(FGM)

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Vibration Analysis of Thin Circular FGM Plate Coupled with Piezoelectric Layers

Farzad Ebrahimi, Abbas Rastgo

ABSTRACT

Analytical investigation of the vibration behavior of thin circular functionally graded (FG) plates integrated with two uniformly distributed piezoelectric actuator layers based on the classical plate theory (CPT) is presented. The material properties of the FG substrate plate are assumed to be graded in the thickness direction according to the power-law distribution. The differential equations of motion are solved analytically for clamped edge boundary condition of the plate. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of varying the gradient index of FG plate on the vibration characteristics of the structure.

KEYWORDS

Functionally graded material, Piezoelectric, Circular plate, Classical plate theory

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$$\varepsilon_{r\theta} = \frac{1}{2} \left(\frac{\partial u_r}{r \partial \theta} + \frac{\partial u_{\theta}}{\partial r} - \frac{u_{\theta}}{r} \right) = z \left(\frac{\partial w}{r^2 \partial \theta} - \frac{\partial^2 w}{r \partial r \partial \theta} \right)$$
(17)

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 $u_z \quad u_\theta \quad u_r$

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$$E(z) = (E_c - E_m)(z/2h_f + 1/2)^g + E_m$$

$$\rho(z) = (\rho_c - \rho_m)(z/2h_f + 1/2)^g + \rho_m$$
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$$\sigma_{rr}^{f} = \frac{E(z)}{1 - v^{2}} (\varepsilon_{rr} + v\varepsilon_{\theta\theta}) = -\frac{zE(z)}{1 - v^{2}} \left[\frac{\partial^{2}w}{\partial r^{2}} + v \left(\frac{\partial^{2}w}{r^{2}\partial \theta^{2}} + \frac{\partial w}{r\partial r} \right) \right]$$
(14)

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$$\sigma_{\theta\theta}^{f} = \frac{E(z)}{1 - v^{2}} (\varepsilon_{\theta\theta} + v\varepsilon_{rr}) = -\frac{zE(z)}{1 - v^{2}} \left[v \frac{\partial^{2} w}{\partial r^{2}} + \frac{\partial^{2} w}{r^{2} \partial \rho^{2}} + \frac{\partial w}{r \partial r} \right]$$
(10)

$$\tau_{r\theta}^{f} = -\frac{zE(z)}{1+\nu} \left(\frac{\partial^{2}w}{r\partial r\partial \theta} - \frac{\partial w}{r^{2}\partial \theta} \right)$$
(19)

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$$\begin{array}{c} :[] \\ \sigma_{rr}^{p} = \overline{C}_{11}^{E} \varepsilon_{rr} + \overline{C}_{12}^{E} \varepsilon_{\theta\theta} - \overline{e}_{31} E_{z} \end{array}$$

$$\sigma_{rr}^{p} = \overline{C}_{11}^{E} \varepsilon_{rr} + \overline{C}_{12}^{E} \varepsilon_{\theta\theta} - \overline{e}_{31} E_{z} \qquad ()$$

$$\sigma_{\theta\theta}^{p} = \overline{C}_{12}^{E} \varepsilon_{rr} + \overline{C}_{11}^{E} \varepsilon_{\theta\theta} - \overline{e}_{31} E_{z} \qquad ()$$

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$$\phi = \left[1 - \left(\left(2z - 2h_f - h_p\right)/h_p\right)^2\right] \varphi(r, \theta, t)$$
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$$\phi(r, \theta, t)$$
$$h_p \qquad h_f$$
$$E$$

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$$D_{r} = \overline{\Xi}_{11}E_{r} = \overline{\Xi}_{11}\left(-\frac{\partial\varphi}{\partial r}\right) \qquad (1 \land) \qquad :[] ()$$

$$D_{r} = \overline{\Xi}_{11}E_{r} = \overline{\Xi}_{11}\left(-\frac{\partial\varphi}{\partial r}\right) \qquad (1 \land) \qquad u_{z} = u_{z}(r,\theta,t) = w(r,\theta,t)$$

$$D_{\theta} = \overline{\Xi}_{11}E_{\theta} = \overline{\Xi}_{11}\left(-\frac{\partial\varphi}{r\partial\theta}\right) \qquad (1 \land) \qquad u_{r} = u_{r}(r,\theta,t) = -z\frac{\partial u_{z}}{\partial r}$$

$$D_{z} = \overline{\Xi}_{33}E_{z} + \overline{e}_{31}\left(\varepsilon_{rr} + \varepsilon_{\theta\theta}\right) \qquad (1 \land) \qquad u_{\theta} = u_{\theta}(r,\theta,t) = -z\frac{\partial u_{z}}{r\partial\theta}$$

$$\overline{\Xi}_{33}, \overline{\Xi}_{11} \qquad \theta \quad r$$

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$$D_{z} = \overline{\Xi}_{33}E_{z} + \overline{e}_{31}(\varepsilon_{rr} + \varepsilon_{\theta\theta}) \qquad (\Upsilon \cdot)$$
$$\overline{\Xi}_{33}, \overline{\Xi}_{11}$$

