

Analysis of Free Vibration of Sandwich Panels Based on Improved High-order Sandwich Panel Theory

B. Eftari, S.M.R. Khalili, A. Jafari, K. Malekzadeh

ABSTRACT

A new improved high-order theory is presented to investigate the dynamic behavior of sandwich panels with flexible core. Shear deformation theory is used for the face sheets while the three-dimensional elasticity theory is used for the core. Displacements in the core are assumed as polynomial with unknown coefficients. Inertia forces, moments of inertia and shear deformations in the core and the face sheets are taken into consideration. Unlike the previous improved theory, the in-plane normal and shear stresses in the core are considered. The governing equations and the boundary conditions are derived by Hamilton's principle. Closed form solution is achieved using the Navier method and solving the eigenvalues. The numerical results of present analysis are compared with the available numerical or theoretical results in the literatures. It indicates that the present new modified theory is more accurate than the other developed theories for sandwich panels. The variations of the non-dimensional natural frequency with respect to the various geometrical and material parameters are investigated.

KEYWORDS : Sandwich Panel, Flexible Core, Improved Higher-Order Theory, Free Vibrations

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h, h_b, h_t, h_c

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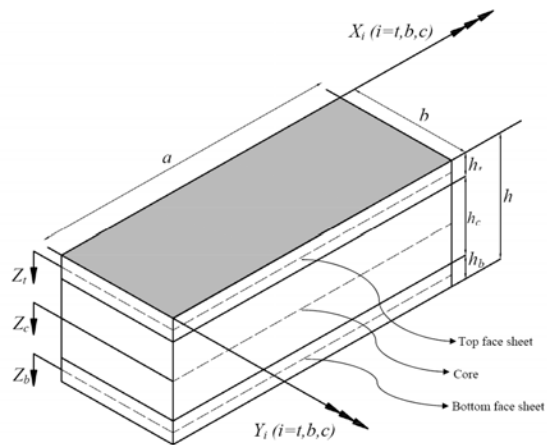
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$$\delta \int_{t_1}^{t_2} (U + V - T) dt = 0$$

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$$\begin{aligned}
 u_j(x, y, z, t) &= u_0^j(x, y, t) + z_j \psi_x^j(x, y, t) \\
 v_j(x, y, z, t) &= v_0^j(x, y, t) + z_j \psi_y^j(x, y, t) \quad (j=t, b) \\
 w_j(x, y, z, t) &= w_0^j(x, y, t)
 \end{aligned}$$



$$\begin{aligned}
 u_c(x, y, z_c, t) &= u_0(x, y, t) + z_c u_1(x, y, t) + z_c^2 u_2(x, y, t) \\
 &+ z_c^3 u_3(x, y, t) \\
 v_c(x, y, z_c, t) &= v_0(x, y, t) + z_c v_1(x, y, t) + z_c^2 v_2(x, y, t) \\
 &+ z_c^3 v_3(x, y, t) \\
 w_c(x, y, z_c, t) &= w_0(x, y, t) + z_c w_1(x, y, t) + z_c^2 w_2(x, y, t)
 \end{aligned} \quad ()$$

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$$\begin{aligned}
 u_c(z_c = -h_c/2) &= u_0^t + \frac{1}{2} h_c \psi_x^t \\
 v_c(z_c = -h_c/2) &= v_0^t + \frac{1}{2} h_c \psi_y^t \\
 w_c(z_c = -h_c/2) &= w_0^t \\
 u_c(z_c = h_c/2) &= u_0^b - \frac{1}{2} h_c \psi_x^b \\
 v_c(z_c = h_c/2) &= v_0^b - \frac{1}{2} h_c \psi_y^b \\
 w_c(z_c = h_c/2) &= w_0^b
 \end{aligned}$$

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$$\begin{aligned}
 u_2 &= (2(u_0^b + u_0^t) - h_b \psi_x^b + h_b \psi_x^t - 4u_0) / h_c^2 \\
 u_3 &= (4(u_0^b - u_0^t) - 2(h_b \psi_x^b + h_b \psi_x^t) - 4h_c u_0) / h_c^3 \\
 v_2 &= (2(v_0^b + v_0^t) - h_b \psi_y^b + h_b \psi_y^t - 4v_0) / h_c^2 \\
 v_3 &= (4(v_0^b - v_0^t) - 2(h_b \psi_y^b + h_b \psi_y^t) - 4h_c v_0) / h_c^3
 \end{aligned}$$

