and $(u,s),(v,s)\in\rho_{\Re TF}(st,st')$.

(3) $\Rightarrow$ (1). Suppose τ is a right congruence on *S*, where S/τ is $\Re$-torsion free and let $(t,t')\in\tau$. Then by assumption, there exist $u,v\in S$ such that $ut=vt'$ and $(u,1),(v,1)\in\rho_{\Re TF}(t,t')$. By Theorem 3.1, $\rho_{\Re TF}(t,t')\subseteq\tau$, and so $(u,1),(v,1)\in\tau$. Thus S/τ satisfies Condition (*P*), by [12, III, 13.4]. □

Theorem 3.3. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (P_E).*

(2) *For any $x,y,t,t'\in S$, there exist $u,v\in S$ and $e,f\in E(S)$ such that $ut=vt'$, $et=t$, $ft'=t'$, $(xe,ue),(yf,vf)\in\rho_{\Re TF}(xt,yt')$.*

Proof. Using [6, Theorem 2.5] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.4. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (P').*

(2) *For any $x,y,t,t',z\in S$, the equality $tz=t'z$ implies that there exist $u,v\in S$ such that $ut=vt'$, $(x,u),(y,v)\in\rho_{\Re TF}(xt,yt')$.*

Proof. Using [10, Theorem 3.1] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.5. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (E).*

(2) *For any $s,t\in S$, there exists $u\in S$ such that $ut=us$ and $(u,1)\in\rho_{\Re TF}(s,t)$.*

Proof. Using [12, III, 14.8] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.6. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (E').*

(2) *For any $s,t,z\in S$, the equality $tz=sz$ implies that there exists $u\in S$ such that $ut=us$ and $(u,1)\in\rho_{\Re TF}(s,t)$.*

Proof. It follows from Theorem 3.1, definition of Condition (*E'*) and using the same argument as in the proof of Theorem 3.2. □

Theorem 3.7. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All $\Re$-torsion free cyclic right S-acts satisfy Condition (E'P).*

(2) *For any $x,y,z\in S$, the equality $xz=yz$ implies that there exist $u,v\in S$ such that $ux=vy$ and $(u,1),(v,1)\in\rho_{\Re TF}(x,y)$.*

(3) *For any $x,t,t',z\in S$, the equality $tz=t'z$ implies that there exist $u,v\in S$ such that $ut=vt'$ and $(u,x),(v,x)\in\rho_{\Re TF}(xt,xt')$.*

Proof. Using [3, Theorem 2.10] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.8. *Let S be a monoid. If all $\Re$-torsion free cyclic right S-acts are flat, then for any left* *congruence* λ *on S and any* $s,t \in S$, *there exist* $u,v \in S$ *such that* $(us,vt) \in \lambda$, $(u,1) \in \rho_{\Re TF}(s,t) \vee s\lambda$ *and* $(v,1) \in \rho_{\Re TF}(s,t) \vee t\lambda$.

Proof. Suppose λ is a left congruence on *S* and let $s,t \in S$. Then the cyclic right *S*-act $S/\rho_{\Re TF}(s,t)$ is $\Re$-torsion free, and so it is flat. Thus by [12, III, 12.11], there exist $u,v \in S$ such that $(us,vt) \in \lambda$, $(u,1) \in \rho_{\Re TF}(s,t) \vee s\lambda$ and $(v,1) \in \rho_{\Re TF}(s,t) \vee t\lambda$. □

Theorem 3.9. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free cyclic right S-acts are weakly flat.*

(2) *For any* $s,t \in S$, *there exist* $u,v \in S$ *such that* $us = vt$, $(u,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_s$ *and* $(v,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_t$.

Proof. (1) $\Rightarrow$ (2). The cyclic right *S*-act $S/\rho_{\Re TF}(s,t)$ is $\Re$-torsion free, and so it is weakly flat. Thus by [12, III, 11.5], there exist $u,v \in S$ such that $us = vt$, $(u,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_s$ and $(v,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_t$.