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$$\begin{array}{c} \Xi_{33} = \overline{\Xi}_{33} + (e_{33}^2/C_{33}^E) & \overline{\Xi}_{11} = \Xi_{11} & (\Upsilon 1) \end{array}$$

$$\Xi_{11}, \Xi_{33}$$

.

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} = -z \frac{\partial^2 w}{\partial r^2} \tag{11}$$

$$\varepsilon_{\theta\theta} = \frac{\partial u_{\theta}}{r\partial\theta} + \frac{u_r}{r} = -z(\frac{\partial^2 w}{r^2 \partial\theta^2} + \frac{\partial w}{r\partial r})$$
(17)

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$$M_{rr} = \int_{-h_f}^{h_f} z \sigma_{rr}^f dz + 2 \int_{h_f}^{h_f + h_p} z \sigma_{rr}^p dz$$
(YY)

$$M_{\theta\theta} = \int_{-h_f}^{h_f} z \sigma_{\theta\theta}^f dz + 2 \int_{h_f}^{h_f + h_p} z \sigma_{\theta\theta}^p dz \tag{YY}$$

$$M_{r\theta} = \int_{-h_f} z \tau_{r\theta}^r dz + 2 \int_{h_f} z \tau_{r\theta}^r dz \qquad (ff)$$

$$a = \frac{\partial M_{rr}}{\partial dt} + \frac{\partial M_{r\theta}}{\partial dt} + \frac{M_{rr}}{\partial dt} - \frac{M_{\theta\theta}}{\partial dt} \qquad (ff)$$

$$q_{r} = \frac{\partial M_{r\theta}}{\partial r} + \frac{\partial M_{\theta\theta}}{r\partial \theta} + \frac{2M_{r\theta}}{r}$$
(10)
$$q_{\theta} = \frac{\partial M_{r\theta}}{\partial r} + \frac{\partial M_{\theta\theta}}{\partial r} + \frac{2M_{r\theta}}{r}$$
(17)

$$\frac{\partial q_r}{\partial r} + \frac{\partial q_{\theta}}{r\partial \theta} + \frac{q_r}{r}$$

$$-\left(\int_{-h_f}^{h_f} \rho_f(z) \frac{\partial^2 u_z}{\partial t^2} dz + 2\int_{h_f}^{h_f + h_g} \rho_g \frac{\partial^2 u_z}{\partial t^2} dz\right) = 0$$

$$()$$

$$(D_r + D_g) \Delta \Delta w + \frac{4}{r} h \ \overline{e}_{2r} \Delta \varphi + P_g \frac{\partial^2 w}{\partial t} = 0$$

$$(YV)$$

$$(D_1 + D_2)\Delta\Delta w + \frac{4}{3}h_p\overline{e}_{31}\Delta\varphi + P_0\frac{\partial}{\partial t^2} = 0$$
 (YA)

$$\widetilde{\rho}_{f} = \frac{1}{2h_{f}} \int_{-h_{f}}^{h_{f}} \rho_{f}(z) dz$$

$$D_{1} = \int_{-h_{f}}^{h_{f}} \frac{z^{2} E(z)}{1 - v^{2}} dz ,$$

$$D_{2} = \frac{2}{3}h_{p} \left(3h_{f}^{2} + 3h_{f}h_{p} + h_{p}^{2}\right) \overline{C}_{11}^{E}$$

$$P_{0} = 2(\widetilde{\rho}_{f}h_{f} + \rho_{p}h_{p})$$

$$ho_p$$
 ho_f

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$$\theta r$$

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 $^{+h_p}\vec{\nabla}.\vec{D}dz = \int_{h_f}^{h_f+h_p} \left(\frac{\partial(rD_r)}{r\partial r} + \frac{\partial D_{\theta}}{r\partial \theta} + \frac{\partial D_z}{\partial z}\right) dz = 0$
(YA)

