

## ***An Investigation of Condensing Steam Flow in a Turbine Cascade with Injection of Water Droplets at Inlet***

A.R. Teymourtash, M.R. Mahpaker, E. Lakzian

### ***ABSTRACT***

During the course of expansion of steam in a Laval nozzle and a cascade of turbine, the state path crosses the saturation line; the steam first supercools and then reverts to equilibrium through the spontaneous formation of droplets or condensation shock, which causes aerodynamics and thermodynamics losses. In this way by formation of droplets and so reducing Gibbs energy, equilibrium is reached. This paper describes a two-phase model and provides an approach for including spontaneous homogeneous nucleation. In order to solve conservation equations, coupled with the equations of formation and growth of the droplets, a 2-D time-marching solution scheme with Baldwin-Lomax

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turbulence model, was used in this study. Pressure distribution and droplets size in a Laval nozzle and a turbine cascade are predicted and compared with empirical results. In the strength of validation, the effect of injection of water droplets into the steam flow in order to control the intensity and location of condensation shock is considered theoretically. A converging nozzle is used to producing droplets at inlet of turbine cascade. The results illustrate that wet steam at inlet of a turbine cascade weakens or delays the condensation shock in the passage of the blades.

**KEYWORDS :** Supercooled, Nucleation, Condensation Shock, Wet Steam, Jameson, Baldwin-Lomax, Injection of Droplets.

[ ]  
CFX-TASC flow

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(A-A )

(A-B)

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$$r^*$$

$$\Delta G = 0$$

[ ]

$$r^* = \frac{2\sigma_r}{\rho_L RT_G \ln\left(\frac{P}{P_s(T_G)}\right)} \quad (1)$$

( )

$$J_{st} = J_{class} = q_c \frac{\rho_G^2}{\rho_L} \sqrt{\left(\frac{2\sigma_r}{\pi M^3}\right)} \exp\left[-\frac{\Delta G^*}{KT_G}\right] \quad (2)$$

[ ]

$$J_{Ka} = \frac{1}{1+\phi} J_{st} \quad (3)$$

$$\phi = \frac{q_c \rho_G}{\alpha_r} \left(\frac{RT_G}{2\pi}\right)^{0.5} \left(\frac{L^2}{RT_G^2} - \frac{L}{2T_G}\right) \quad (4)$$

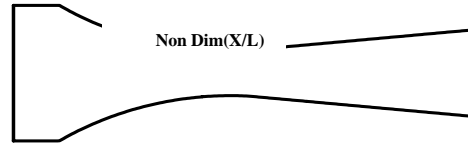
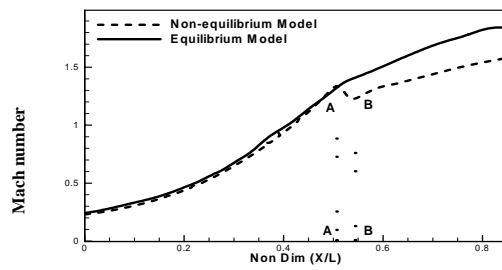
$$L = h_G - h_L \quad (5)$$

$$\alpha_r = \frac{\lambda}{r(1+3.18K_n)} \quad (6)$$

$$\alpha_r = \frac{\lambda}{K_n} \quad (7)$$

[ ]

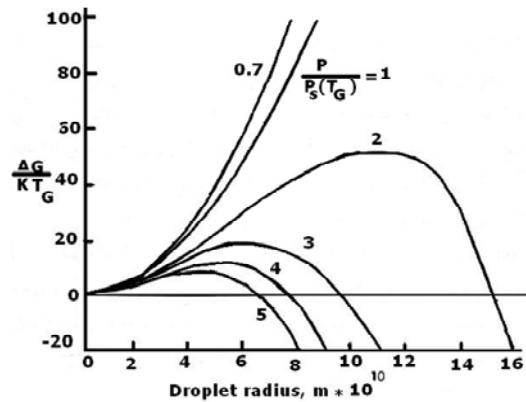
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$$n_g = n_1 \exp\left(\frac{-\Delta G}{KT_G}\right) \quad (8)$$

