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Strapdown Attitude Estimation Using IMU and Altimeter Integration for Maneuvering Vehicles

H. ghanbarpour asl, S. H. Pourtakdust

ABSTRACT

In this paper, a new algorithm for attitude estimation in maneuvering flight, utilizing a combination of inertial measuring unit (IMU) and altimeter information is presented. Attitude estimation using a single IMU is possible only for near cruise flights, however for non-cruise flights, very large errors are obtained. In this paper, attitude estimation error is stabilized using an integrated IMU and altimeter system. The

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altimeter output being affected by gravity and the specific forces projected into the vertical plane bears insufficient information regarding the attitude states. Being a function of the roll and pitch angle, the specific forces will be in error, due to errors estimation of the attitude angles. Subsequently the vehicle vertical acceleration, speed and attitude will be inaccurate. In addition, due to a weak observability between the altitude measurements and the attitude angles to be estimated. For this reason and having a better estimate of the attitudes, the nonlinear attitude equations are converted into linear space, which will be beneficial for the estimation algorithm. Finally, simulation results using linear and unscented Kalman filters are carried out. A Monte Carlo simulation reveals that the newly suggested linear filter has a better performance in comparison with the non-linear unscented Kalman filter.

KEYWORDS

Attitude Estimation, Nonlinear Filter, Strapdown attitude estimation, Integrated navigation

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GPS

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IMU

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$$C_b^n = C_b^n \Omega_{nb}^b \quad (1)$$

$$C_b^n = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}, \quad \Omega_{nb}^b = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (2)$$

$$\omega_{nb}^b = [\omega_x \quad \omega_y \quad \omega_z]^T$$

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$$\omega_{ib}^b = \omega_{ie}^b + \omega_{en}^b + \omega_{nb}^b \quad (3)$$

$$\omega_{en}^b$$

$$\dot{V}_e^n = C_b^n f^b - [2\omega_{ie}^n + \omega_{en}^n] \times V_e^n + g^n \quad (1)$$

ω_{ie}^b

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$$g^n$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \text{tg } \theta \sin \phi & \text{tg } \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (2)$$

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$$\begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \\ \dot{d} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad (3)$$

$$q_b^n = [a \ b \ c \ d]^T$$

$$f^b \cong -C_n^b g^n = g \begin{bmatrix} \sin \theta \\ -\cos \theta \sin \phi \\ -\cos \theta \cos \phi \end{bmatrix} \text{ if } \text{abs}(\|f^b\| - g) < \varepsilon \quad (4)$$

ε
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$$C_b^n(3,:) = [-\sin \theta \ \sin \phi \cos \theta \ \cos \phi \cos \theta] \\ = [2(bd-ac) \ 2(cd+ab) \ (a^2-b^2-c^2+d^2)] \quad (5)$$

$$\phi = \text{arc tag } 2(f_y, f_z) \quad (6)$$

$$\theta = \text{arc sin}(f_x / g) \quad (7)$$

ε

$$F_t^n = \dot{V}_e^n + [2\omega_{ie}^n + \omega_{en}^n] \times V_e^n = C_b^n f^b + g^n \quad (8)$$

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$$\dot{C}_{31} = \omega_z C_{32} - \omega_y C_{33} \quad (9)$$

$$\dot{C}_{32} = \omega_x C_{33} - \omega_z C_{31} \quad (10)$$

$$\dot{C}_{33} = \omega_y C_{31} - \omega_x C_{32} \quad (11)$$

$$c = [C_{31} \ C_{32} \ C_{33}]^T \quad (12)$$

$$C_{31}^2 + C_{32}^2 + C_{33}^2 = 1 \quad (13)$$

$$\dot{c} = [-\omega_{nb}^b \times] c \quad (14)$$

$$\omega_{nb}^b \quad [-\omega_{nb}^b \times]$$

ϕ, θ

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V_e

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$$c_m = \frac{s^2 c_g + k_p c_a s + k_i c_a}{s^2 + k_p s + k_i} \quad ()$$

