

## ***Modification of a Critical-state Constitutive Model for the Prediction of Inherent Anisotropy in Sands***

R. Rasouli; S. M. R. Imam and V. Masoumifard

### ***ABSTRACT***

A critical state constitutive model for sands was previously developed with emphasis on capturing the main aspects of the behavior of loose liquefiable sands. The model, which was presented in detail in previous publications, was formulated and verified for various drained and undrained loadings of sands under monotonic conditions. However, in order to enable the model to predict the behavior of in-situ soils, which often exhibit strong inherent anisotropy, it was found important to extend the model formulation such that it will also be able to predict the behavior of sands with strong anisotropy.

In this paper, it is shown that by adding a new anisotropy parameter to the model, it is possible to simulate the behavior of strongly anisotropic sands. The anisotropy parameter depends on sand fabric and loading condition. Ability of the modified model to account for soil inherent anisotropy is verified by comparing observed and predicted responses of inherently anisotropic sand subjected to various loadings.

### ***KEYWORDS***

Constitutive modeling, Anisotropy, Numerical analysis, Sand behavior

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rouzbeh\_rasouli63@yahoo.com

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rimam@aut.ac.ir

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vmasomi@gmail.com

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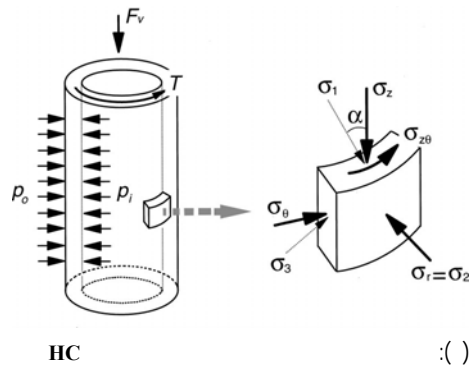
( $q = \sigma_1 - \sigma_3$ )

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( $M_p$ )

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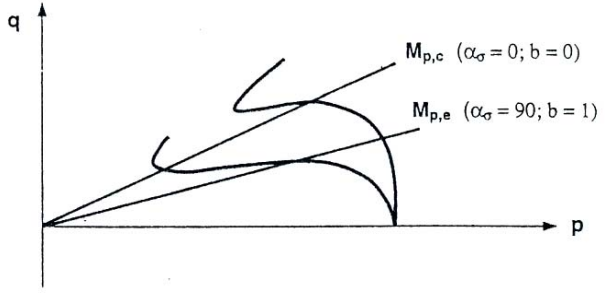
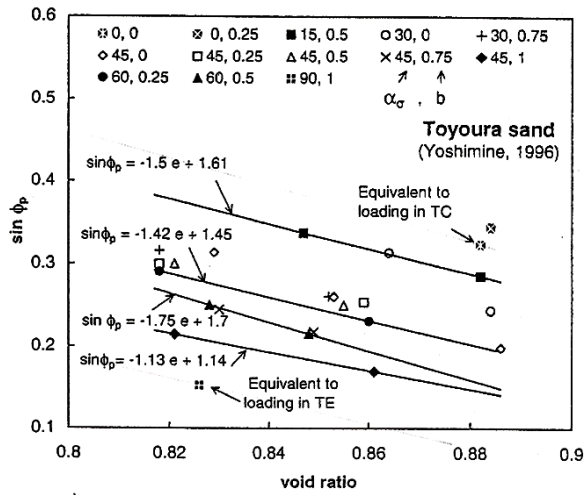
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$M_p$





$$b = \frac{(\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)}{\alpha}$$

$$M_p = \frac{6(1 - b + b^2)^{1/2} \sin \varphi_p}{3 + (2b - 1) \sin \varphi_p}$$

$$F_{ij} = \frac{1}{2N} \sum_{k=1}^{2N} n_i^k n_j^k$$

= N

$$A = n_j^k n_i^k$$

( )

$$F_3 \quad F_2 \quad F_1$$

$$A = \frac{\tilde{R}}{M_c g(\tilde{\theta})} - 1 \quad ( )$$

$$\theta \quad \tilde{R} \quad \tilde{R} \quad ( ) \quad F_3 \quad F_2$$

$M_c$

$$F_1 = 1 - (F_2 + F_3) = 1 - 2F_3 : \quad F_3 \quad F_2$$

A =

b  $\alpha$  A

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$\alpha =$  b =

$\alpha =$  b =

$$F_{ij}' = \frac{1}{3 + \Delta} \begin{pmatrix} 1 - \Delta & 0 & 0 \\ 0 & 1 + \Delta & 0 \\ 0 & 0 & 1 + \Delta \end{pmatrix} \quad ( )$$

