JAST, Vol. 10, No. 1, pp. 17- 26

Rotor Sizing of Helicopters Using Statistical Approach

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This paper is concerned with the statistical model development issues, necessary for rapid estimation of the rotor sizing for single main rotor helicopters at the preliminary design stage. However, Central Composite Design (CCD) method, simulation-based data collection, linear regression analysis, mathematical modelsdevelopmentand validations through the analysis of variance (ANOVA) were performed as central themes in this approach. The CCD enforced the use of replicated central points and some star points, added to the basic factorial design space, required for constructing the test plan matrix. This matrixwas used to developed mathematical models in the form of quadratic polynomials (second-order), that represented the physical size of rotor as functions of the helicopter gross weight, maximum forward flight speed, main and tail rotor blade number and their interactions.The validations were examined by ANOVA and comparing against data for a general single rotor configuration. Using this approach, improvements in physical sizing of both main and tail rotor of the single rotor were obtained using minimum number of data, provided by CCD test plan.The obtained results of this work support the ongoing researches for the development of rapid prototyping, especially, main and tail rotor sizing of helicopters.

Keywords: Helicopter, Rotor, Statistical Design, Central Composite Design

INTRODUCTION[1]

In general, physical sizing of the air vehicles is considered as a master part of the design cycle entitled as the preliminary design process.In an overall view, the preliminary design process for all vehicles, both fixed-wing aircraft and helicopters are almost identical, while in helicopters due to dynamical systems such as rotors, drive systems and the irregularly aerodynamic shaped of the fuselage, this process is about more complicated than fixed- wing, and thus it takes longer to modify or to perform a new alternate for design refinements.

Returning to helicopter design literature shows that the design process has been speeded up through the advanced flight dynamics simulation programs such as 2GCHS, CAMRAD, TECH-01 and UMARC [1-8], but to date, this process has been supported definitely by statistical information, collected for helicopters with the similar missions [9]. However, the comprehensive database should be in access for adequately supplying the statistical data for our design problem. Even presuming the presence of such a database, any decision about the purpose of design improvement is rarely possible, so the simple and reliable design trends should be formulated for making decisions quickly and easily about the sizing problem.

The design trend studies based on the population of helicopters have been also shown that the accuracies of the thesetrends are not yet equally distributed across the database, as a resultinformation providedfor new design problems is rarely appropriate [10]. Thisis probably due to a lack of an appropriate data exploration tool for choosingthe desired data from the original database (population). Consequently, these trends consist of only simple fitted curves showing the influence of major parameters with a different level of accuracy across the database. Furthermore, on most trends,becausethe effects of the secondary parameters and their interactions have been neglected, the generated trends cannot be accountedas optimal trendsat the preliminary design stage.

The more recent investigations have been shown that CCD is an efficient methodthan traditional methods such as factorial, fractional factorial and

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Placket-Burmanfor data exploration from an original database[11-14]. This is due to the addition of accessory points such as central and star (axial) points inserted into the factorial designthatresult in a rotatable space which isindependent of any coordinate direction.

In this study, to address and remove the mentioned drawbacks, an optimal space was generated using CCD, then the main, secondary parameters and their interactions weredescribedthrough the generated quadratic polynomials (predicted responses) in which their coefficients were estimated by the linear regression analysis. This subsequently followed by representation of the results and discussions on their validations and improvements.

MATHEMATICAL MODELING

The main assumption for presenting the mathematical models as represented by Equation 1 is quadratic polynomials (responses) that should be chosen before starting data collection from the available database.

$$y = \beta_0 + \sum \beta_i x_i + \sum \beta_{ii} x_i^2 + \sum\sum \beta_{ij} x_i x_j + \varepsilon_i \quad (1)$$

In Equation 1, the x_i terms are the parameters that influence the actual response y, β_i and β_{ij} are regression coefficients. The cross terms $x_i x_j$ and square terms x_i^2 represent two-parameter interaction and second-order nonlinearity, respectively.

Constructing a second-order model requires that kparameters have to be studied at least three levels, so that the regression coefficients in Equation 1 can be necessarily estimated by 3^k factorial experiments. For small values of k such as two or three, this approach works well,however, when many parameters are under study, the number of observations required for a factorial experiment may become excessive. Fortunately, a second-order approximation model can be constructed efficiently by using CCD from design of experiment literature. CCDis the first-order 2^k supported by additional center and star points to allow estimation of the coefficients of the second-order model [11].

CCD offers an efficient alternative to 3^k designs for building second-order polynomials as shown by Equation 1. For example, a problem involving fiveparameters requires only 42 CCD experiments to construct a second-order polynomial as opposed to 243 (3^5) required by a factorial experiment.In a matrix form, Equation 1 can be rewritten as;

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \quad (2)$$

where $\mathbf{y}$ is an $(n \times 1)$ vector of actual responses, $\mathbf{X}$ is an $(n \times p)$ matrix, β is a $(p \times 1)$ vector of regression coefficients (unknowns) and ε is an $(n \times 1)$ vector of random errors. In the case of k parameters the value of p is corresponded to $(1 + 2k + k(k-1)/2)$. Accordingly;

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix}^T \quad (3)$$

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{11}x_{12} & x_{11}^2 & x_{22}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n1}x_{12} & x_{n1}^2 & x_{n2}^2 \end{bmatrix} \quad (4)$$

$$\beta = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 & \beta_{12} & \beta_{11} & \beta_{22} \end{bmatrix}^T \quad (5)$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{bmatrix}^T \quad (6)$$