: $t \rightarrow \infty$

$$\lim_{s \rightarrow 0} c_m = c_a \quad ()$$

c_m

k_p, k_i

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$$k_i = \omega^2, \quad k_p = 2\xi\omega \quad ()$$

$$k_p = \xi = 1 \quad ()$$

$$k_p = \sqrt{2}\omega \quad ()$$

PI

$$\omega \quad \frac{rad}{sec} \quad \frac{rad}{sec}$$

$$\dot{V}_d = c^T f^b + g \quad ()$$

V_d
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$$\dot{h} = -V_d \quad ()$$

$$\dot{c} = k\gamma c - \Omega_{nb}^b c = [k\gamma I - \Omega_{nb}^b] c \quad ()$$

$$\gamma = 1 - (c^T c) \quad ()$$

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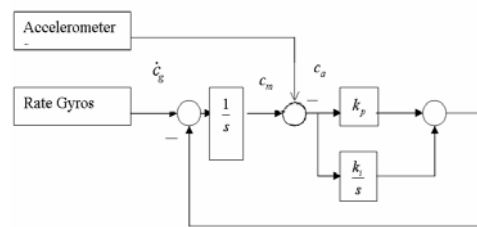
k_p PI

k_i

$$f^b \approx -C_n^b g^n = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = -g \begin{bmatrix} C_{31} \\ C_{32} \\ C_{33} \end{bmatrix} = -gc \quad ()$$

$$\hat{c} = \frac{-1}{g} f^b \quad ()$$

$$c_a = \frac{-1}{\|f^b\|} f^b \quad ()$$



PI ()

$$\begin{aligned} & \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{V}_d \\ \dot{h} \end{array} \right] = \left[\begin{array}{ccccc} k\gamma & \tilde{\omega}_z & -\tilde{\omega}_y & 0 & 0 \\ -\tilde{\omega}_z & k\gamma & \tilde{\omega}_x & 0 & 0 \\ \tilde{\omega}_y & -\tilde{\omega}_x & k\gamma & 0 & 0 \\ \tilde{f}_x & \tilde{f}_y & \tilde{f}_z & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array} \right] \left[\begin{array}{c} c_1 \\ c_2 \\ c_3 \\ V_d \\ h \end{array} \right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right] g \\ \dot{x} &= A(t)x + Bu + C(x)w \\ z &= Hx + v \end{aligned} \quad ()$$

$$\begin{aligned} & \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{V}_d \\ \dot{h} \end{array} \right] = \left[\begin{array}{ccccc} k\gamma & \tilde{\omega}_z & -\tilde{\omega}_y & 0 & 0 \\ -\tilde{\omega}_z & k\gamma & \tilde{\omega}_x & 0 & 0 \\ \tilde{\omega}_y & -\tilde{\omega}_x & k\gamma & 0 & 0 \\ \tilde{f}_x & \tilde{f}_y & \tilde{f}_z & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array} \right] \left[\begin{array}{c} c_1 \\ c_2 \\ c_3 \\ V_d \\ h \end{array} \right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right] g \\ & \left[\begin{array}{ccccc} 0 & -c_3 & c_2 & 0 & 0 \\ c_3 & 0 & -c_1 & 0 & 0 \\ -c_2 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & c_2 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{c} n_{wx} \\ n_{wy} \\ n_{wz} \\ n_{ax} \\ n_{ay} \\ n_{az} \end{array} \right] \\ & \tilde{\omega} = [\tilde{\omega}_x \quad \tilde{\omega}_y \quad \tilde{\omega}_z]^T \end{aligned} \quad ()$$

$$\begin{aligned} x_{k+1} &= F_k x_k + L_k u_k + G(x_k)w_k \\ & \vdots \\ F_k &= \exp\left(\int_{t_k}^{t_{k+1}} A(t)dt\right), \quad G(x_k) \approx \Delta C(x_k), \quad L_k \approx \Delta B \end{aligned} \quad ()$$

$$\begin{aligned} \tilde{f}^b &= [\tilde{f}_x \quad \tilde{f}_y \quad \tilde{f}_z]^T \\ n_w &= [n_{wx} \quad n_{wy} \quad n_{wz}]^T \\ n_a &= [n_{ax} \quad n_{ay} \quad n_{az}]^T \end{aligned} \quad ()$$

