

Logical Content and Its Malcontents*

David Miller

Emeritus Reader, University of Warwick, Coventry, CV4 7AL UK. E-mail: d.w.miller@warwick.ac.uk

Article Info

ABSTRACT

Article type:

Research Article

Article history:

Received 29 January 2023

Received in revised form 23 April 2023

Accepted 15 May 2023

Published online 18 June 2023

Keywords:

logical content, assertoric content, Karl Popper, William Minto, disjunction, objective knowledge, material conditional, partial truth, Ross's paradox

The doctrine that the content of the conclusion of a deductively valid argument is included in the content of its premises, taken jointly, is a familiar one. It has important consequences for the question of what value valid arguments possess, since it indicates the poverty of three traditional answers: that arguments may and should be used as instruments of persuasion, that they may and should be used as instruments of justification; and that they may and should be used to advance knowledge. The truth is, however, that in each of these cases the argument has only a managerial role and, if there is any work done, it is the premises that do it. It will be maintained that this point has little force against the critical rationalist answer, which I shall defend, that the principal purpose of deductive reasoning from an assemblage of premises is the exploration of their content, facilitating their criticism and rejection. That said, the main aim of the present paper is not to promote critical rationalism but to consider some published objections to the doctrine that a statement asserts every statement that is validly deducible from it. The alleged counterexamples to be considered fall roughly into two groups: statements that emerge with time from a rich mathematical or empirical theory, but were originally unformulated and are deducible from the theory only in a non-trivial way (Frederick 2011, 2014; Williamson 2012); and statements, notably disjunctions, that are easily formulated and are deducible from a theory in a trivial way (Schurz & Weingartner 1987; Mura 1990, 2008; Gemes 1994; Yablo 2014). Each of these counterexamples will be evaluated and dismissed.

Cite this article Miller, D. (2023). Logical Content and Its Malcontents. *Journal of Philosophical Investigations*, 17(42), 281-297. <http://doi.org/10.22034/jpiut.2023.16588>



© The Author(s).
<http://doi.org/10.22034/jpiut.2023.16588>

Publisher: University of Tabriz

* With some variations, mostly minor but including a change of title, this paper was presented, with several cuts, at the colloquium *Does Valid Reasoning Matter?* which was held from September 13 to September 16, 2018 at the CONGRESS CENTER ACADEMIA, Stará Lesná, The High Tatras, Slovakia. The respondent at this colloquium was the late Danny Frederick, whose revised comments appear in Chapter 7 of Frederick (2020). For the most part these comments ignore the present paper and revert to the predominantly subjectivistic understanding of human knowledge, rejected in §3 below, that critical rationalism is concerned to overcome. The main features of the theory of objective knowledge presented in Popper (1972) are accepted here with little explicit comment. In quotations from the writings of others I have quietly brought their notation into line with my own.

Introduction

In this paper I shall try to defend, though in an indirect way, the critical rationalist doctrine that the most important intellectual use of deductive arguments (which may be distinguished from deductive inferences and reasoning) and deductive inferences and reasoning (which may be distinguished from deductive arguments) is to enable us to investigate what our hypotheses and assumptions assert, that is, to elucidate aspects of their content, and thereby, if we are lucky, to bring to light in what ways those hypotheses and assumptions are defective. As part of this defence I shall reject three common claims about the proper purposes of deductive inferences: that they may and should be used as instruments of persuasion; that they may and should be used as instruments of justification or validation; and that they may and should be used to advance knowledge. These points were made succinctly in Miller (2005) with regard to the seemingly unstoppable educational fashion known as *critical thinking*, and they were made at greater length, and more generally, in Chapter 3 of Miller (2006) (see also Miller 1995). I pointed out that in each case it cannot be the argument (whether valid or invalid) that does the work that is to be done (if indeed any real work is done) — the persuading, the justifying, the contributing to knowledge — but the premises of the argument. If a valid argument plays any part in persuading an agent that its conclusion is true, it is only by revealing that the agent must be already persuaded that the premises are true. The argument itself has no persuasive power. If a valid argument plays any part in justifying its conclusion, it is only because the premises are already justified. The argument itself has no authenticating power. If a valid argument plays any part in contributing to our knowledge of the world, it is only by revealing that the premises implicitly contain that contribution. Why validity is so valuable is that a valid argument may be used critically, to reveal that a hypothesis or theory that has been endorsed conflicts with some other item of information (or in some cases, with itself). A valid argument that is used critically, and is successful, does no more than any other argument in uncovering something that is already present, but if that something is an inconsistency, we can use it to reject at least one of the premises from which it was drawn. Our knowledge is not advanced when we learn more by means of reasoning; it is abridged.