The regression coefficients are estimatedusing the least square method whenthe norm of therandom errorvector (residual), assuming normal distribution, constant variance and zero mean, is minimized as;

$$\frac{\partial L}{\partial \beta} = 0 \Rightarrow \hat{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \mathbf{X}^T\mathbf{y} \quad (7)$$

where;

$$L = \sum \varepsilon_i^2 = \varepsilon^T\varepsilon = (\mathbf{y} - \mathbf{X}\beta)^T (\mathbf{y} - \mathbf{X}\beta) \quad (8)$$

where $\mathbf{X}^T\mathbf{X}$ is a $(p \times p)$ symmetric matrix, $\mathbf{X}^T\mathbf{y}$ is a $(p \times 1)$ column matrix and $\hat{\beta}$ is the vector of regression coefficients. Thusthe quadratic polynomials or predicted responsescan be expressed as;

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\beta} \quad (9)$$

TEST PROBLEM

In this problem, 4 parameters such as w_0, v_m, N and N_{tr} representing gross weight, maximum forward flight speed, main rotor and tail rotor blade number for examination of 10 responses of single rotor helicopters were selected (Table1). The interval changes of each parameter was considered base on the experience, (i.e., $10^3 \le w_0 \le 10^4$ kg, $200 \le v_m \le 350$ km / hr, $2 \le N \le 6$ and $2 \le N_{tr} \le 4$). On the other hand, All10 responses were chosen such that the physical size of main and tail rotor could be adequately determined. Accordingly, the finalized test plan sheet based on CCD was designed as;

Table 1. Data measurement for CCD test plan matrix.

Parameter				Actual Response									
				Main rotor				Tail rotor					
x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
w_0	v_m	N	N_{tr}	D	c	v_{tip}	Ω	D_{tr}	c_{tr}	v_{tiptr}	$Ω_{tr}$	P_{ava}	T/A
kg	km/h			m	m	m/s	rpm	m	m	m/s	rpm	hp	kg/m^2
-1	+1	+1	-1	10.4	0.27	209	385	1.7	0.18	200	2207	1303	29.4
+1	-1	+1	-1	16.5	0.43	226	262	2.9	0.28	218	1444	2380	39.5
+1	+1	-1	+1	14.4	0.62	221	292	2.5	0.22	213	1614	3070	39.5
+2	0	0	0	16.9	0.57	227	256	3	0.28	220	1394	3505	43.1
-1	-1	-1	-1	11.9	0.39	214	344	2	0.18	205	1975	1010	29.4
+1	-1	-1	+1	16.5	0.62	226	262	2.9	0.22	218	1444	2380	39.5
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-1	+1	-1	-1	10.4	0.39	209	385	1.7	0.18	200	2207	1303	29.4
0	0	0	+2	13.5	0.42	218	309	2.3	0.17	210	1736	1944	35.2
0	+2	0	0	12	0.42	214	341	2.1	0.21	206	1915	2434	35.2
+1	+1	-1	-1	14.4	0.62	221	292	2.5	0.28	213	1614	3070	39.5
-1	+1	+1	+1	10.4	0.27	209	385	1.7	0.14	200	2207	1303	29.4
+1	+1	+1	-1	14.4	0.43	221	292	2.5	0.28	213	1614	3070	39.5
0	0	-2	0	13.5	0.68	218	309	2.3	0.21	210	1736	1944	35.2
0	0	0	0	13.5	0.42	218	309	2.3	0.21	210	1736	1944	35.2
-1	+1	-1	+1	10.4	0.39	209	385	1.7	0.14	200	2207	1303	29.4
0	0	0	-2	13.5	0.42	218	309	2.3	0.27	210	1736	1944	35.2
0	0	0	0	13.5	0.42	218	309	2.3	0.21	210	1736	1944	35.2
0	-2	0	0	15.7	0.42	224	272	2.7	0.21	216	1528	1450	35.2
-1	-1	-1	+1	11.9	0.39	214	344	2	0.14	205	1975	1010	29.4
-1	-1	+1	+1	11.9	0.27	214	344	2	0.14	205	1975	1010	29.4
-2	0	0	0	7.1	0.17	196	529	1.1	0.09	186	3152	362	19.5
0	0	+2	0	13.5	0.31	218	309	2.3	0.21	210	1736	1944	35.2

In this Table, the first four responses were related tothe main rotor and four second responses were used to measure the tail rotor physical size. Moreover, because of the importance of power available $\left(P_{ava}\right)$ and disc loading $\left(T/A\right)$ in design process of helicopters, these responses were also investigated. The actual responses in Table 1 were collected through the open source database, but in cases where the data were unavailable the simulated responses were used instead. As seen in Table 1,all theparameters were coded as;

$$x_i = \frac{x_i - \frac{1}{2}\left(x_h + x_L\right)}{\frac{1}{2}\left(x_h - x_L\right)}, \qquad x_h \le x \le x_L \tag{10}$$

wherethe "h" and "L"subscripts involve the highest and the lowest values of aparameter, respectively. As seen in Table 1, factorialpoints can be altered from the lowest (-1) to the highest level (+1), the value of the star points is set to (±2) and the center points illustrated by zero value in the table. Thus the star pointsdesribe the rotatable property of the test plan sheet (constant standard error variance in any coordinate directions).

The regression coefficients of the quadratic polynomials were ultimately generated as;

Table 2. Coefficients of the quadraticpolynomials.