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Top face sheet
Core
Bottom face sheet

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V, T, U

δ

t₂-t₁

ε_{ii} σ_{ii} (i=x, y, z)

γ_{iz} τ_{iz} (i=x, y)

ε_{zz} σ_{zz}

V_c, V_b

, V_t

σ_{yy}^c δε_{yy}^c σ_{xx}^c δε_{xx}^c ()

τ_{xy}^c δγ_{xy}^c

$$\begin{aligned}
 \delta U &= \int_{V_t} (\sigma_{xx}^t \delta \epsilon_{xx}^t + \sigma_{yy}^t \delta \epsilon_{yy}^t + \tau_{xy}^t \delta \gamma_{xy}^t + \tau_{xz}^t \delta \gamma_{xz}^t + \tau_{yz}^t \delta \gamma_{yz}^t) dv \\
 &+ \int_{V_b} (\sigma_{xx}^b \delta \epsilon_{xx}^b + \sigma_{yy}^b \delta \epsilon_{yy}^b + \tau_{xy}^b \delta \gamma_{xy}^b + \tau_{xz}^b \delta \gamma_{xz}^b + \tau_{yz}^b \delta \gamma_{yz}^b) dv \\
 &+ \int_{V_c} (\sigma_{zz}^c \delta \epsilon_{zz}^c + \sigma_{yy}^c \delta \epsilon_{yy}^c + \sigma_{xx}^c \delta \epsilon_{xx}^c + \tau_{xy}^c \delta \gamma_{xy}^c + \tau_{xz}^c \delta \gamma_{xz}^c + \tau_{yz}^c \delta \gamma_{yz}^c) dv
 \end{aligned} \quad ()$$

$$\left[\begin{array}{cccc} / & / & / & / \\ / & / & / & / \\ / & / & / & / \\ / & / & / & / \end{array} \right] \left[\begin{array}{cccc} / & / & / & / \\ / & / & / & / \\ / & / & / & / \\ / & / & / & / \end{array} \right]$$

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$$\bar{\omega} = \frac{\omega \alpha^2}{h} \sqrt{\frac{\rho_0}{E_0}}$$

$h_c/h = /$)

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HEREX-C70,130PVC

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$E_1 = 24.51 \text{ GPa}$, $E_2 = 7.77 \text{ GPa}$, $G_{12} = G_{13} = 3.34 \text{ GPa}$,
 $G_{23} = 1.34 \text{ GPa}$, $\nu_s = 0.078$, $\rho_s = 1800 \text{ Kg/m}^3$

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$E_c = 103.63 \text{ MPa}$, $G_c = 50 \text{ Mpa}$, $\nu_c = 0.32$,
 $\rho_c = 130 \text{ Kg/m}^3$

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$$\begin{aligned} & N'_{xx,xx} + N'_{xy,yy} + \bar{n}'_{xt} + \frac{1}{6} h_c E_c u_{0,xx} + \frac{1}{40} h_c^3 E_c u_{2,xx} \\ & + \frac{1}{6} h_c G_{xye} (u_{0,yy} + v_{0,xy}) + \frac{1}{40} h_c^3 G_{xye} (u_{2,yy} + v_{2,xy}) \\ & - \frac{1}{3} h_c G_{xzc} (2u_2 + w_{1,x}) - \frac{1}{40} h_c^2 E_c u_{1,xx} - \frac{1}{112} h_c^4 E_c u_{3,xx} \\ & - \frac{1}{20} h_c^2 G_{xye} (u_{1,yy} + v_{1,xy}) - \frac{1}{112} h_c^4 G_{xye} (u_{3,yy} + v_{3,xy}) \\ & + G_{xzc} (u_1 + w_{0,x}) + \frac{1}{40} h_c G_{xzc} (3u_3 + w_{2,x}) \\ & = m' u'_{0,\mu} - \frac{2}{h_c^2} I_2^c u_{0,\mu} + \frac{4}{h_c^2} I_4^c u_{1,\mu} - \frac{2}{h_c^2} I_4^c u_{2,\mu} + \frac{4}{h_c^2} I_6^c u_{3,\mu} \end{aligned} \quad ()$$