I have nothing more to say about persuasion and justification, except to ask why either is regarded as valuable. No doubt arguments, like bribery and physical violence, can be used to persuade, or to justify, but we should question whether they should be. A rational, open-minded person does not need to be persuaded; nor does he need justification for what he thinks. In sum, it is only in connection with the third rejected claim above, the one about the advancement of knowledge, that there is, I think, much more to be said. But something does have to be said. In recent years there have been put forward several related criticisms of the view, which this paper endorses, that the content of a hypothesis, that is, what it asserts or says, should be identified with the class of statements that are deducible from it. It will be necessary to explain the various ways in which each of these criticisms is mistaken.

There are two short preliminary sections to be endured before I turn to my main topic, the correctness or incorrectness of the identification of what a statement says with the class of statements that can be deduced from it; that is, everything that it logically implies. In §1, I shall bring to your attention, but not dwell on further, a noteworthy historical prevision of the central critical rationalist teaching about deductive reasoning, that its primary purpose is inquisitive and inquisitorial. In §2, I shall report and discuss some definitions, familiar to logicians, of logical content. They ought not to be contentious, unless it is supposed that I am planning to settle a philosophical question by mere stipulation. I have no intention of doing this. The issue that is waiting to be discussed is whether the *content* of a statement or theory or hypothesis h (which may, for the sake of clarity, sometimes be called its *assertoric content*) always includes the *logical content* of h , namely $Cn(h)$.

In §3, I shall analyse two fairly recent attempts to bolster the not uncommon view that the existence of mathematical discoveries, and of surprising deductive connections between previously unconnected ideas, shows that deductive reasoning, if undertaken with enthusiasm and perseverance, has a miraculous ability to advance our knowledge. In §4, I shall turn to a rather different type of objection to the thesis that everything that can be deduced from a statement is asserted by it; according to this line of argument, from almost any statement we may deduce, in a transparent way (in particular, by a single use of the rule VI of disjunction introduction) a statement (indeed, any number of statements) that cannot realistically be regarded as part of what the original statement says, its assertoric content. My main response to this objection is that it has little force when disjunctions are rewritten as material conditionals. My response to most of the ways in which the objection has been developed into theories of partial truth and verisimilitude is not dissimilar, but I shall say little on that subject on this occasion. For the most part these theories ignore, or brush aside, the problem of language dependence that has been around, and not properly answered, for over 40 years. I have discussed this problem at length in Chapter 11 of my (2006), and shall revert to it, but quite briefly, in §4.2 below.

1. A 19th century anticipation

Only recently have I come to appreciate to what extent earlier writers have advocated the critical rationalist thesis that the primary role of deductive reasoning is not to extend our knowledge but to criticize and control it. This thesis is really not at all original with Karl Popper, though it is a decisive component (see §VII of his 1957, and the fifteenth thesis of his 1962) of the critical rationalist approach to knowledge of which he was the architect. What is original to Popper is not the thesis that *conjectures* and *refutations* must be sharply separated, but the thesis that they comprise the totality of our intellectual activity; that is, that all reasoning (but not all intelligent thinking) is deductive reasoning.

Without in any way installing the author as an authority, I should here like to quote an extended passage from §II of the Introduction to *Logic, Inductive and Deductive* by William Minto (1893).

Minto, whose name is not too well known, was the successor in 1880 to Alexander Bain (the founder of *Mind*) in the Regius Chair of Logic and English Literature at the University of Aberdeen.

Why describe logic as a system of defence against error? Why say that its main end and aim is the organization of reason against confusion and falsehood? Why not rather say, as is now usual, that its end is the attainment of truth? Does this not come to the same thing?