	Predicted Response									
	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
Intercept	13.48339	0.42	218.44	309.31	2.31	0.21	210.22	1736.09	1943.75	35.15
x_1	2.271876	0.099	6.72	-51.83	0.44	0.045	7.39	-333.69	784.7	5.35
x_2	-0.90742	0	-2.52	17.65	-0.16	0	-2.44	99.33	245.76	0
x_3	0	-0.082	0	0	0	0	0	0	0	0
x_4	0	0	0	0	0	-0.025	0	0	0	0
x_1x_2	-0.14576	0	-0.07	-2.43	-0.029	0	-0.078	-15.59	99.24	0
x_1x_3	0	-0.018	0	0	0	0	0	0	0	0
x_1x_4	0	0	0	0	0	-5.33E-03	0	0	0	0
x_2x_3	0	0	0	0	0	0	0	0	0	0
x_2x_4	0	0	0	0	0	0	0	0	0	0
x_3x_4	0	0	0	0	0	0	0	0	0	0
x_1^2	-0.35956	-0.012	-1.69	19.38	-0.059	-5.62E-03	-1.74	124.8	-2.56	-0.92
x_2^2	0.107214	-2.41E-04	0.28	-2.12	0.018	1.44E-04	0.27	-13.13	-0.34	0.041
x_3^2	0.014189	0.02	0.094	-1.47	2.23E-03	1.44E-04	0.095	-9.47	0.051	0.041
x_4^2	0.014189	-2.41E-04	0.094	-1.47	2.23E-03	4.04E-03	0.095	-9.47	0.051	0.041

The sensitivity of the all predicted responses to each parameter was investigated bydrawing the perturbation plots about the selected arbitrary reference pointshown typically in Fig. 1 through (4).A steep slope and the slight curvature in a parameter showed that the predicted response is sensitive to that parameter. As seen in Figures, the size of rotors significantly depends on gross weight and maximum flight speed, denoted by A and B, respectively. As a result, the predicted responses were transferred into the reduced and simple forms as;

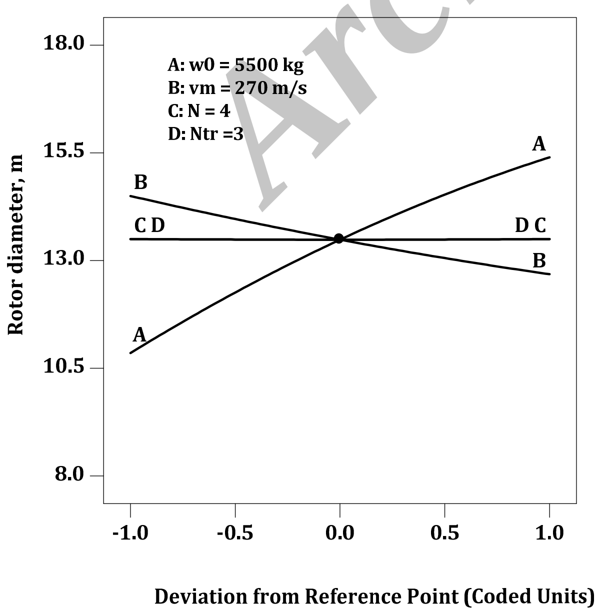

Figure 1. Sensitivity of main rotor diameter to 4 parameters.

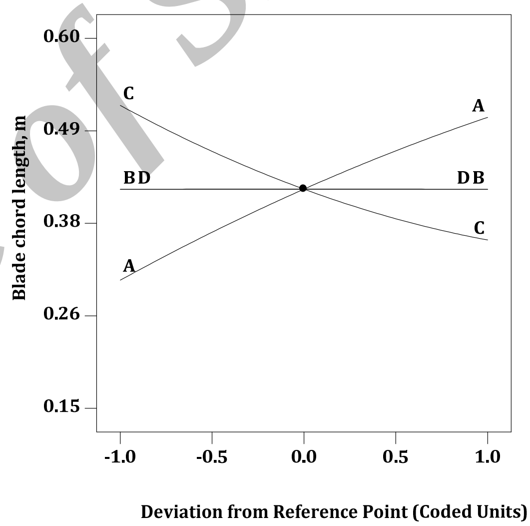

Figure 2. Sensitivity of blade chord length to 4 parameters.

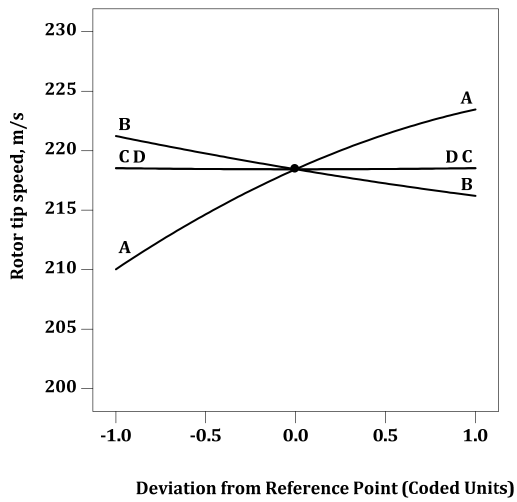

Figure 3. Sensitivity of main rotor tip speed to 4 parameters

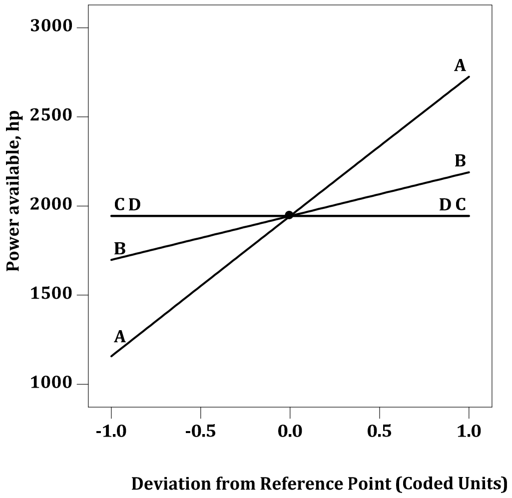

Figure 4. Sensitivity of power available to 4 parameters.