=  $\Delta$

A

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$\alpha$

b  $\alpha$

b

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$\sin \varphi_p$  b  $\alpha$

A

$$\Delta = \frac{1}{2N} \sqrt{\left( \sum_{k=1}^{2N} \cos 2\varphi_k \right)^2 + \left( \sum_{k=1}^{2N} \sin 2\varphi_k \right)^2} \quad ( )$$

n k =  $\varphi_k$

(  $X_2' - X_3'$  )

$\Delta$

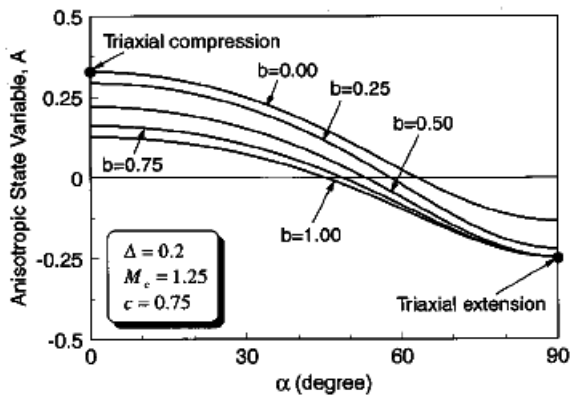
$\Delta$

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$X_2' - X_3'$

A

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b  $\alpha$

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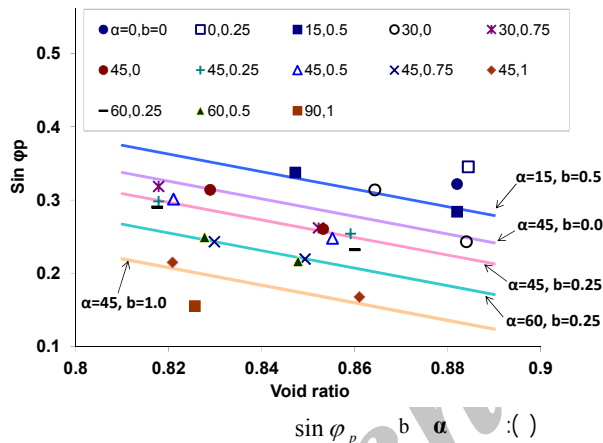
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$\sigma_{ij}$



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$A_c$   
 $A_e$   
 $A$   
 $b$   $\alpha$   
 $a_p$   
 $( )$   
 $( )$   
 $\sin \varphi_p$   
 $( )$   
 $b$   $\alpha$   
 $b$   $\alpha$   
 $M_p$   $( )$   $( )$   $( )$   
 $M_p$   
 $\sin \varphi_p$   $( )$   
 $( )$



$$\eta^2 - 5M_p^2[1 - (p/p_c)^{0.5}] = 0 \quad ( )$$

$$P_c \quad \eta = q/p$$

$$M_p \quad q =$$

$$M_{p,c} = \frac{6 \sin \varphi_{p,c}}{3 - \sin \varphi_{p,c}} \quad ( )$$

$$M_{p,e} = \frac{6 \sin \varphi_{p,e}}{3 + \sin \varphi_{p,e}} \quad ( )$$

$$\varphi_{p,e} \quad \varphi_{p,c}$$

$$\sin \varphi_{p,c} = \sin \varphi_{\mu} - k_p \psi_p \quad ( )$$

$$\sin \varphi_{p,e} = \sin \varphi_{\mu} - k_p \psi_p + a_p \quad ( )$$

$$\psi_p = \varphi_{\mu}$$

$$a_p \quad k_p$$

$$\psi_p$$

$$M_p \quad ( ) \quad ( )$$

$$( ) \quad M_p$$

$$( ) \quad ( )$$

$$\sin \varphi_p = \sin \varphi_{\mu} - k_p \psi_p + a_p(A) \quad ( )$$

$$a_p(A) = \frac{A_c - A}{A_c - A_e} \times a_p \quad ( )$$

$\alpha$  b = % %

( ) ( )

b =

%

$\alpha$  b =

$\alpha$

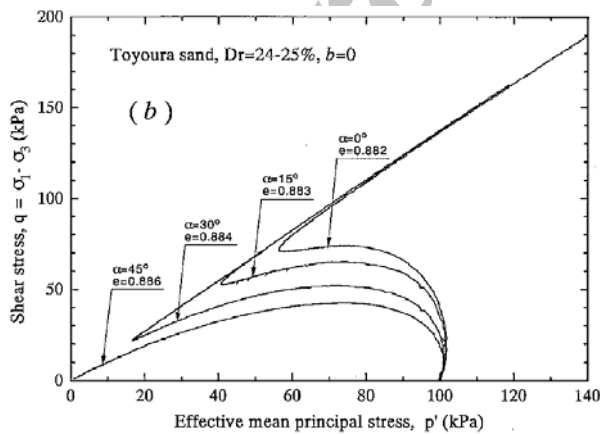
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$\alpha$  b =