Substantially, the meaning is the same, but the latter expression is more misleading. To speak of logic as a body of rules for the investigation of truth has misled people into supposing that logic claims to be an art of Discovery that it claims to lay down rules by simply observing which investigators may infallibly arrive at new truths. Now, this does not hold even of the Logic of Induction, still less of the older Logic, the precise relation of which to truth will become apparent as we proceed. It is only by keeping men from going astray and by disabusing them when they think they have reached their destination that logic helps men on the road to truth. Truth often lies hid in the centre of a maze, and logical rules only help the searcher onwards by giving him warning when he is on the wrong track and must try another. It is the searcher's own impulse that carries him forward: Logic does not so much beckon him on to the right path as beckon him back from the wrong. In laying down the conditions of correct interpretation, of valid argument, of trustworthy evidence, of satisfactory explanation, Logic shows the inquirer how to test and purge his conclusions, not how to reach them.

This quotation, along with similar but much shorter quotations from the writings of Cicero and of Russell, may be found in §8 of my (2015). The quotations from Cicero and Minto, and not dissimilar thoughts of Condillac and of Hamilton (1859–1860, Volume III), may be found on the pages devoted to these writers in David Marans's entertaining *Logic Gallery* (Marans 2018).

2. Logical content

Throughout this paper, it will be assumed that, unless otherwise stated, the logic of statements includes classical elementary logic in such a way that the familiar soundness and completeness theorems hold. It will therefore be possible systematically to blur the distinction between the relation $k \vdash h$ (h is deducible from k) and the relation $k \models h$ (h is a logical consequence of k). Nothing of importance will hang on this, but it is useful to be able to supplement the vocabulary of deducibility with the often more flexible vocabulary of logical consequence. In particular, we shall be able to use without discomfort the term *consequence class of h* , and Tarski's notation $Cn(h)$, to refer to the class of statements that can be deduced from h .¹

¹ Logical truths, which are consequences of any statement whatever, are sometimes excluded from every logical content, but this is a technical artifice that, contrary to what Frederick (2014) seems to suggest, is without deep significance. It ensures that the

It is true that some writers (including Popper 1976a, §7) use the word *implication* exactly as I wish to use the word *consequence*, but I prefer to maintain a clean distinction between the relation of implication, for which some writers use the word *entailment*, and its second term (its relatum). But using *implication* as a synonym of *logical consequence* is greatly to be preferred to using it as a synonym for *material conditional*.

It is standard to define the *logical content* $I(h)$ of a statement or hypothesis h as its consequence class $Cn(h)$. (See, for example, Carnap 1935, §6, or Popper 1934, §35, or the abstract of Gemes 1994, who repudiates the definition.) This is the definition that will be adopted here. h is occasionally called the *informative content* of h , but note that this term has a different connotation in Popper *op.cit.*, which will be mentioned in §3.1 below. It is to be observed that set-theoretical relations among logical contents reflect logical relations among statements only in an upside-down and somewhat distorted (but not inaccurate) manner. The logical content $I(h \vee k)$ of the *disjunction* $h \vee k$ of two statements h and k is identical with the *intersection* $h \cap k$ of their individual logical contents, but $I(h \wedge k)$, the logical content of the *conjunction* $h \wedge k$ of h and k , may not be the same as the *union* $I(h) \cup I(k)$ of their logical contents; unless one of h and k is deducible from the other (so that $h \wedge k$ is equivalent to either h or k), the logical content $I(h \wedge k)$ includes, but is not included in, $I(h) \cup I(k)$. This mismatch is avoided by a popular alternative definition of logical content I , a definition that is expressed in terms of sets of models (or structures), rather than sets of statements. On this semantic (or model-theoretic) definition, which was proposed by Carnap & Bar-Hillel (1952) and Bar-Hillel & Carnap (1954), the logical content $I(h)$ of the statement h is identified with the class $M(h)$ of models (thinned down, by the relation of elementary equivalence, to set-hood) in which h is false. The two definitions of the function I agree at a crucial point: the logical contents of the statements h and k are set-theoretically comparable if and only if h and k are logically comparable, that is to say, $I(k) \subseteq I(h)$ if and only if k is deducible from h . But they do not agree in every particular. Set-theoretic relations among the various $M(h)$ reflect much more cleanly the logical relations among statements than do set-theoretic relations among the $Cn(h)$.