$$\mathbf{D} = +13.64 + 2.27\mathbf{x}_1 - 0.91\mathbf{x}_2 - 0.15\mathbf{x}_1\mathbf{x}_2 - 0.39\mathbf{x}_1^2 \quad (11)$$

$$\mathbf{c} = +0.42 + 0.099\mathbf{x}_1 - 0.082\mathbf{x}_3 - 0.018\mathbf{x}_1\mathbf{x}_3 - 0.012\mathbf{x}_1^2 + 0.020\mathbf{x}_3^2 \quad (12)$$

$$\mathbf{v}_{\mathbf{tip}} = +218.97 + 6.72\mathbf{x}_1 - 2.52\mathbf{x}_2 - 1.80\mathbf{x}_1^2 \quad (13)$$

$$\Omega = +303.53 - 51.83\mathbf{x}_1 + 17.65\mathbf{x}_2 + 20.58\mathbf{x}_1^2 \quad (14)$$

$$\mathbf{D}_{\mathbf{tr}} = +2.32 + 0.44\mathbf{x}_1 - 0.16\mathbf{x}_2 - 0.029\mathbf{x}_1\mathbf{x}_2 - 0.060\mathbf{x}_1^2 + 0.017\mathbf{x}_2^2 \quad (15)$$

$$\mathbf{c}_{\mathbf{tr}} = +0.21 + 0.045\mathbf{x}_1 - 0.025\mathbf{x}_4 - 5.33\times 10^{-3}\mathbf{x}_1\mathbf{x}_4 - 5.714\times10^{-3}\mathbf{x}_1^2 + 3.95\times10^{-3}\mathbf{x}_4^2 \quad (16)$$

$$\mathbf{v}_{\mathbf{tiptr}} = +210.75 + 7.39\mathbf{x}_1 - 2.44\mathbf{x}_2 - 1.85\mathbf{x}_1^2 \quad (17)$$

$$\Omega_{\mathbf{tr}} = +1699.43 - 333.69\mathbf{x}_1 + 99.33\mathbf{x}_2 + 132.43\mathbf{x}_1^2 \quad (18)$$

$$\mathbf{P}_{\mathbf{ava}} = +1943.48 + 784.70\mathbf{x}_1 + 245.76\mathbf{x}_2 + 99.24\mathbf{x}_1\mathbf{x}_2 - 2.51\mathbf{x}_1^2 \quad (19)$$

$$\mathbf{T} / \mathbf{A} = +35.29 + 5.35\mathbf{x}_1 - 0.95\mathbf{x}_1^2 \quad (20)$$

The predicted responses in terms of actual parameters were therefore reducedto;

$$\mathbf{D} = 9.9 + 2.36\times10^{-3}\mathbf{w}_0 - 0.016\mathbf{v}_{\mathbf{m}} - 1.85\times10^{-6}\mathbf{w}_0\mathbf{v}_{\mathbf{m}} - 7.74\times10^{-8}\mathbf{w}_0{}^2 \quad (21)$$

$$\mathbf{c} = 0.58 + 10^{-4}\mathbf{w}_0 - 0.2\mathbf{N} - 7.81\times 10^{-6}\mathbf{w}_0\mathbf{N} - 2.28\times10^{-9}\mathbf{w}_0{}^2 + 0.02\mathbf{N}^2 \quad (22)$$

$$\mathbf{v}_{\mathbf{tip}} = 211 + 6.89\times10^{-3}\mathbf{w}_0 - 0.07\mathbf{v}_{\mathbf{m}} - 3.55\times10^{-7}\mathbf{w}_0{}^2 \quad (23)$$

$$\Omega = 417 - 0.07\mathbf{w}_0 + 0.5\mathbf{v}_{\mathbf{m}} + 4.06\times10^{-6}\mathbf{w}_0{}^2 \quad (24)$$

$$\mathbf{D}_{\mathbf{tr}} = 2.54 + 4.24\times10^{-4}\mathbf{w}_0 - 9.85\times10^{-3}\mathbf{v}_{\mathbf{m}} - 3.62\times10^{-7}\mathbf{w}_0\mathbf{v}_{\mathbf{m}} - 1.18\times10^{-7}\mathbf{w}_0{}^2 + 1.37\times10^{-5}\mathbf{v}_{\mathbf{m}}{}^2 \quad (25)$$

$$\mathbf{c}_{\mathbf{tr}} = 0.27 + 4.68\times10^{-5}\mathbf{w}_0 - 0.12\mathbf{N}_{\mathbf{tr}} - 4.74\times10^{-6}\mathbf{w}_0\mathbf{N}_{\mathbf{tr}} - 1.13\times10^{-9}\mathbf{w}_0{}^2 + 0.016\mathbf{N}_{\mathbf{tr}}{}^2 \quad (26)$$