$n_g$   
K (Clusters/m<sup>3</sup>)



[ ] (P/P<sub>s</sub>(T<sub>G</sub>)) 100°C

100°C

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial R}{\partial x} + \frac{\partial S}{\partial y}$$

$$K_n = \frac{1.88\mu_G}{2r\rho_G\sqrt{RT_G}} \quad ( )$$

$$G = \begin{bmatrix} \rho v \\ \rho v u \\ P + \rho v^2 \\ \rho v h_0 \end{bmatrix} \quad F = \begin{bmatrix} \rho u \\ P + \rho u^2 \\ \rho u v \\ \rho u h_0 \end{bmatrix} \quad W = \begin{bmatrix} \rho \\ \rho v \\ \rho u \\ \rho e_0 \end{bmatrix} \quad ( )$$

$$R = \begin{bmatrix} 0 \\ \sigma_x \\ \tau_{yx} \\ \sigma_x u + \tau_{xy} v + k \frac{\partial T}{\partial x} \end{bmatrix} \quad S = \begin{bmatrix} 0 \\ \tau_{xy} \\ \sigma_y \\ \sigma_y v + \tau_{xy} u + k \frac{\partial T}{\partial y} \end{bmatrix}$$

$$L \frac{dm_r}{dt} = 4\pi r^2 \alpha_r (T_L - T_G) \quad ( )$$

$$k, h_0, e_0$$

$$\sigma_x, \sigma_y, \tau_{xy}, \tau_{yx}$$

$$T_L = T_G + \left[ 1 - \frac{r^*}{r} \right] [T_s(p) - T_G] \quad ( )$$

$$\sigma_x = -\frac{2}{3} \mu_{eff} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu_{eff} \frac{\partial u}{\partial x} \quad ( )$$

$$\sigma_y = -\frac{2}{3} \mu_{eff} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu_{eff} \frac{\partial v}{\partial y} \quad ( )$$

$$\tau_{xy} = \tau_{yx} = \mu_{eff} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad ( )$$

$$\frac{a}{2} (r^2 - r_1^2) + b(r - r_1) + c \ln \left( \frac{r - r^*}{r_1 - r^*} \right) = d \delta t \quad ( )$$

$$\mu_{eff} = \mu + \mu_t \quad ( )$$

$$a = h_G - c_L (T_s(P) - T_D)$$

$$b = (r^* + 1.59\bar{l}) \times [h_G - c_L (T_s(P) - T_D) + c_L r^* (T_s(P) - T_G)] \quad ( )$$

$$c = r^* (r^* + 1.59\bar{l}) [h_G - c_L (T_G - T_D)]$$

$$\frac{\mu(\rho, T)}{\mu^*} = \psi(\delta, \theta) = \psi_0(\theta) \psi_1(\delta, \theta) \quad ( )$$

$$d = \frac{\lambda}{\rho_L} (T_s(P) - T_G)$$

$$T_s(P) \quad c_L \quad h_G$$

$$T_D \quad \bar{l} = 273.16 K$$

$$T_G$$

$$\delta = \frac{\rho}{\rho^*}, \quad \theta = \frac{T}{T^*}, \quad \mu^* = 1 \times 10^{-6} Pa s \quad ( )$$

$$\psi_0, \psi_1 \quad \psi$$

$$\psi_0$$



$$\psi_1(\delta, \theta) = \exp \left[ \delta \sum_{i=1}^{21} n_i (\delta - 1)^{i_i} (\theta - 1)^{j_i} \right] \quad (1)$$

$$\psi_0(\theta) = \theta^{0.5} \left[ \sum_{i=1}^4 n_i^0 \theta^{1-i} \right]^{-1} \quad (2)$$