% %

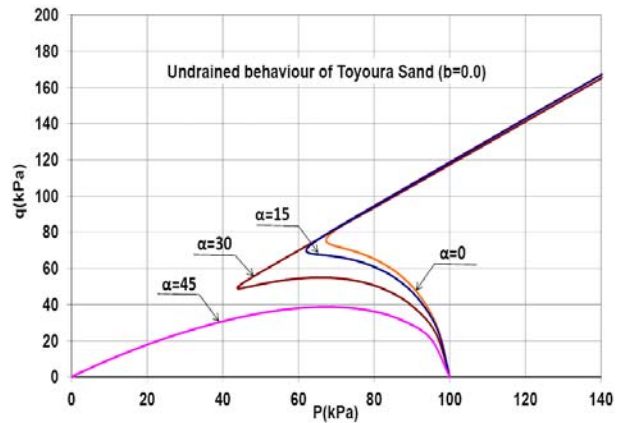
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Parameter type	Parameter name	
Peak state	$k_p$	/
	$\varphi_p$	
	$a_p$	/
Stress-dilatancy	$\varphi_{cs}$	
	$k_{pT}$	/
	$a_{pT}$	/
Plastic stiffness	$h$	
Elasticity	$G_a$	
	$K_a$	
Anisotropy	$\Delta$	/
	$c$	/
	$M_c$	/



Dr = %

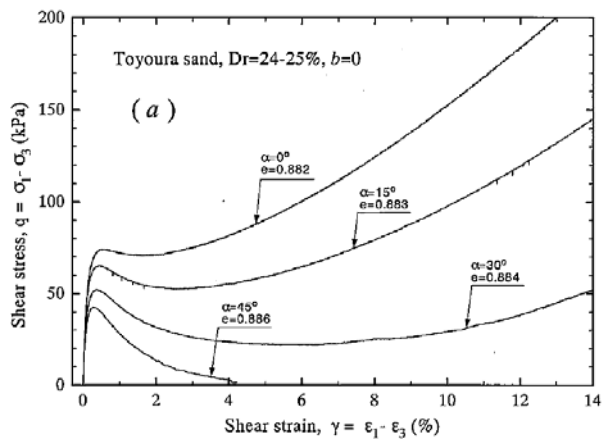
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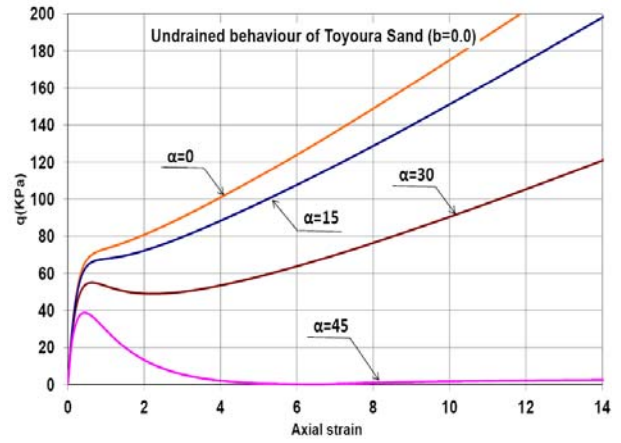


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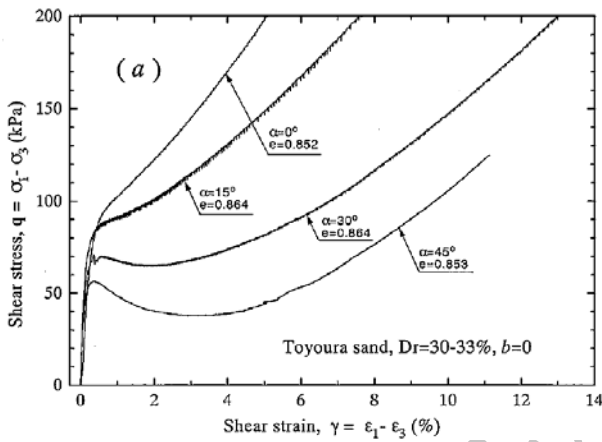