$$\frac{h_p^2 \overline{\Xi}_{11}}{12 \overline{\Xi}_{33}} \Delta \varphi - \varphi + \frac{h_p^2 \overline{e}_{31}}{8 \overline{\Xi}_{33}} \Delta w = 0 \qquad (\mathfrak{r} \cdot)$$

$$\phi$$
 () ()
: () w
 $(D + D)h \overline{\Xi}$

$$\begin{split} \varphi(r,\theta,t) &= -\frac{(D_1 + D_2)n_p \Sigma_{11}}{16\overline{e}_{31}\overline{\Xi}_{33}} \Delta \Delta w \\ &+ \frac{h_p^2 \overline{e}_{31}}{8\overline{\Xi}_{33}} \Delta w - \frac{h_p (\widetilde{\rho}_f h_f + \rho_p h_p) \overline{\Xi}_{11}}{8\overline{e}_{31}\overline{\Xi}_{33}} \frac{\partial^2 w}{\partial t^2} \end{split} \tag{(71)}$$

 $: \qquad ()$ $P_{3}\Delta\Delta\Delta w - P_{2}\Delta\Delta w + P_{1}\Delta(\frac{\partial^{2}w}{\partial t^{2}}) - P_{0}\frac{\partial^{2}w}{\partial t^{2}} = 0 \qquad (\forall \forall)$ $P_{3}\Delta\Delta\Delta w - P_{2}\Delta\Delta w + P_{1}\Delta(\frac{\partial^{2}w}{\partial t^{2}}) - P_{0}\frac{\partial^{2}w}{\partial t^{2}} = 0 \qquad (\forall \forall)$

$$P_{1} = h_{p}^{2} \overline{\Xi}_{11} P_{0} / 12 \overline{\Xi}_{33} \qquad P_{2} = D_{1} + D_{2} + h_{p}^{3} \overline{e}_{31}^{2} / 6 \overline{\Xi}_{33}$$

$$P_{3} = (D_{1} + D_{2})h_{p}^{2} \Xi_{11}/12\Xi_{33}$$
(11)
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$$P_{3}\overline{\Delta\Delta\Delta}w_{1} - P_{2}\overline{\Delta\Delta}w_{1} - \omega^{2}P_{1}\overline{\Delta}w_{1} + \omega^{2}P_{0}w_{1} = 0$$
(Ya)

$$\overline{\Delta} = d^2/dr^2 + d/rdr - m^2/r^2$$

$$w_1 = \sum_{n=1}^3 A_{nm} Z_{nm}(\alpha_n r)$$
(79)

$$\begin{array}{c} \alpha_{1} = \sqrt{|x_{1}|}, \alpha_{2} = \sqrt{|x_{2}|}, \alpha_{3} = \sqrt{|x_{3}|} \\ () \\ () \\ () \\ () \\ (x_{3}, x_{2}, x_{I} \\ \int_{h_{I}}^{h_{I} + 1} dx_{I} \\ (x_{2}) \\ (x_{3}) \\ (x_{2}) \\ (x_{3}) \\ (x_{2}) \\ (x_{3}) \\ (x_$$

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$$P_{3}x^{3} - P_{2}x^{2} - \omega^{2}P_{1}x + \omega^{2}P_{0} = 0 \tag{7A}$$

$$Z_{im}(\alpha_i r) = Z_{im}(\alpha_i, r) = \begin{cases} J_m(\alpha_i r) &, x_i < 0\\ I_m(\alpha_i r) &, x_i > 0 \end{cases}$$

$$I_m(\alpha_i r) & J_m(\alpha_i r) & i = \end{cases}$$
(rq)

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$$+ \left(\frac{\alpha_{1}Z_{3m}(\alpha_{3}r_{0})Z'_{1m}(\alpha_{1}r_{0}) - \alpha_{3}Z_{1m}(\alpha_{1}r_{0})Z'_{3m}(\alpha_{3}r_{0})}{\alpha_{2}Z_{1m}(\alpha_{1}r_{0})Z'_{2m}(\alpha_{2}r_{0}) - \alpha_{1}Z_{2m}(\alpha_{2}r_{0})Z'_{1m}(\alpha_{1}r_{0})}\right) \times Z_{2m}(\alpha_{2}r)$$

$$\times \left[h_{p}(2s_{2}\alpha_{2}^{2}h_{p}\overline{e}_{31}^{2} - (D_{1} + D_{2})\alpha_{2}^{4}\overline{\Xi}_{11} + P_{0}\omega^{2}\overline{\Xi}_{11})\right] \left[16\overline{e}_{31}\overline{\Xi}_{33}\right]^{-1}$$