$$\delta = \frac{\rho}{\rho^*}, \rho^* = \rho_c = 322 \text{ kg m}^{-3} \quad (3)$$

$$\theta = \frac{T^*}{T}, T^* = T_c = 647.096 \text{ K} \quad (4)$$

$$\psi_1 \quad [ ] \psi_0 \quad (5)$$

$i$	$n_i^0$	$i$	$n_i^0$
/	*	/	*
/	*	/	*

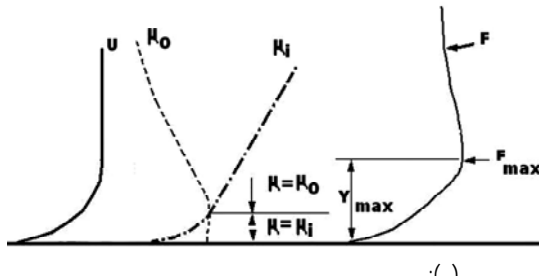
$$[ ] \psi_1 \quad (6)$$

$i$	$I_i$	$J_i$	$n_i$	$i$	$I_i$	$J_i$	$n_i$
/			/	/			/
/			*	/			/
/			*	/			/
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/			/	/			*
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$$\mu_i = \rho (KYD)^2 |\omega| \quad (7)$$

$$Y \quad K = 0.4 \quad \rho \quad D \quad (8)$$

$$D = 1 - \exp \left( \frac{-Y^+}{26} \right) \quad (9)$$



$$\mu_i = \min(\mu_i, \mu_o) \quad (10)$$

$$\mu_o \quad \mu_i \quad (11)$$

$$( ) \quad Y^+ = \frac{Y}{\mu_w} \sqrt{\rho_w |\tau_w|} \quad ( )$$

$$\omega \quad \mu_w \quad \tau_w$$

$$\mu_0 = \rho k C_{cp} F_{wake} F_{kleb} \quad ( )$$

$$k = 0.0168, C_{cp} = 1.6$$

$$F_{kleb} = \left( 1 + 5.5 \left( C_{kleb} \frac{Y}{Y_{max}} \right)^6 \right)^{-1} \quad ( )$$

$$C_{kleb} = 0.3$$

$$F_{wake} = \min \left( Y_{max} F_{max}, C_{wk} Y_{max} \frac{U_{diff}^2}{F_{max}} \right) \quad ( )$$

$$F \quad F_{max} \quad C_{wk} = 0.25$$

$$( )$$

$$( )$$

$$( )$$

$$h = wh_L + (1-w)h_G \quad ( )$$

$$F(Y) = Y |\omega| D \quad ( )$$

$$\frac{1}{\rho} = \frac{w}{\rho_L} + \frac{(1-w)}{\rho_G} \quad ( )$$

$$( )$$

$$F(Y)$$

$$Y_{max}$$

$$( )$$

$$U_{diff}$$

$$U_{diff} = \left( \sqrt{u^2 + v^2} \right)_{max} - \left( \sqrt{u^2 + v^2} \right)_{min} \quad ( )$$

$$v_{min} \quad v_{max}$$

$$u$$

$$v_L = \frac{v_C + a(T_C - T_S)^3 + b(T_C - T_S)^4 + c(T_C - T_S)^4}{1 + d(T_C - T_S)^3 + e(T_C - T_S)} \quad ( )$$

$$w$$

$$647.27^\circ K$$

$$(T_C) \quad 3.1975 \text{ cm}^3/\text{g}$$

$$w = \frac{M_L}{M_L + M_G} \quad ( )$$

$$L \quad G \quad ( )$$

$$a = -0.3151548$$

$$b = -1.203374 \times 10^{(-5)}$$

$$c = 7.48908 \times 10^{(-13)}$$

$$d = 0.1342489$$

$$e = -3.946263 \times 10^{(-3)}$$

$$\text{in SI units}$$

$$( ) \quad (T_L)$$

$$h_L = c_L(T_L - T_D) \quad ( )$$

$$( )$$

$$273.15^\circ K$$

$$T_D$$



$$e = e_0 - \frac{u^2 + v^2}{2}$$

$$(۴۳) \quad \left( \frac{\partial h_G}{\partial P_G} \right) = v_G - T_G \left( \frac{\partial v_G}{\partial T_G} \right)_P \quad ( )$$