$$\hat{x}_k \quad z_k \quad ()$$

$$\begin{aligned} \hat{x}_{k|k-1} &= F_{k-1} \hat{x}_{k-1|k-1} + L_{k-1} u_{k-1} \\ P_{k|k-1} &= F_{k-1} P_{k-1|k-1} F_{k-1}^T + G(\hat{x}_{k-1|k-1}) Q_{k-1} G^T(\hat{x}_{k-1|k-1}) + \Gamma(Q_{k-1} \otimes P_{k-1|k-1}) \Gamma^T \\ K_k &= P_{k|k-1} H^T (H P_{k|k-1} H^T + R_k)^{-1} \\ P_{k|k} &= P_{k|k-1} - P_{k|k-1} H^T (H P_{k|k-1} H^T + R_k)^{-1} H P_{k|k-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (z_k - H \hat{x}_{k|k-1}) \end{aligned} \quad ()$$

$$\begin{aligned} & R_k \quad Q_{k-1} \\ & \hat{x}_{k|k} \\ & \hat{x}_{k|k-1} \\ & \otimes \\ & \Gamma, G_{ci}, i=1...6 \\ & [A] \end{aligned} \quad ()$$

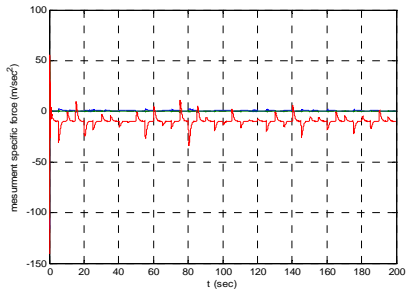
$$\Gamma = [G_{c1} \quad G_{c2} \quad \dots \quad G_{c6}] \quad ()$$

$$\begin{aligned} & \hat{x}_{k|k} \\ & \hat{x}_{k|k-1} \\ & \otimes \\ & \Gamma, G_{ci}, i=1...6 \\ & [A] \end{aligned} \quad ()$$

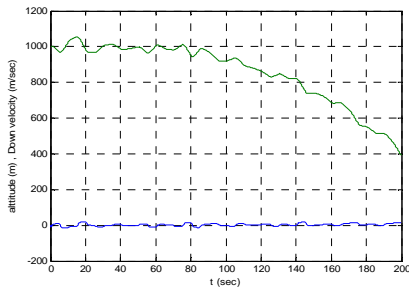
$$G_{ci} x_k = G(x_k) e_i \quad ()$$

$$\begin{aligned} z &= Hx + v, \quad H = [0 \quad 0 \quad 0 \quad 0 \quad 1] \\ & \vdots \\ & R_k \\ & G(x)w \\ & x \\ & A \end{aligned} \quad ()$$

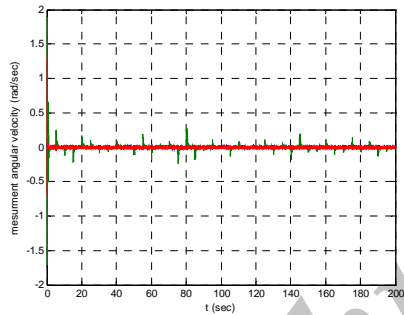




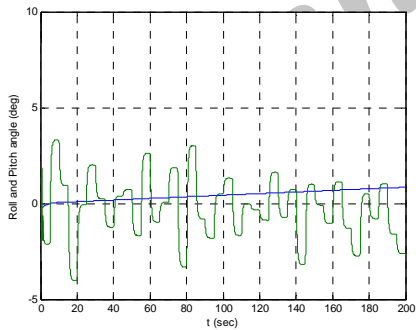
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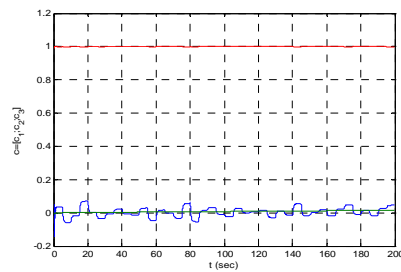


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Unscented Kalman Filter (UKF)