$$+ \left[h_{p}(2s_{3}\alpha_{5}^{2}h_{p}\overline{e}_{31}^{2} - (D_{1} + D_{2})\alpha_{3}^{4}\overline{\Xi}_{11} + P_{0}\omega^{2}\overline{\Xi}_{11})\right] \left[16\overline{e}_{31}\overline{\Xi}_{33}\right]^{-1} Z_{3m}(\alpha_{5}r)\right]$$

$$(\$\Lambda)$$

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$$\varphi(r,\theta,t) = \varphi_1(r)e^{i(m\theta - \alpha x)}$$

$$(f \cdot)$$

$$(g(r) - \alpha x)$$

$$(f \cdot)$$

$$(f \cdot)$$

$$(f \cdot)$$

$$(f \cdot)$$

$$\vdots \qquad \varphi_1(r) \qquad ()$$

$$\begin{split} \varphi_{1}(r) &= \left[16\overline{e}_{31}\overline{\Xi}_{33} \right]^{-1} \sum_{n=1}^{3} \left[A_{nm} h_{p} (2s_{n} \alpha_{n}^{2} h_{p} \overline{e}_{31}^{2} - (\mathbf{f}) \right] \\ &(D_{1} + D_{2}) \alpha_{n}^{4} \overline{\Xi}_{11} + P_{0} \omega^{2} \overline{\Xi}_{11} \right] \times Z_{nm} (\alpha_{n} r) \end{split}$$

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$$w_1 = dw_1/dr = d\varphi_1/dr = 0$$
 at $(r = r_0)$ (*Y)
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$$\begin{array}{c|c} & : & () \\ c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{array} = 0 \qquad c_{1i} = Z_{im}(\alpha_i r_0) \\ c_{2i} = \alpha_i r_0 Z'_{im}(\alpha_i r_0) \qquad (\red{equation})$$

$$c_{3i} = \left(\frac{h_p^2 r_0 s_i \alpha_i^3}{8} - \frac{(D_1 + D_2) h_p r_0 \alpha_i^5 \overline{\Xi}_{11}}{16 \overline{e}_{31}^2} + \frac{1}{16 \overline{e}_{31$$

$$\frac{(D_1 + D_2)h_p \alpha_i \lambda^4 \Xi_{11}}{16\overline{e}_{31}^2 r_0^3} \bigg) Z'_{im}(\alpha_i r_0)$$

$$(\mathbf{f} \Delta)$$

$$\lambda = r_0 \left[\frac{2(\tilde{\rho}_f h_f + \rho_p h_p) \omega^2}{D_1 + D_2} \right]^{\frac{1}{4}}$$
⁽⁴²⁾

$$w_I$$
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 $w_1(r) = A_{3m} \times$

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$$\begin{bmatrix} \left(\frac{\alpha_{3}Z_{2m}(\alpha_{2}r_{0})Z'_{3m}(\alpha_{3}r_{0}) - \alpha_{2}Z_{3m}(\alpha_{3}r_{0})Z'_{2m}(\alpha_{2}r_{0})}{\alpha_{2}Z_{1m}(\alpha_{1}r_{0})Z'_{2m}(\alpha_{2}r_{0}) - \alpha_{1}Z_{2m}(\alpha_{2}r_{0})Z'_{1m}(\alpha_{1}r_{0})}\right) \\ \times Z_{1m}(\alpha_{1}r) + \end{bmatrix}$$

$$\begin{pmatrix} \alpha_{1}Z_{3m}(\alpha_{3}r_{0})Z'_{1m}(\alpha_{1}r_{0}) - \alpha_{3}Z_{1m}(\alpha_{1}r_{0})Z'_{3m}(\alpha_{3}r_{0})\\ \alpha_{2}Z_{1m}(\alpha_{1}r_{0})Z'_{2m}(\alpha_{2}r_{0}) - \alpha_{1}Z_{2m}(\alpha_{2}r_{0})Z'_{1m}(\alpha_{1}r_{0}) \end{pmatrix} \times Z_{2m}(\alpha_{2}r) + Z_{3m}(\alpha_{3}r) \end{bmatrix}$$