$(T_G)$

:

$(T_G, P)$

$$h_G = \int \left[ v_G - T_G \left( \frac{\partial v_G}{\partial T_G} \right) \right] dp + F_h(T_G) \quad ( )$$

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$$h_G = \frac{RT_G}{2} \left( \sqrt{1 + \frac{4PB}{RT_G}} - 1 \right) \left( 1 - \frac{T_G}{B} \frac{dB}{dT_G} \right) + F_h(T_G) \quad ( )$$

$T_G$

( )

$$F_h(T_G) = a \ln T_G + b T_G + c T_G^2 \quad ( )$$

$$- d T_G^3 + e T_G^4 - f T_G^5 + const.$$

$$F(T_G) = \left( e_0 - \frac{u^2 + v^2}{2} \right) - \left( h_G - \frac{P}{\rho_G} \right)$$

(۴۴)

( )

$$a = 46$$

$$b = 1.47276$$

$$c = 0.419465 \times 10^{-3}$$

$$d = 7.33297 \times 10^{-8}$$

$$e = 6.16548 \times 10^{-11}$$

$$f = 1.94063 \times 10^{-14}$$

$$const. = 1782.24 \text{ in SI units}$$

$$\Delta T_G = \frac{-F(T_G)}{dF(T_G)/dT_G}$$

(۴۵)

[ ]

$$T_{G(new)} = T_{G(old)} + \Delta T_G$$

(۴۶)

$$|\Delta T_G / T_G| \leq \varepsilon$$

$10^{-4}$

$\varepsilon$

( )

$h_G$

$T_G$

$P$

$(\rho, \rho u, \rho v, \rho e_0)$

$$J_{min} = 10^{18} \text{ Nuclei/kg.sec}$$

[ ]

جریان جوانه‌زایی II

I. جریان خشک

( )

$D$

$\rho_G$

$(\rho)$

$h_G$

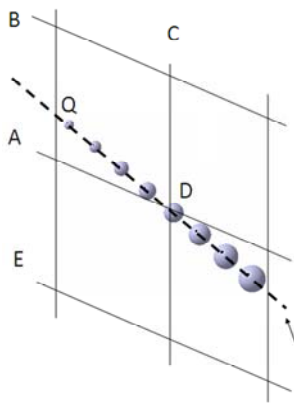
$(h)$

( )

$$e = h_G - \frac{P}{\rho_G}$$

(۴۲)

$w$



Calculated streamline flow

$$\delta t = \frac{\Delta X_D}{((u_Q + u_D)/2)} \quad (47)$$

$$F_1 = e_D - e'_D = \left[ (1-w)h_G + wh_L - \frac{P}{\rho} \right] - e'_D$$

$$F_2 = \frac{1}{\rho_D} - \frac{1}{\rho'_D} = [(1-w)v_G + wv_L]_D - \frac{1}{\rho'_D} \quad (48)$$

$$F_3 = r_D - r'_D$$

D Q

$$N_Q = r_Q \quad (49)$$

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$J_{min}$

$$r'_D, \rho'_D, e'_D$$

$$T_L, T_G, T_S(P)$$

$$\rho_L, \rho_G, h_G$$

$$r = -1.59\bar{I} + \sqrt{(1.59\bar{I})^2 + r_1(r_1 + 2(1.59\bar{I})) + 2\lambda \left( \frac{T_L - T_G}{h_G - h_L} \right) \delta t} \quad (49)$$

$$\frac{\partial F_1}{\partial T_G}, \frac{\partial F_1}{\partial T_L}, \frac{\partial F_1}{\partial T_S(P)}$$