In contrast to those philosophers of science who laud the supposed benefits of the *semantic view of theories* (scientific theories are sets of models) over what is disparagingly referred to as the *syntactic view* (scientific theories are sets of statements), logicians tend not to discriminate fussily between the two definitions of logical content $I(h)$, the definition as $Cn(h)$ and the definition as $M(h)$. Even though the senses are distinct, anything said about logical contents in one sense can easily be translated into statements about logical contents in the other sense. Both reformulate, in an alternative vocabulary based on elementary set theory, statements about logical consequence or deducibility. Since the semantic definition of h is technically the more docile, it may be thought that it should be preferred. In the context of the present paper, however, it seems to be quite unhelpful. Some imagination is required to entertain the thought that what a statement asserts is

smallest logical content of all is the empty set, rather than the class of logical truths, and also that no statement h shares any logical consequences with its contradictory $\neg h$.

the set of models in which it is false. Indeed, a statement h never belongs to $M(h)$, its own content, whereas, of course, h always belongs to $Cn(h)$. We shall accordingly retain the definition of logical content $I(h) = Cn(h)$ proposed above.

3. Novelty

3.0 Summary

Let us now look at two fairly recent attempts to controvert the thesis that is being defended here, the thesis that a hypothesis or theory asserts every one of its consequences, that its assertoric content extends as far as its logical content does. Frederick (2011) offers a counterexample, distilled from §7 of Popper's *Unended Quest* (1976a), of a theory in the empirical sciences that has remote logical consequences that were not known, and perhaps could not have been known, to the author of the theory. In a slightly different, more conventional, line of attack, Williamson (2012) reminds us that mathematics is full of surprising and unexpected theorems that were proved only many years after the formulation of the axioms and assumptions from which they were eventually deduced. It is maintained by each of these authors that the examples displayed show that the assertoric content of a theory, what it says, may well be less comprehensive than its logical content.

To the interventions of both Frederick and Williamson the critical rationalist (or anyway, this critical rationalist) replies that, although the newly discovered consequences may not have been known to earlier scientists and mathematicians in any subjective or even intersubjective sense — they were neither personal knowledge nor common knowledge —, they were part of the *objective scientific knowledge* of those earlier times; that is, part of knowledge in the only sense that critical rationalism is concerned with. The existence of the psychological states and processes that feature so largely in traditional, and contemporary, subjectivist epistemology is not denied by critical rationalism, and it is appreciated that without subjective knowledge there would be no objective knowledge. (Without shoemaking subjects there would be no shoes, but shoes are not subjective entities.) But the world of objective knowledge consists of linguistically formulated hypotheses, theories, problems (and much else besides, but that is enough for today); that is, of items that have discarded as much as possible of their psychological patrimony. This answer, that the deduction of new consequences does not extend our objective knowledge, is pretty much the answer that has been given by many authors, at least since Mill (1843). Does either Frederick or Williamson say anything that overturns it?

In both authors' presentations, the disputed thesis that statements assert all their logical consequences, a thesis that I judge to be true, is committed to trial in the company of another thesis, a different one for each author, that is judged by its sponsor to be false; and although neither associate thesis seems to be a logical consequence of the thesis in dispute, each is adduced as providing an argument against that thesis. In Frederick's case, I agree with him that the associate thesis is false, but I deny that this has any repercussions for the thesis that assertoric content is the same as logical content. Contrary to Williamson, on the other hand, I think that the associate thesis that he interposes is true and that, if it has any repercussions for the disputed thesis that assertoric

content is the same as logical content, it is to indicate that it is true. In short, once we disentangle the thesis in dispute from the rogue theses with which it is associated, we shall see that neither author really makes a compelling objection to the objectivist view that a theory asserts everything that in principle may be deduced from it. Both presentations, it seems to me, have an unwelcome justificationist flavour that needs to be exposed and expunged. We shall consider each in turn, first Frederick, and second Williamson.