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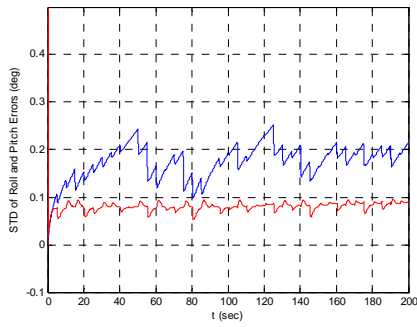
$$\frac{m}{\text{sec}^2}$$

$$\frac{\text{deg}}{\text{sec}}$$

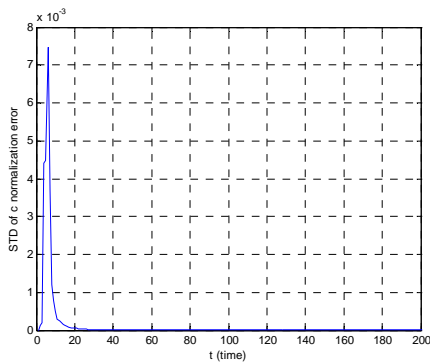
m

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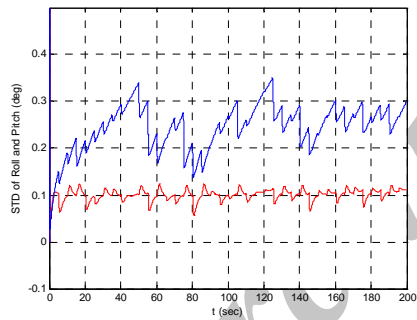
(UKF)



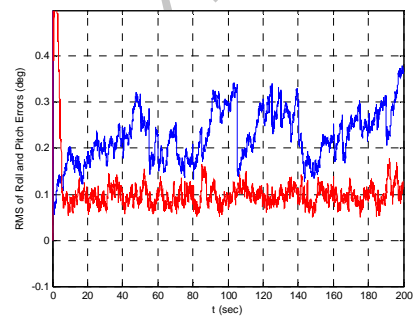
RMS : ()



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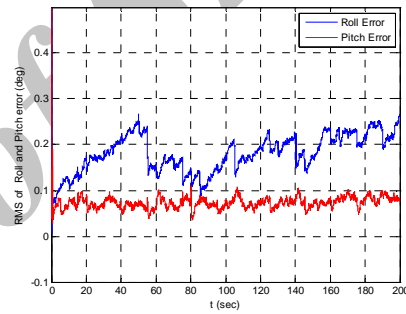
RMS : ()

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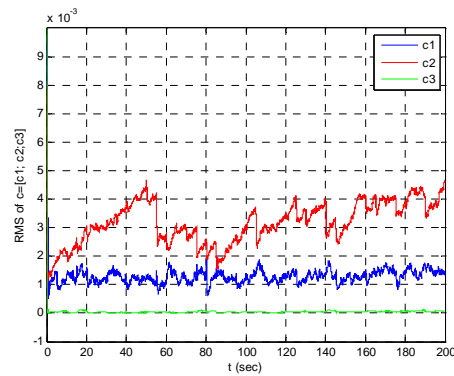
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RMS : ()



RMS : ()



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Root Mean Square

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(RMS)

RMS

RMS

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y_k

$$x_k \in R^n \quad w_k \in R^m$$

$$y_k = F_k x_k + G(x_k) w_k \quad (a)$$

$$E(w_k) = 0 \quad E(x_k) = \hat{x}_k$$

$$E(w_k w_k^T) = Q_k \quad E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = P_k \quad (a) \quad [11]$$

$$\begin{matrix} G(x_k) & x_k & w_k \\ G(x_k) w_k & x_k & \end{matrix} \quad (a)$$

$$z_k = G(x_k) w_k = G(x_k) \left(\sum_{i=1}^m w_{i,k} e_i \right) \quad (a)$$

$$w_{i,k} \quad (a-)$$

$$: \quad (a)$$

$$z_k = G(x_k) w_k = \sum_{i=1}^m w_{i,k} [G(x_k) e_i] = \quad (a)$$

$$\sum_{i=1}^m w_{i,k} (G_{ci} x_k) = \left(\sum_{i=1}^m w_{i,k} G_{ci} \right) x_k$$

$$(a) \quad z_k$$

[]