$$\mathbf{v}_{\mathbf{tiptr}} = 200 + 7.31\times10^{-3}\mathbf{w}_0 - 0.07\mathbf{v}_{\mathbf{m}} - 3.66\times10^{-7}\mathbf{w}_0{}^2 \quad (27)$$

$$\Omega_{\mathbf{tr}} = 2540 - 0.43\mathbf{w}_0 + 2.84\mathbf{v}_{\mathbf{m}} + 2.61\times10^{-5}\mathbf{w}_0{}^2 \quad (28)$$

$$\mathbf{P}_{\mathbf{ava}} = -14 + 0.01\mathbf{w}_0 + 0.09\mathbf{v}_{\mathbf{m}} + 1.26\times10^{-3}\mathbf{w}_0\mathbf{v}_{\mathbf{m}} - 4.95\times10^{-7}\mathbf{w}_0{}^2 \quad (29)$$

$$\mathbf{T} / \mathbf{A} = 16.5 + 4.45\times10^{-3}\mathbf{w}_0 - 1.88\times10^{-7}\mathbf{w}_0{}^2 \quad (30)$$

MODEL VALIDATION STUDY

The accuracy of the predicted responseswas the question of interest that answered through the calculationof model sum of square (SSR), residual sum of square (SSE), F-value, and R-squared estimationin this section. From the statistical point of view, it can be shown that;

$$\mathrm{SSR} = \sum(\hat{y}_i - \bar{y})^2 = \hat{\beta}^T\mathbf{X}^T\mathbf{y} - n(\bar{y})^2$$
$$\mathrm{SSE} = \sum(y_i - \hat{y}_i)^2 = \mathbf{y}^T\mathbf{y} - \hat{\beta}^T\mathbf{X}^T\mathbf{y} \quad (31)$$

$$\mathrm{SSR} = \sum(\hat{y}_i - \bar{y})^2 = \hat{\beta}^T\mathbf{X}^T\mathbf{y} - n(\bar{y})^2$$
$$\mathrm{SSE} = \sum(y_i - \hat{y}_i)^2 = \mathbf{y}^T\mathbf{y} - \hat{\beta}^T\mathbf{X}^T\mathbf{y} \quad (32)$$

and thus, the F-value given by F distribution can be written as;

$$F = \frac{\mathrm{SSR} / k}{\mathrm{SSE} / (n-k-1)} = \frac{\mathrm{MSR}}{\mathrm{MSE}} \quad (33)$$

where$k$ is the polynomial and $(n-k-1)$ is the residual degrees of freedom, respectively. Moreover, the R-squared of each response that shows how well the actual responses were correlated to the predictions were ultimatelye stimatedas;

$$R^2 = 1 - \frac{\mathrm{SSE}}{(\mathrm{SSR} + \mathrm{SSE})} \quad (34)$$

As seen in Table 2, the R-squared values reaches to unity, so the difference between the actual data and the predictionsare properly small.

Table 2. Comparison of the predicted responses in CCD design space

Source	SSE	DOF	MSR	F-value	R^2
Equation 21	147.9444	4	36.9861	1036.38	0.995
Equation 22	0.42	5	0.084	1067.86	0.9963
Equation 23	1319.74	3	439.91	494.5	0.9854
Equation 24	82907.55	3	27635.85	158.67	0.9558
Equation 25	5.38	5	1.08	1432.4	0.9976
Equation 26	0.067	5	0.013	2596.96	0.9985
Equation 27	1540.54	3	513.51	563.09	0.9871
Equation 28	3.36E+06	3	1.12E+06	155.31	0.9549
Equation 29	1.64E+07	4	4.10E+06	5414000	1
Equation 30	710.09	2	355.04	2076.54	0.994492

On the other hand, the model F-valueof each response shows that there are little chancedue to noise that affects eachresponse. Consequently, fairly high quality polynomials were probablydevelopted in this manner. Furtherresults were achieved through the direct comparison of predicted to actual responses as shown typically in Figure (5) through (7).