3.1 Deductive validity and deductive reasoning

Frederick (2011) associates what he calls the 'hoary claim about deductive *validity*' (in his words: 'every deductively valid argument is a *petitio principii*') with

an equally hoary theory of deductive *reasoning*, which is found more or less explicitly in both empiricist and rationalist philosophers of the modern period and which is still popular today. The hoary theory says that a deductive reasoner arrives at a conclusion of an argument by analysing the content of its premises.

Frederick tells us that 'the hoary theory of deductive reasoning may seem to entail the hoary claim about deductive validity', but he soon questions this imagined entailment and writes that, if we were to be granted a distinction between

the objective content of a theory and the part of that content that is available to a particular reasoner in a particular situation, we could consistently retain the hoary theory of deductive reasoning while rejecting the hoary claim about deductive validity. For we could maintain that ... any deductively valid conclusion from a set of premises to which we can reason deductively must be contained in that part of the content of the premises that is already available to us [but there are other parts of the objective content from which] we are currently (and, in some cases, perhaps forever) unable to reason deductively and which would therefore tell us something new even though they are part of the objective content of the premises.

But, going into reverse once more, he then concludes that 'the falsity of the hoary claim about deductive validity casts doubt on the hoary theory of deductive reasoning'. This hardly constitutes a forceful objection to 'the hoary claim about deductive validity'.

Few people who have tried to prove a serious mathematical theorem, and have reflected on what they were doing, can be expected to accredit the theory of deductive reasoning that Frederick says is so hoary. Mathematical work of any depth is far from a smooth process of starting with axioms, or other assumptions, and gaily deducing consequence after consequence. It is much more a business of incessant trial and error in which both the final theorem, if there is one, and its negation, have appeared as trials, perhaps many times (Miller 2006, Chapter 3, §3). This is not to say that there are not algorismic moments in the midst of the fits and starts. It would be perverse, for

example, to resort to trial and error in order to solve a quadratic equation that turns up during an attempt to solve a deeper problem, or to evaluate a familiar integral. But unexpected theorems of interest are indeed uncommon. What is much more common, at the end of a long chain of calculation, is the disappointing conclusion $0 = 0$. Those who forcefully reject the hoary claim about deductive reasoning, as Frederick does, but retain the hoary claim about validity, may wonder why Frederick gives the former claim such prominence. Its falsity tells us nothing about the truth of the latter. Only if it is supposed to furnish a central argument in favour of that claim can its falsity appear to be damaging.¹

The above distinction between the *objective content* of a theory and its *available* (or *accessible*) content is attributed by Frederick to Popper's remark that, since the logical content of a theory is usually infinite, '*we never know what we are talking about*' (*op.cit.*, §7). Popper continued:

For when we propose a theory, or try to understand a theory, we also propose, or try to understand its logical implications [consequences]; that is, all those statements which follow from it. But this, as we have just seen, is a hopeless task: there is *an infinity of unforeseeable nontrivial statements belonging to the informative content of any theory*. We can therefore never know or understand all the implications [consequences] of any theory, or its full significance

and he suggested that if N is Newton's theory of gravitation and E is Einstein's, then '*non-E* belongs to the *logical content* of N , a fact which, obviously, could not have been known to Newton, or anybody else, before E was discovered'. But this is not quite true. If the theories N and E are spelt out in some common mathematical language (such as the language of real analysis) in sufficient detail for it to be possible to assess the validity of the argument $N \therefore \neg E$, then its validity could have been discovered, though perhaps not appreciated, before Einstein.

Popper *loc.cit.* provided a simple proof of his assertion that '*there is an infinity of unforeseeable nontrivial statements belonging to the informative content of any theory*', but he did not specify the conditions under which the proof is valid. One corollary that he did not consider explicitly is that since what holds for a complex and wide-ranging theory must hold also for every one of its consequences (except logical truths), we may not be able to understand these consequences any better than we understand the theory itself (Miller 1998, §2.6, p.19). It is not just Newton's theory of gravitation that we can never fully understand, but singular statements such as 'The Moon is smaller than the Earth' (though this particular statement is not a consequence of Newton's theory, which is here assumed to be consistent). Like the peace of God, almost any theory you care to mention 'surpasses all understanding' (Holy Bible, New King James version, *Philippians* 4:7).