$$(\texttt{FV})$$

$$\hat{\varphi}(r) = A_{3m} \times \left[\left(\frac{\alpha_3 Z_{2m}(\alpha_2 r_0) Z'_{3m}(\alpha_3 r_0) - \alpha_2 Z_{3m}(\alpha_3 r_0) Z'_{2m}(\alpha_2 r_0)}{\alpha_2 Z_{1m}(\alpha_1 r_0) Z'_{2m}(\alpha_2 r_0) - \alpha_1 Z_{2m}(\alpha_2 r_0) Z'_{1m}(\alpha_1 r_0)} \right] \times Z_{1m}(\alpha_1 r)$$

$$\times \left[h_{p}(2s_{1}\alpha_{1}^{2}h_{p}\overline{e}_{31}^{2} - (D_{1} + D_{2})\alpha_{1}^{4}\overline{\Xi}_{11} + P_{0}\omega^{2}\overline{\Xi}_{11})\right] \left[16\overline{e}_{31}\overline{\Xi}_{33}\right]^{-1}$$

$: ()$ FGM $E_{c} = Gpa, v = , E_{m} = \rho_{c} = (kg/m^{3}) \rho_{m} = \rho_{c} = \rho$			() ()
FGM Plate $E_{c} = Gpa, v = , E_{m} = \rho_{c} = (kg/m^{3}) \rho_{m} = \rho_{c} = (kg/m^{3}) \rho_{m} = \rho_{31} (C/m^{2}) = , e_{33} = , \rho_{p} = (kg/m^{3}) e_{15} = , \rho_{p} = (kg/m^{3}) e_{15} = , P_{12} = E_{11} = , (nF/m) = E_{33} = , C_{12} = E_{11} = , (nF/m) = E_{33} = , C_{13} =$:()	
$PZT4 \\ Layers \\ PZT4 \\ Layers \\ C_{11}^{E} = Gpa \\ C_{12}^{E} = \\ \Xi_{11} = , (nF/m) \\ C_{13}^{E} = , C_{55}^{E} = C_{33}^{E} = \\ C_{13}^{E} = , C_{55}^{E} = C_{33}^{E} = \\ () \\ () \\ . \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ / \\ . \\ .$	FGM Plate	$E_c = Gpa, v = ,$ $\rho_c = (kg/m^3)$	$E_m = ho_m =$	
$\begin{array}{c} PZT4\\ Layers \\ C_{11}^{E} = Gpa \\ \Xi_{11} = , (nF/m) \\ C_{13}^{E} = , C_{55}^{E} = C_{33}^{E} = \\ C_{13}^{E} = , C_{55}^{E} = C_{33}^{E} = \\ \end{array}$		$e_{31}(C/m^2) = ,$	$e_{33} = ,$	0
	PZT4 Layers	$C_{11}^E = Gpa$	$C_{12}^{E} =$	K
		$\Xi_{11} = , (nF/m)$ $C_{13}^E = , C_{55}^E =$	$\Xi_{33} = , \\ C_{33}^E = ,$	
	/			

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	۲	471,19	424,11	•,••	-
V	•	180,09	149,74	1,40	-
	١	779,89	۲۹۰,۸۳	1,47	-
	۲	۴۷۰,۳۰	472,90	• ,٧٧	-
٩	•	187,98	139,00	1,01	-
	١	۲۸۶,۴۰	19.,04	1,87	-
	۲	499,18	477,19	•,••	-
۱۰	•	۱۳۷,۵۷	189,81	1,89	-
	١	۲۸۹,۳۳	19.,41	1,47	-
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Power	Mode	FGM plate			
Index g	no. m	Present Method	Present (FEM)	Diff. (%)	Wang et al.[14]
		1	/	/	1
		1	/	/	1
		1	/	/	/
		,	,	/	-
		/	/	,	-
		,	1	/	-
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Power Index	Mode no. m	FGM plate			
g		Present Method	Present (FEM)	Diff. (%)	Wang et al.[14]
		148,58	144,59	• ,٧٣	145,01
	١	191,91	r,49	۰,۵۲	799,•V
	۲	49.,WV	497,87	۴۶,	49.,87
١		14.,79	147,77	١,٣٨	-
	١	191,19	190,11	۱,۳۳	-
	۲	۴۷۸,۸۴	422.04	۶۷, ۶۷	-
٣		187,04	14.,8.	1,49	-
	١	۲۸۸,۳۳	797, f V	1,47	-
	۲	471,99	475,51	۰,٧۶	-
۵		۱۳۸,۰۱	14.,.v	1,47	-
	١	۲۸۷,۲۱	191,79	1,47	-

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