[A]

$$E(z_k) = E \left[\left(\sum_{i=1}^m w_{i,k} G_{ci} \right) x_k \right] = \sum_{i=1}^m E(w_{i,k}) G_{ci} E(x_k) = 0 \quad (a)$$

$w_{i,k}$

/ / / /

$$\begin{aligned}
&= \sum_{i=1}^m \sum_{j=1}^m G_{ci} Q_{ij} P_k G_{cj}^T \\
&= \sum_{i=1}^m \sum_{j=1}^m G_{ci} [Q \otimes P_k] G_{cj}^T \\
&= [G_{c1} G_{c2} \dots G_{c6}] (Q \otimes P_k) [G_{c1} G_{c2} \dots G_{c6}]^T \\
&= \Gamma (Q \otimes P_k) \Gamma^T
\end{aligned}$$

E

$$\sum_{x_k, w_k}$$

$$y_k \quad (a) \quad \Gamma \quad (a) \quad (a)$$

$$P_{y_k} = F_k P_k F_k^T + G(\hat{x}_k) Q G(\hat{x}_k) + \Gamma (Q \otimes P_k) \Gamma^T \quad (a)$$

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$$(a) \quad y_k \quad (a)$$

$$\begin{aligned}
P_{y_k} &= E[(y_k - \hat{y}_k)(y_k - \hat{y}_k)^T] = F_k E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] F_k^T \\
&+ F_k E[(x_k - \hat{x}_k)w_k^T G^T(x_k)] + E[G(x_k)w_k(x_k - \hat{x}_k)^T] \\
&+ E[G(x_k)w_k w_k^T G^T(x_k)]
\end{aligned} \quad (a)$$

(a-)

$$P_{y_k} = F_k P_k F_k^T + E[G(x_k)w_k w_k^T G^T(x_k)] \quad (a)$$

$$x_k \quad (a-)$$

$$x_k = \hat{x}_k + (x_k - \hat{x}_k) \quad (a)$$

$$(a-)$$

$$\begin{aligned}
E[G(x_k)w_k w_k^T G(x_k)] &= [G(\hat{x}_k)w_k w_k^T G(\hat{x}_k)] \\
&+ E[G(\hat{x}_k)w_k w_k^T G^T(x_k - \hat{x}_k)] \\
&+ E[G(x_k - \hat{x}_k)w_k^T G^T(\hat{x}_k)] \\
&+ E[G(x_k - \hat{x}_k)w_k w_k^T G^T(x_k - \hat{x}_k)]
\end{aligned} \quad (a)$$

$$w_k \quad x_k \quad E(x_k - \hat{x}_k) = 0 \quad (a)$$

$$\begin{aligned}
E[G(x_k)w_k w_k^T G(x_k)] &= [G(\hat{x}_k)E(w_k w_k^T)G(\hat{x}_k)] \\
&+ E[G(x_k - \hat{x}_k)w_k w_k^T G^T(x_k - \hat{x}_k)] \\
&= [G(\hat{x}_k)Q_k G(\hat{x}_k)] + E[G(x_k - \hat{x}_k)w_k w_k^T G^T(x_k - \hat{x}_k)]
\end{aligned} \quad (a)$$

$$\begin{aligned}
&E[G(x - \hat{x}_k)w_k w_k^T G^T(x_k - \hat{x}_k)] \\
&= E\left[\sum_{i=1}^m \sum_{j=1}^m G_{ci} w_{i,k} w_{j,k} (x_k - \hat{x}_k)(x_k - \hat{x}_k)^T G_{cj}^T\right] \\
&= \sum_{i=1}^m \sum_{j=1}^m G_{ci} E[w_{k,i} w_{j,k} (x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] G_{cj}^T \\
&= \sum_{i=1}^m \sum_{j=1}^m G_{ci} E[w_{k,i} w_{j,k}] E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] G_{cj}^T
\end{aligned} \quad (a)$$



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C_b^n

ω_{nb}^b

ω_{en}^b

ω_{ie}^b

Ω_{nb}^b

a,b,c,d

ϕ, θ, ψ

ϕ, θ, ψ ()

V_e^n

V_d

F_t^n

$\tilde{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$

$\tilde{f}^b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$

Resolution

Vertical

Level Switches

Gimbals

Gain

Direction Cosine Matrix (DCM)

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