$F_1, F_2$

$r'_D$

( )

( )

[ ]

C

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( )

$F_3$

$T'_{LD}$

$T_{LD}$

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(

$$F_3 = T_{LD} - T'_{LD}$$

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[ ]

C

(CFL)

( )

C

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$$\Delta t = FT \cdot CFL \cdot \frac{\Delta x}{(|V| + a)}$$

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[ ]

|V|

$\Delta x$

CFL

FT

a

$$P_{0in} = 25 \text{ Kpa}$$

$$T_{0in} = 358.6 \text{ K}$$

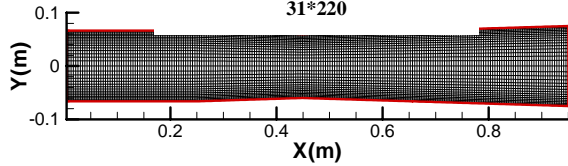
$$P_{OUT} = 8 \text{ Kpa}$$

$$CFL = 2\sqrt{2}$$

[ ]

Nozzle C- P0=25 kPa-T0=358.6 K  
[19]  
31\*220

0.4 FT



C

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$(\frac{P}{P_{0in}})$

( )

$$\left| \frac{\Delta u}{u} \right| < 0.01\%$$

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$$R_{error} = \frac{1}{N_{node}} \left[ \sqrt{\sum_{ij} (\delta \rho_{ij}^2)} \right]$$

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[ ]

$\delta \rho$

$N_{node}$

$$R_{error} \leq 1 \times 10^{-4}$$

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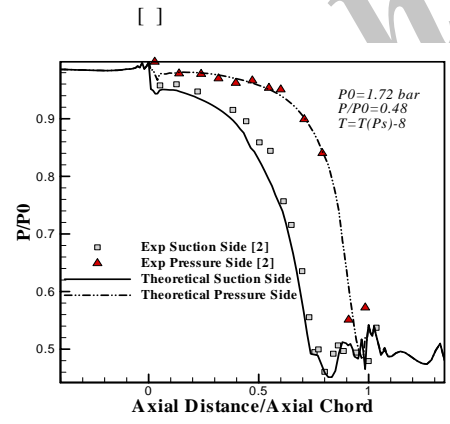
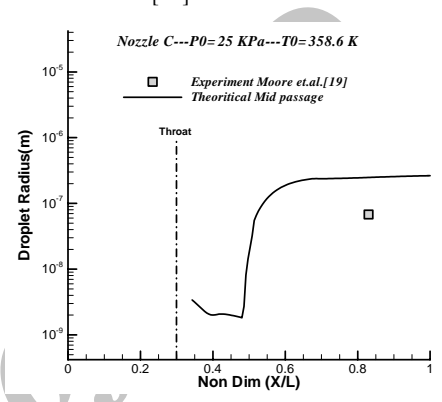
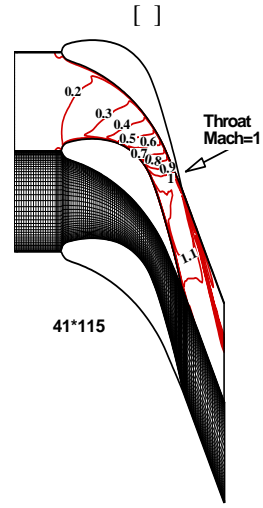
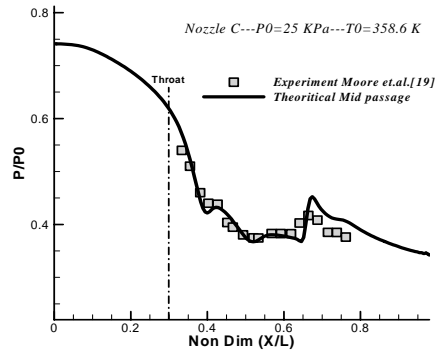
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$M = 1$