(2) $\Rightarrow$ (1). Suppose τ is a right congruence on *S*, where S/τ is $\Re$-torsion free and let $(s,t) \in \tau$. By Theorem 3.1, $\rho_{\Re TF}(s,t) \subseteq \tau$ and by assumption, there exist $u,v \in S$ such that $us = vt$, $(u,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_s$ and $(v,1) \in \rho_{\Re TF}(s,t) \vee \ker \rho_t$. Thus $(u,1) \in \tau \vee \ker \rho_s$ and $(v,1) \in \tau \vee \ker \rho_t$, and so S/τ is weakly flat, by [12, III, 11.5]. □

Theorem 3.10. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free cyclic right S-acts satisfy Condition (PWP).*

(2) *For any* $x,y,t \in S$, *there exist* $u,v \in S$ *such that* $ut = vt$ *and* $(u,x),(v,y) \in \rho_{\Re TF}(xt,yt)$.

Proof. Using [13, Lemma 2.7] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.11. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free cyclic right S-acts satisfy Condition (*PWP_E*).*

(2) *For any* $x,y,t \in S$, *there exist* $u,v \in S$ *and* $e,f \in E(S)$ *such that* $ut = vt$ *and* $(ue,xe),(vf,yf) \in \rho_{\Re TF}(xt,yt)$.

Proof. Using [7, Theorem 3.7] and Theorem 3.1, it is similar to the proof of Theorem 3.2. □

Theorem 3.12. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free cyclic right S-acts are principally weakly flat.*

(2) *For any* $u,v,s \in S$, $(u,v) \in \rho_{\Re TF}(us,vs) \vee \ker \rho_s$.

Proof. (1) $\Rightarrow$ (2). Suppose $u,v,s \in S$. The cyclic right *S*-act $S/\rho_{\Re TF}(us,vs)$ is $\Re$-torsion free, and so it is principally weakly flat. Since $(us,vs) \in \rho_{\Re TF}(us,vs)$ we have $(u,v) \in \rho_{\Re TF}(us,vs) \vee \ker \rho_s$, by [12, III, 10.7].

(2) $\Rightarrow$ (1). Suppose τ is a right congruence on *S*, where S/τ is $\Re$-torsion free and let $(us,vs) \in \tau$. Then by Theorem 3.1, $\rho_{\Re TF}(us,vs) \subseteq \tau$. By assumption, $(u,v) \in \rho_{\Re TF}(us,vs) \vee \ker \rho_s$, and so $(u,v) \in \tau \vee \ker \rho_s$. Thus S/τ is principally weakly flat, by [12, III, 10.7]. □

Theorem 3.13. *Let S be a monoid. Then:*

(1) $\rho_{\Re TF}(s,t) \subseteq \rho_{TF}(s,t)$.

(2) *All* $\Re$*-torsion free cyclic right S-acts are torsion free if and only if* $\rho_{\Re TF}(s,t) = \rho_{TF}(s,t)$.

Proof. (1) . $\rho_{TF}(s,t)$ is the right congruence containing (s,t), where $S/\rho_{TF}(s,t)$ is torsion free. Thus by Theorem 3.1, $\rho_{\Re TF}(s,t) \subseteq \rho_{TF}(s,t)$, since torsion freeness implies $\Re$-torsion freeness.

(2). Using [12, III, 8.4], Theorem 3.1, and (1), it is similar to the proof of Theorem 3.2. □

Theorem 3.14. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$*-torsion free cyclic right S-acts are free.*

(2) *All* $\Re$*-torsion free cyclic right S-acts are are projective generators.*

(3) *All* $\Re$*-torsion free cyclic right S-acts are generators.*

(4) *All* $\mathfrak{R}$ *-torsion free cyclic right S-acts are faithful.*

(5) *All* $\mathfrak{R}$ *-torsion free cyclic right S-acts are strongly faithful.*

(6) $S = \{1\}$.

Proof. It follow from Theorem 2.13. □

4. Characterization by $\mathfrak{R}$ ***-torsion freeness of right acts***

In this section we characterize monoids by $\mathfrak{R}$ -torsion freeness of right acts.

Lemma 4.1. *Let S be a monoid and (U) be a property of S-acts which implies torsion freeness. Then the following statements are equivalent:*

(1) *All right S-acts satisfy (U).*

(2) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy (U).*

Proof. (1) $\Rightarrow$ (2). It is obvious.