$Dr = \%$

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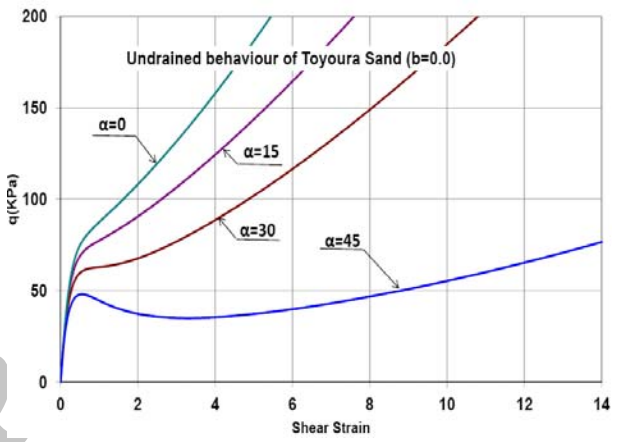


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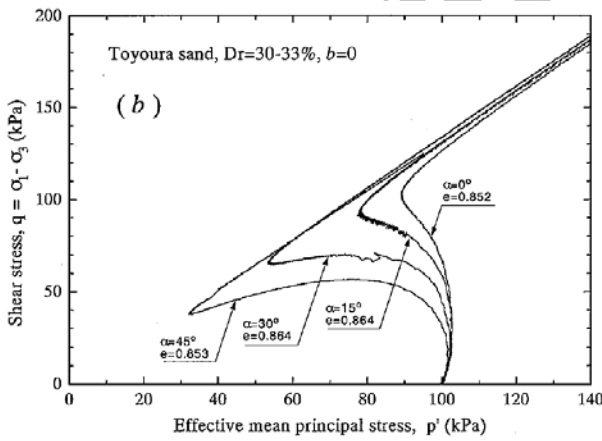


$Dr = \%$

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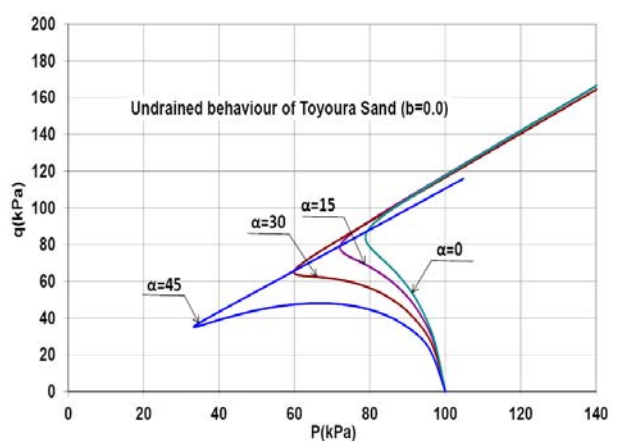


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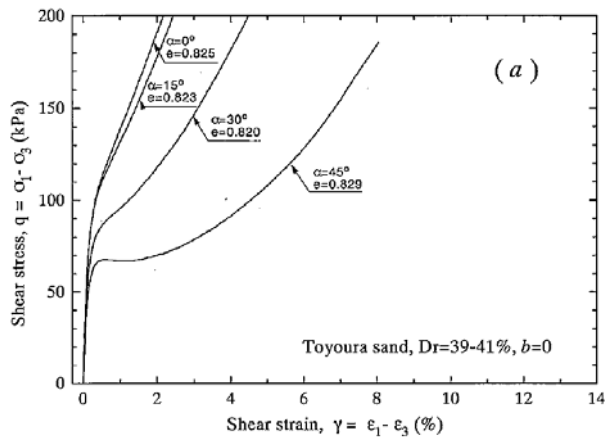