$\left(\frac{P}{P_{0in}}\right)$

$x/c = 0.7$



$P_{0in} = 172 \text{ Kpa}$

$T_{0in} = 380 \text{ K}$

$P_{OUT} = 82.56 \text{ Kpa}$

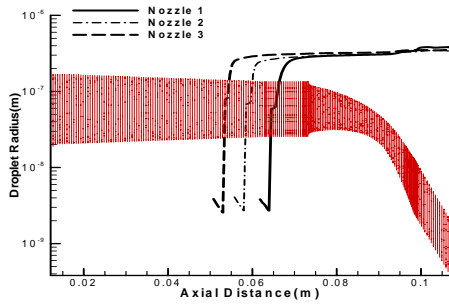
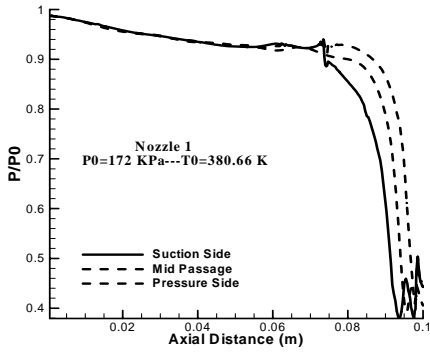
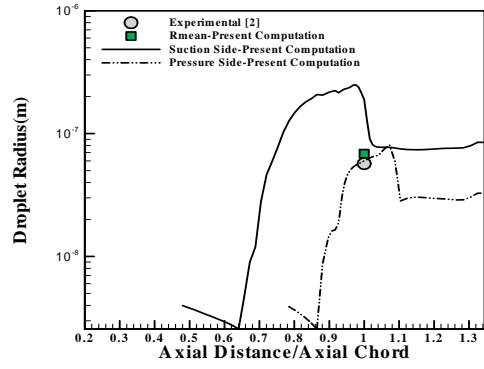
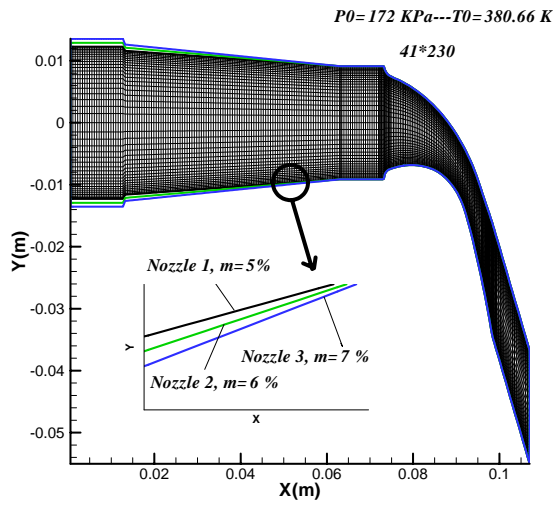
$\theta_{in} = 0^\circ$



X=0.053 m

X=0.065 m

X=0.058 m



(X=0.065 m)

(X=0)

(X>0.075 m)

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$(P/P_{0in})$

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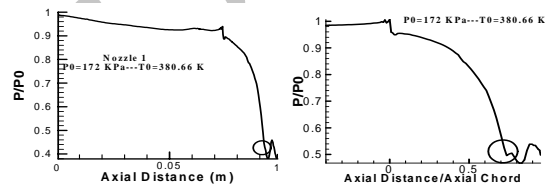
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$S_c = \mu_G / \rho_G D$

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$m_r$



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$\alpha_r$   
 $\mu_G$   
 $\mu_{eff}$   
 $\rho$   
 $\lambda$   
 $\sigma$   
 $\rho_s(T_L, r)$

$T_L$

$r$

$r$

$T$

$T_S(P)$

$P$

$\Delta T$

$[T_s(P) - T_G]$

$t$

$u$

$x$

$v$

$y$

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