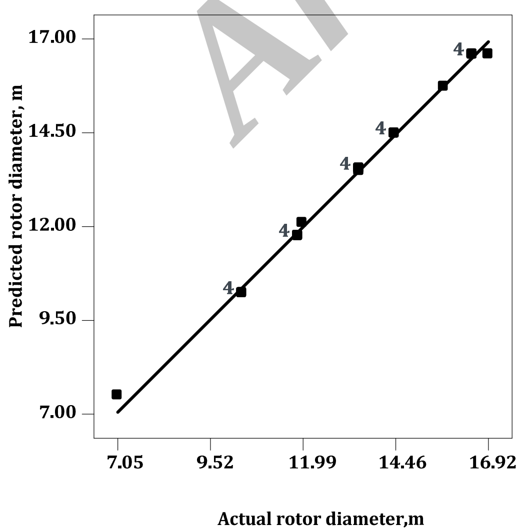

Figure 5. Predicted main rotor diameter versus actual data

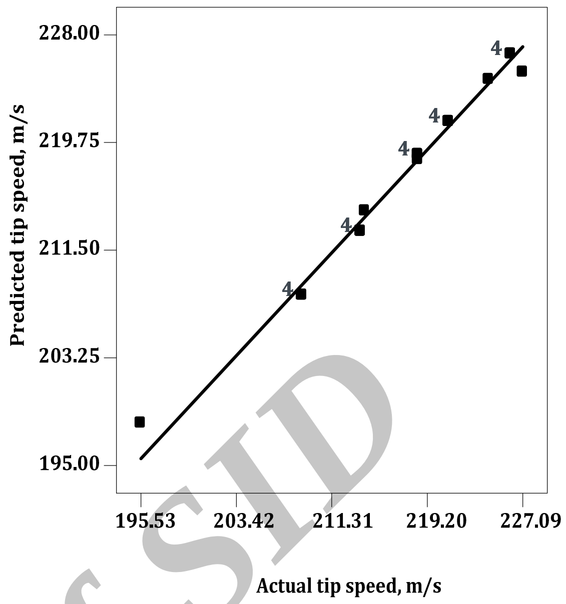

Figure 6. Predicted main rotor tip speed versus actual data

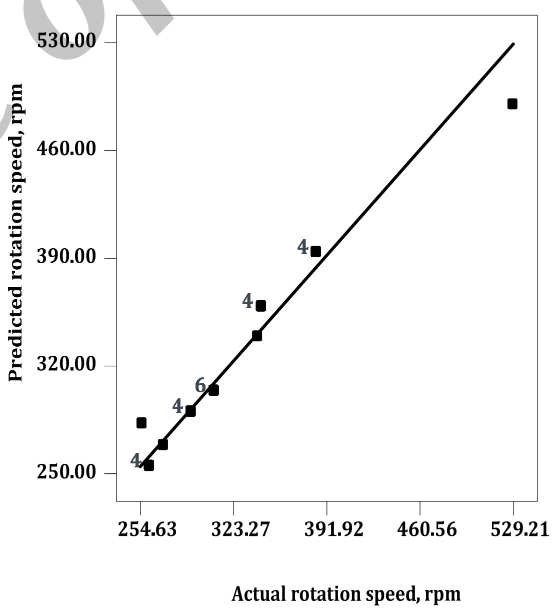

Figure 7. Predicted main rotor rotational speedversus actual data

Additionally, residual plots versus predicted responsesin Figure 8. Through 11 show that the error has been distribute drandomly. This result is in consistant with the earlier assumption for the error vector with a random nature. In other words, thenature of errors during the modeling processis basically the sameas the natural properties of the actual data about the mean values. The late result together with that stated above was them aincause in which all predictions could beaccepted.

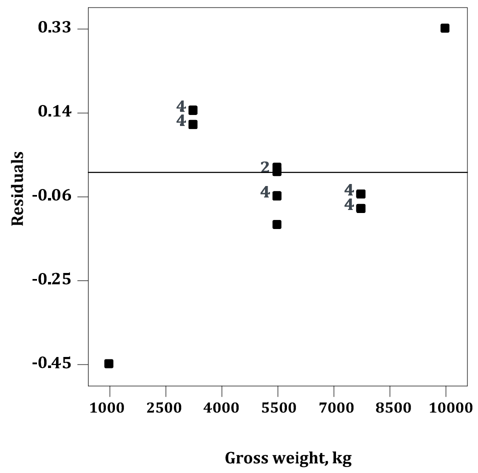

Figure 8. Residual distribution of helicopter gross weight

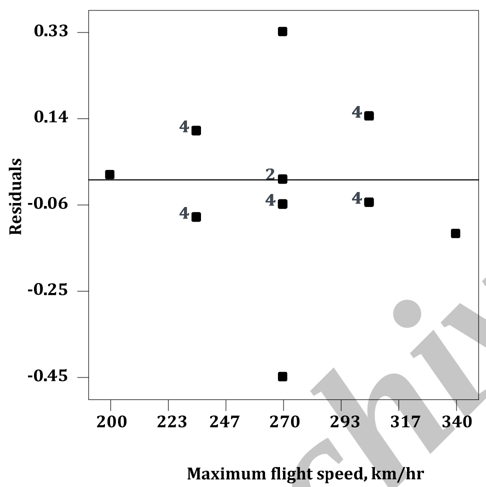

Figure 9. Residual distribution of maximumflight speed

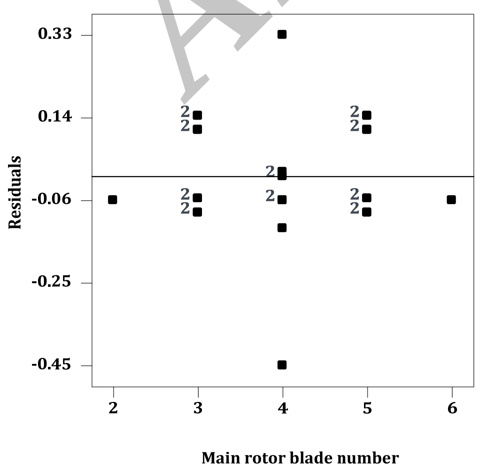

Figure 10. Residual distribution of main rotor blade number

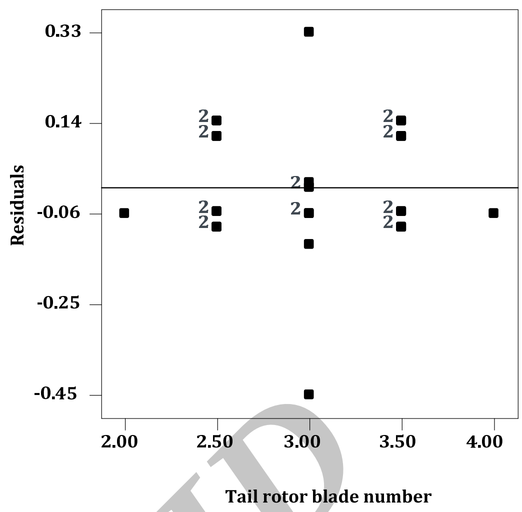

Figure 11. Residual distribution of tail rotor blade number

DISSCUSSION

The variations of the main rotor diameter versus w_0 and $v_m$has been illusstrated in Figure12. As seen in this Figure,for a given gross weight when the maximum forward flight speed is increased the main rotor diameter is subsequently decreased. Consequently, it can be found that the main rotor size isapproximately proportional to $D \propto v_m^{-0.2}$.