$Dr = \%$

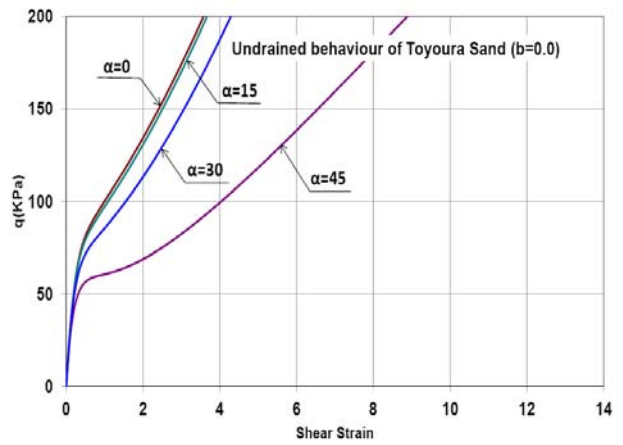
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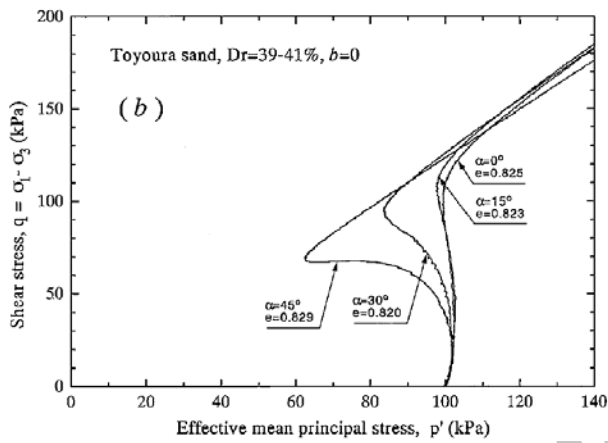
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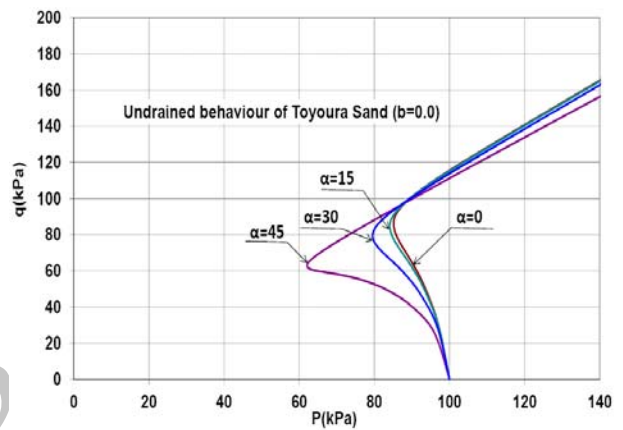
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$$\sin \varphi_{p,c} = \sin \varphi_{\mu} - k_p \psi_p$$

$$\sin \varphi_{p,e} = \sin \varphi_{\mu} - k_p \psi_p + a_p$$

$$\psi_p = 0$$

$$a_p = k_p$$

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$$\sin \varphi_f = \sin \varphi_{cs} - k_f \psi_p$$

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$$k_f \varphi_{cs} ($$

$\varphi_{\mu}$

$$f = (\eta - \alpha)^2 - M_{\alpha}^2 \left[ 1 - (p/p_c)^2 \right] = 0$$

$$M_{\alpha}^2 = (5M_p - \alpha)(M_p - \alpha)$$

$$M_{p,c} = \frac{6 \sin \varphi_{p,c}}{3 - \sin \varphi_{p,c}}$$

$$M_{p,e} = \frac{6 \sin \varphi_{p,e}}{3 + \sin \varphi_{p,e}}$$

$$\varphi_{p,e} \quad \varphi_{p,c}$$



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$$G = G_r \frac{h}{(p_f - p_c)} \left( \frac{p_f - p_c}{(p_f - p_c)_{ini}} \right)$$

$$G = G_r \frac{(2.973 - e)^2}{1 + e} (p/p_a)^{1/2}$$

$$K = K_r \frac{(2.973 - e)^2}{1 + e} (p/p_a)^{1/2}$$

$K_r$   $G_r$

$$d = \frac{d\varepsilon_p^p}{d\varepsilon_q^p} = A (M_{cs} - \eta)$$

A

$$A_c = 9/(9 - 2M_{PT,c}\eta + 3M_{PT,c})$$

$$A_e = 9/(9 - 2M_{PT,e}\eta - 3M_{PT,e})$$

$M_{PT,e}$   $M_{PT,c}$

$$\sin\phi_{PT,c} = \sin\phi_{cs} + k_{PT} \psi$$

$$\sin\phi_{PT,e} = \sin\phi_{cs} + a_{PT} + k_{PT} \psi$$

$$\frac{\partial p_c}{\partial \varepsilon_q^p} = \frac{hG}{(p_f - p_c)_{ini}} (p_f - p_c)$$

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Transversely isotropic  
trace  
vector magnitude  
Lode Angle  
state parameter  
flow rule

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