¹ Supporters of 'the hoary claim about deductive validity' do not deny that, although all valid arguments are circular, some are more patently (or immediately) circular than others are. There have been occasions in the history of mathematics when much work was needed to isolate the point or points at which, in a purported demonstration, a suspect axiom (such as the axiom of parallels or the axiom of choice) was silently called on. Kaye (2014) investigates the interesting contrast between the soundness theorem for elementary logic, which is rather obviously assumed in any valid proof of the soundness theorem that uses elementary logic, and the completeness theorem, whose precise role in the proof of the completeness theorem is unearthed only with some care.

In conclusion, it may be suggested that what Frederick's thesis boils down to is that the *available content* of a statement (which is in a constant state of flux) is seldom identical with its logical content (which is more or less fixed, at least if the object language is fixed). But this distinction is as trite as the distinction between a country and the part of that country that is accessible to a particular explorer at a particular time. I hope that there is more to it than that.

3.2 The debatability of logic

'Logic is just not a controversy-free zone', observes Williamson (2012). 'As in the rest of science, no principle is above challenge. Principles of logic can themselves be debated, and often are, just like principles of any other science.' He cites intuitionistic logic, quantum logic, and paraconsistent logic as domains in which principles dear to classical logic have been contested and rejected, and of course there are numerous other theatres of protest and dissent, notably in modal logic. But if we do not resort to logical reasoning or logical inference — in short, if we do not use principles of logic — we cannot argue rationally about principles of logic, but only squabble about them. From this truism Williamson infers that '[t]he conception of logic as a neutral umpire of debate fails to withstand scrutiny. Whichever side is right, logical theories are players in these debates, not neutral umpires.' We must, he says, reject the idea that

logic has no substantive content, for otherwise the correctness of that content could itself be debated, which would impugn the neutrality of logic. One way to develop this idea is by saying that logic supplies no information of its own, because the point of information is to rule out possibilities, whereas logic only rules out inconsistencies, which are not genuine possibilities. On this view, logic in itself is totally uninformative, although it may help us extract and handle non-logical information from other sources.

Williamson thinks that this doctrine of the uninformative nature of logic is mistaken, and goes on:

The power of logic becomes increasingly clear when we chain together such elementary steps into longer and longer chains of reasoning, and the idea of logic as uninformative becomes correspondingly less and less plausible. Mathematics provides the most striking examples, since all its theorems are ultimately derived from a few simple axioms by chains of logical reasoning, some of them hundreds of pages long, even though mathematicians usually don't bother to analyze their proofs into the most elementary steps.

He cites as an example of an informative mathematical discovery Wiles's proof of Fermat's Last Theorem, the conjecture that there exists no integer $m > 2$ for which the equation $x^m + y^m = z^m$ has a solution in the positive integers. He says that 'what matters is that together the accepted axioms suffice [to] imply Fermat's Last Theorem' and asks rhetorically 'If logic is uninformative, shouldn't it be uninformative to be told that the accepted axioms of mathematics imply Fermat's Last

Theorem? But that last statement is not, as it stands, 'a truth of pure logic', since the term 'the accepted axioms' is logically indeterminate. When a sufficient set of axioms is fully spelt out, we do obtain 'a truth of pure logic', but not a convincing example of a genuine discovery. Fermat himself knew that the conjecture is true (though he did not manage to exhibit a justification or proof), and before Wiles got to work on it, it had been extensively tested and never refuted. (That no counterexample has been found does not of course imply that it is true, and it may even be undecidable from any consistent set of arithmetical axioms so far proposed.) But this is a digression, and it cannot be pursued further.

Williamson takes the thesis that he initially considers, that logic is 'an umpire, a neutral arbitrator between opposing theories, imposing some basic rules on all sides in a dispute', to be false, and the thesis with which he ends, that 'logic is informative', to be true. To the extent that he has something new to say about the informativeness of logic (that is, something beyond the appeal to mathematical surprises, which is decidedly old hat), it is that it is a consequence of the true thesis, with which I fully agree, that *we can debate rationally the correctness of logical principles*, and in order to do so we must make use of logical principles. Where I disagree is with Williamson's assertion that *rational debate about logic is impossible if logic is neutral*. This seems to me to be a characteristic justificationist misapprehension of rationality.