(2) $\Rightarrow$ (1). We claim that $cS = S$, for any right cancellable $c \in S$. Otherwise, $cS \neq S$, for some right cancellable $c \in S$. Then the right *S*-act $S_S \coprod^{cS} S_S$ satisfies Condition (*E*), and so by (2) of Proposition 1.2, it is $\mathfrak{R}$ -torsion free. Thus by assumption, $S_S \coprod^{cS} S_S$ is torsion free, and so the equality $(1,x)c = (1,y)c$, implies $(1,x) = (1,y)$, which is a contradiction. Thus $cS = S$, and so all right cancellable elements of *S* are right invertible. Thus all right *S*-acts are torsion free, by [12, IV, 6.1], and so all right *S*-acts satisfy (*U*), as required. □

Theorem 4.1. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right S-acts are free.*

(2) *All* $\mathfrak{R}$ *-torsion free right S-acts are projective generators.*

(3) *All* $\mathfrak{R}$ *-torsion free right S-acts are projective.*

(4) *All* $\mathfrak{R}$ *-torsion free right S-acts are strongly flat.*

(5) *All* $\mathfrak{R}$ *-torsion free right S-acts are generators.*

(6) *All* $\mathfrak{R}$ *-torsion free right S-acts are faithful.*

(7) *All* $\mathfrak{R}$ *-torsion free right S-acts are strongly faithful.*

(8) $S = \{1\}$.

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) and (8) $\Rightarrow$ (1) are obvious.

(4) $\Rightarrow$ (8). Since strong flatness and pullback flatness coincide, it follows from Lemma 4.1 and [15, Theorem 3.4].

(5) $\Leftrightarrow$ (6) $\Leftrightarrow$ (7) $\Leftrightarrow$ (8). The same argument can be used as in the proof of Theorem 2.13. □

Theorem 4.2. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right S-acts are weakly pullback flat.*

(2) *All* $\mathfrak{R}$ *-torsion free right S-acts are weakly kernel flat.*

(3) *All* $\mathfrak{R}$ *-torsion free right S-acts are principally weakly kernel flat.*

(4) *All* $\mathfrak{R}$ *-torsion free right S-acts are translation kernel flat.*

(5) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy Condition (P).*

(6) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy Condition (WP).*

(7) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy Condition (PWP).*

(8) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy Condition (P').*

(9) *S is a group.*

Proof. Implications (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5) $\Leftrightarrow$ (6) $\Leftrightarrow$ (7) $\Leftrightarrow$ (9) follow from Lemma 4.1, and [1, Proposition 9].

(8) $\Leftrightarrow$ (9). It follows from Lemma 4.1, and [10, Theorem 2.5]. □

Theorem 4.3. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All right S-acts are flat.*

(2) *All* $\mathfrak{R}$ *-torsion free right S-acts are flat.*

Proof. Since flatness implies torsion freeness, it follow from Lemma 4.1. □

Theorem 4.4. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\mathfrak{R}$ *-torsion free right S-acts satisfy Condition (*P_E*).*

(2) *All* $\mathfrak{R}$ *-torsion free right S-acts are weakly flat.*

(3) *S is regular and satisfies Condition: (R): for all* $s,t \in S$ *there exists* $w \in Ss \cap St$ *such that* $(w,s) \in \rho(s,t)$.

Proof. (1) $\Rightarrow$ (2). It follows from [2, Theorem 2.3].

(2) $\Rightarrow$ (3). It follows from Lemma 4.1, and [12, IV, 7.5].

(3) $\Rightarrow$ (1). It follows from [6, Theorem 2.1]. □

Theorem 4.5. *Let S be a monoid. Then the following* *statements are equivalent:*

(1) *All* $\Re$ *-torsion free right S-acts are principally weakly flat.*

(2) *All* $\Re$ *-torsion free right S-acts satisfy Condition* (PWP_E).

(3) *S is regular.*

Proof. (1) $\Leftrightarrow$ (3). It follows from Lemma 4.1, and [12, IV, 6.6].