On the other hand, at a given maximum flight speed when the helicopter gross weight is increased (due to modifications), the size of rotor have to be increased ($D \propto w_0^{0.4}$). In addition, the small amount of interaction between w_0 and $v_m$is sensed, but it can be neglected from the main rotor diametersizing process at the preliminary design stage (Figure 13).

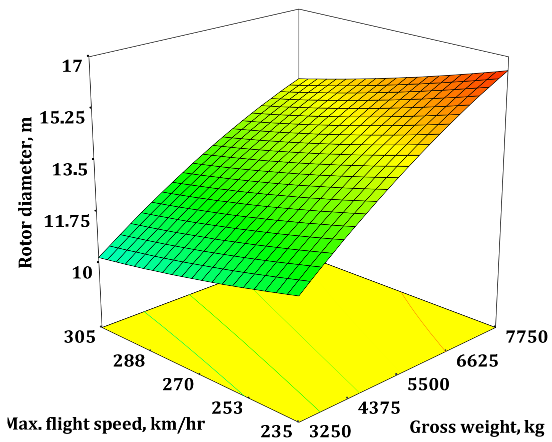

Figure 12. Variationof main rotor diameter versus w_0 and v_m

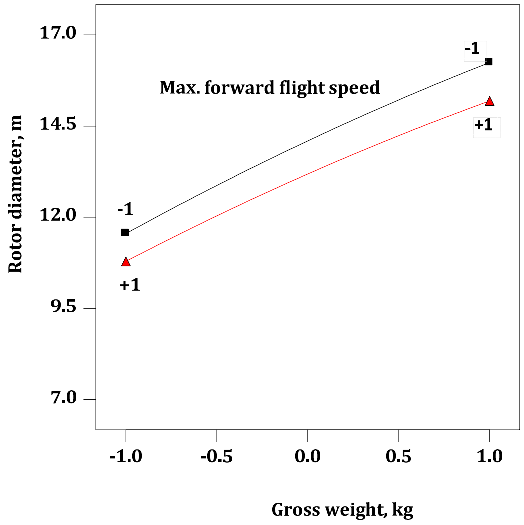

Figure 13. Effect of w_0 and v_m on rotor diameter

As shown in Figure14, for a given gross weight and maximum forward flight speed the blade chord length can be estimated using $c \propto N^{-0.75}$. However, in a 4-bladed helicopter as the number of blades is increased $(N=5)$, the blade chord length should be reduced about 15%. Furthermore, it can be found that if the purpose of design optimization problem is to decrease the helicopter gross weight with the same number of main rotor blades and forward flight speed, the blade chord length should be approximated by $c = w_0^{0.467} v_m^{-0.69} N^{-0.757}$.

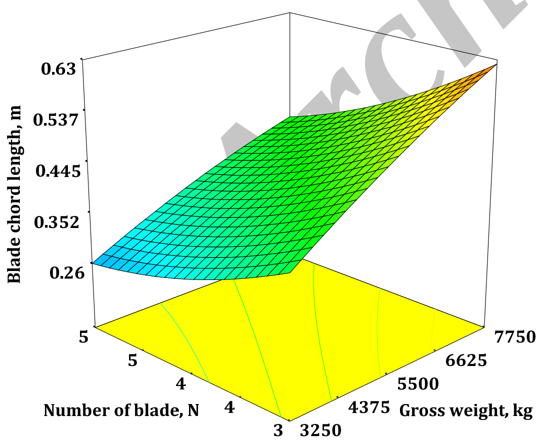

Figure 14. Variation of main rotor blade number versus w_0 and v_m

Main rotor tip speed $(v_{tip} = R\Omega)$ is generally used as a main factorinrotor sizing at the preliminary design stage.Lowtip speeds have the advantage of low noise and good hovering performance (high power loading).Sincehelicopters spend a wide proportion of their missions in hover or low speed forward flight, hover is considered as a start point of the design process.The results based on the present work shows that helicopter gross weight is a significant parameter that influences on the rotor tip speed. As seen in Figure15, larger gross weight leads togreater rotor tip speed $\left(v_{tip} \propto w_0^{0.148} v_m^{0.735}\right)$. The opposite is seen in the main rotor rotational speed given in Figure16 that shows higher gross weight is proportional tosmall rotational speed $\left(\Omega \propto w_0^{-0.22} v_m^{1.36}\right)$.