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$$\{u'_0, \psi'_x, v'_0, \psi'_y, w'_0, u'_0, \psi'_x, v'_0, \psi'_y, w'_0, u_1, v_0, v_1, w_0\} \quad ()$$

$$([k] - [M] \times \omega^2) \{X_0^*\} = \{0\} \quad ()$$

[M] [K] ()

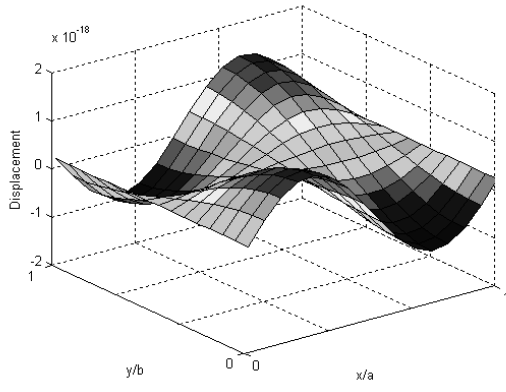
() [M] [K]

$$K_{(1,1)} = \frac{47G_c}{15h_c} + \alpha^2 [A'_{(1,1)} + \frac{3h_c E_c}{35}] + \beta^2 [B'_{(3,3)} + \frac{3h_c G_c}{35}]$$

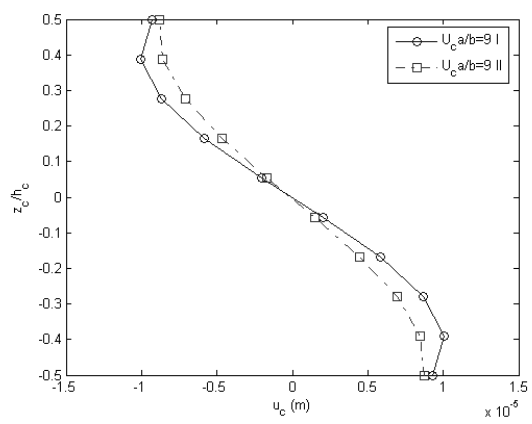
$$M_{(1,1)} = m' + \frac{m^c \rho_c^4}{20} + \frac{m^c \rho_c^6}{28} \quad ()$$

$$m' \quad m^c \quad \beta_n = n\pi/b \quad \alpha_m = m\pi/a \quad ()$$





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 E^c/E_{11}^t
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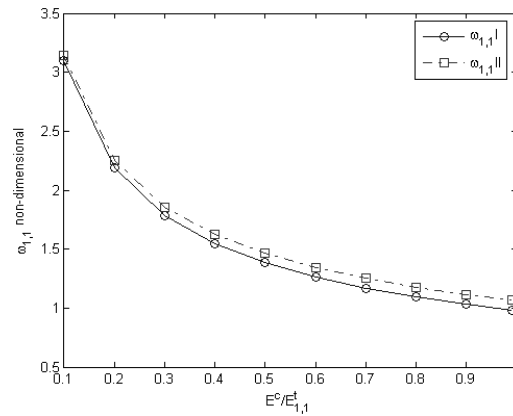
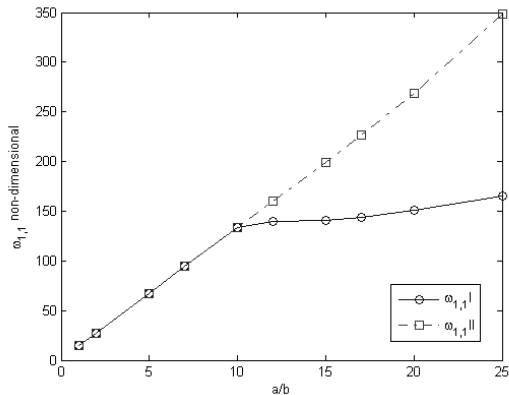
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Hamilton Principle

Navier method

Higher-Order Sandwich Plate Theory

Third-Order Shear Deformation

Mixed layerwise

The Higher-Order Sandwich Panels Theory

Improved Higher-Order Sandwich Plate Theory

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