(2) $\Leftrightarrow$ (3). It follows from Lemma 4.1, and [7, Theorem 3.1]. □

Theorem 4.6. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$ *-torsion free right S-acts are torsion free.*

(2) *Every right cancellable element of S is right invertible.*

Proof. It follows from Lemma 4.1, and [12, IV, 6.1]. □

Theorem 4.7. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$ *-torsion free right S-acts are regular.*

(2) *All* $\Re$ *-torsion free finitely generated right S-acts are regular.*

(3) *All* $\Re$ *-torsion free cyclic right S-acts are regular.*

(4) *All* $\Re$ *-torsion free monocyclic right S-acts are regular.*

(5) $S = \{1\}$ *or* $S = \{0,1\}$.

Proof. Implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) are obvious.

(4) $\Rightarrow$ (5). It follows from [5, Theorem 1.8].

(5) $\Rightarrow$ (1). It follows from [12, IV, 14.4]. □

Theorem 4.8. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$ *-torsion free right S-acts are divisible.*

(2) *All* $\Re$ *-torsion free finitely generated right S-acts are divisible.*

(3) *All* $\Re$ *-torsion free cyclic right S-acts are divisible.*

(4) S_S *is divisible.*

(5) *Every left cancellabe element of S is left invertible.*

Proof. Implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (4). Since S_S is $\Re$ -torsion free, it is clear.

(4) $\Rightarrow$ (5). It follows from [12, III, 2.2].

(5) $\Rightarrow$ (1). It follows from [12, III, 2.2]. □

Theorem 4.9. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All* $\Re$ *-torsion free right S-acts are principally weakly injective.*

(2) *All* $\Re$ *-torsion free finitely generated right S-acts are principally weakly injective.*

(3) *All* $\Re$ *-torsion free cyclic right S-acts are principally weakly injective.*

(4) *S is regular.*

Proof. Implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (4). All principal right ideals of *S* are $\Re$ -torsion free, by (2) and (3) of Proposition 1.1. Thus all principal right ideals of *S* are principally weakly injective, and so *S* is regular, by [12, IV, 1.6].

(4) $\Rightarrow$ (1). By [12, IV, 1.6], it is obvious. □

It is not yet known that when all right (Rees factor, cyclic) acts are $\Re$ -torsion free, but here we give some equivalents of that.

Theorem 4.10. *Let S be a monoid. Then the following statements are equivalent:*

(1) *All right S-acts are* $\Re$ *-torsion free.*

(2) *All divisible right S-acts are* $\Re$ *-torsion free.*

(3) *All principally weakly injective right S-acts are* $\Re$ *-torsion free.*

(4) *All fg-weakly injective right S-acts are* $\Re$ *-torsion free.*

(5) *All weakly injective right S-acts are* $\Re$ *-torsion free.*

(6) *All injective right S-acts are* $\Re$ *-torsion free.*

(7) *All cofree right S-acts are* $\Re$ *-torsion free.*

Proof. (1) $\Rightarrow$ (2). It is obvious.

Since cofreeness $\Rightarrow$ injectivity $\Rightarrow$ weak injectivity $\Rightarrow$ fg-weak injectivity $\Rightarrow$ principal weak injectivity $\Rightarrow$ divisibility, implications (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (7) follow.

(7) $\Rightarrow$ (1). Every right *S*-act can be embeded into a cofree right *S*-act. Thus by (3) of Proposition 1.1, all right *S*-acts are $\Re$ -torsion free. □

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$\mathfrak{R}$-torsion free Acts Over Monoids

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سیستم‌های $\mathfrak{R}$ – بدون تابی روی تکواره‌ها

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چکیده

در این مقاله ابتدا تعمیمی از خاصیت بدون تابی سیستم‌ها روی تکواره‌ها را که $\mathfrak{R}$ – بدون تابی نامیده می‌شود معرفی می‌کنیم. سپس در بخش یک از نتایج برخی از خواص عمومی و در بخش‌های دو، سه و چهار به مشخص سازی تکواره‌هائی می‌پردازیم که این خاصیت از سیستم‌های به ترتیب خارج قسمتی ریس، دوری و در حالت کلی آنها دیگر خواص را نتیجه می‌دهد.

واژه‌های کلیدی: $\mathfrak{R}$ – بدون تابی؛ سیستم فاکتور ریس؛ سیستم دوری