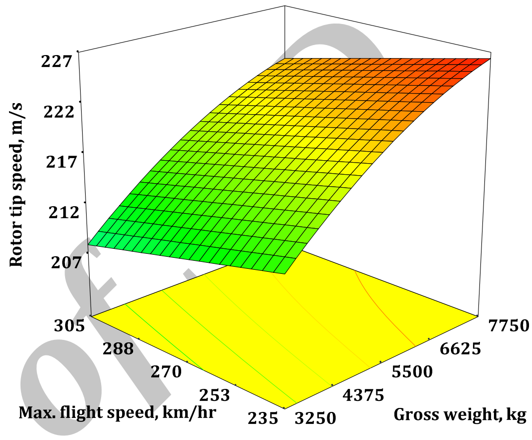

Figure 15. Variation of main rotor tip speed versus w_0 and v_m

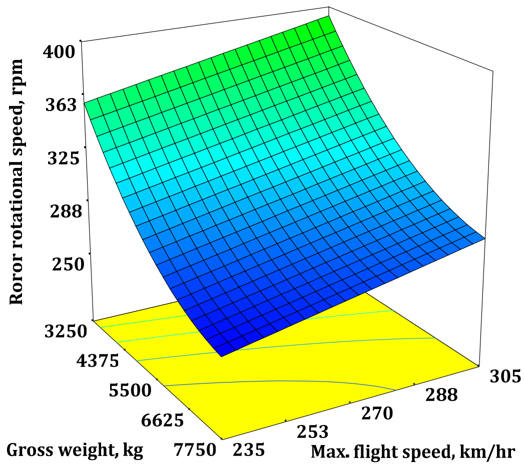

Figure 16. Variation of main rotor rotational speed versus w_0 and v_m

The results of tail rotor have been shown that as the forward flight speed of a helicopter is increased (constant gross weight) the tail rotor diametershould be thereforedecreased(Figure 17). In contrast,at the constant forward speed, the largergross weight leads to the larger tail rotor diameter,so it can be realizedthat $\left(D_{tr} \propto w_0^{0.438} v_m^{-0.525}\right)$.

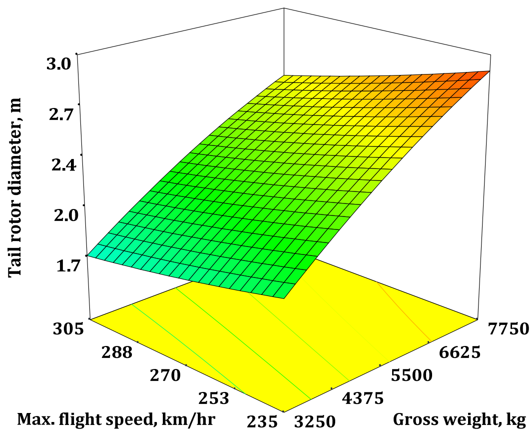

Figure 17. Variation of tai rotor diameter versus gross weight w_0 and v_m

As a summary, it should be emphasised that CCD approach associated with quadratic polynomials can be accounted as an efficient tool for prediction of the helicopter sizing at the preliminary stage and thus;

$$D = w_0^{0.412} / v_m^{0.172}$$
$$c = w_0^{0.467} v_m^{-0.69} N^{-0.757}$$
$$v_{tip} = w_0^{0.148} v_m^{0.735}$$
$$\Omega = w_0^{-0.218} v_m^{1.362}$$
$$D_{tr} = w_0^{0.438} v_m^{-0.525}$$
$$c_{tr} = w_0^{0.425} v_m^{-0.78} N_{tr}^{-0.728}$$
$$v_{tiptr} = w_0^{0.155} v_m^{0.718}$$
$$\Omega_{tr} = w_0^{-0.225} v_m^{1.68}$$
$$P_{ava} = w_0^{0.887} v_m^{0.02} N^{0.06} N_{tr}^{-0.12}$$
$$T / A = w_0^{0.352} v_m^{0.092}$$

In this section, an optimization problem for a helicopter with 4 bladed main rotor and two bladed tail rotor was also examined. The solution was obtained for the conditions when the maximum gross weight, maximum forward flight speed and minimum rotor tip speed (noise consideration and compressibility avoidance) in the range of each parameter were of our interest. The solution method based on the steepest descent/ascent approach was used through guessing the start point [15]. Regardless of details, in Table 4and 5 the summary of the problem associated with the possible constraints and the iterative solution are presented.

Table 4. Summary of the optimization problem

	Name	Condition s	Goal	Lowe r Limit	Upper Limit
1	Gross weight	constraint	Minimize	1000	10000 kg
2	Max. flight speed	constraint	Maximiz e	200	340 km/hr
3	Main rotor blade	constraint	4		
4	Tail rotor blade	constraint	2		
5	Blade tip speed	problem	Minimize		

Finally, the iterative solution was converged as suggested in Table 5.

Table 5. Summary of the optimization problem

Solution												
w_0	v_m	N	N_{tr}	D	c	v_{tip}	Ω	D_{tr}	c_{tr}	v_{tiptr}	Ω_{tr}	T/A
3606 kg	303 km/h	4	2	10.7m	0.32m	210m/s	378 rpm	1.8m	0.22m	201m/s	2167 rpm	30.1 kg/m^2

CONCLUSIONS

Practical formulationsfor the statistical rotor sizing based on empirical data were developed for the ease of conceptual design stage. Empirical data were taken from both a native database with more than 180 single main rotor helicopters and a homemade design software used for cases in which sufficient data were not available. Design space were constructed based on CCD rule included central , star, and factorial design points that were necessary for the quadratic expression development. Thus, the total number of observations were limited to 26 that were found adequate and cost-effectiveness than the conventiaonal approaches traditionaly applied for rotor sizings. In addition, a multiple response optimiazation problem for minimum gross weight, maximum level flight speed, and also minimum tip speed in range of considered tail rotor and main rotor blade number were solved. The obtained results in this paper showed that CCD observations and 10 quadratic expressions can be sufficiently useful for the rotor sizing estimation in design phase.

ACKNOWLEDGEMENTS

The author wishes to express gratefullyacknowledge JAST and the referees for valuable guidance and their comments.

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