According to critical rationalism, as I said above (and in many other places), the principal purpose of logic is to criticize and to falsify. A rational debate about a logical system typically consists of an attempt to show that the system contains a putative law or rule that is, in some sense, unsatisfactory, not that the system is valid. There are no logical means by which logical axioms and logical rules can be justified, and the history of philosophy is littered with failed attempts to construct such justifications. But errors can be detected. Russell's paradox is a famous example.¹ More generally, the rules embodied in a logical system can be directed against the system itself, and provided that it restricts its ambition remorselessly to self-denial and self-sacrifice, the system preserves its neutrality. 'No one should be a judge in his own case' (*Nemo iudex in causa sua*) is a legal principle that is adamantly enforced in many jurisdictions, but it does not prevent a judge with an interest in a case from voluntarily disqualifying himself, that is, from handing down a judgement that may be seriously detrimental to his interest. It is conceivable that, using classical logic, someone might demonstrate that a well thought-of classical principle, for example the rule of *reductio ad absurdum* or the rule of *modus tollens*, has a counterexample. If such a demonstration were to exist, it would not be circular (Miller 1994, Chapter 4, §3e; 2008, §4). It would use the rule of *reductio ad absurdum* to reduce the rule of *reductio ad absurdum* to absurdity, nothing more. I do not expect any such demonstration ever to emerge, but I do not pretend to prove

¹ Two less famous examples that are not trivial are the proof by Kleene & Rosser (1935) of the inconsistency of the system of postulates of Church (1932), and the criticism by Montague & Henkin (1956), notes 5f. and text, of the definition of the term *formal deduction* in Church (1942). There are of course many discussions of logical systems that are not themselves logical but combinatorial, for example proofs that classical sentential calculus based on the operations of conditional and negation can be axiomatized using only five distinct variables (Meredith 1953). Logic's neutrality is not compromised in any of these proofs.

that it is impossible. Pierre Boutroux may have been right to say that 'in order to combat logic it is necessary to use logic' (Marans *op.cit*), but his conclusion that '[I]logic is invincible' does not follow (which is not to say that it is not true).

In this discussion, it is taken for granted that the system or systems being referred to by the word 'logic' contain a rule that is something like the rule of *reductio ad absurdum*; that is, a rule that allows the rejection of one or more assumptions. Despite the protestations of Bunge (1987), §6, most, perhaps all, systems of paraconsistent logic fulfil this requirement. They may deny the universal validity of the law $h \wedge \neg h \vdash k$ of explosion, but they do admit some cases of it. A logical system that cannot, by some means or other, simulate *reductio ad absurdum* would be of no service to critics, and would be discarded for its methodological, as much as for its logical, failings.

3.3 Science and technology

As a parting shot, it may be remarked that many who discuss technological, rather than scientific, innovation, take a view directly contrary to that defended here. That the world is full of wonderful, and sometimes less wonderful, technological surprises is rightly recognized, but technology is often belittled as mere applied science, as no more than the working out of the deductive consequences of our best tested scientific theories. It is nothing of the kind. What successful technological advances depend on is the imaginative realization of previously un contemplated initial conditions, and these genuinely do go beyond the logical content of theoretical science (Miller 1998, §3; 2006, Chapter 5, §3; Petroski 2010). Could it be that it was the mistaken idea that in technology deduction can generate novelty that emboldened the mistaken, but differently mistaken, idea that in science novelty can be generated by deduction?

4 Disjunctions

4.0 Another line of criticism

In §3 of Chapter 10 of (1963) Popper proposed to partition the logical content $Cn(h)$ of a statement or theory h into the set of its true consequences, which he called its *truth content* and the set of its false consequences, which he called its *falsity content*. It is easily seen that truth contents are always logical contents, but only empty falsity contents are logical contents (and then only if we adopt the convention that $Cn(\top) = \emptyset$). He went on to define the comparative *verisimilitude* of two theories h and k in terms of these subsets of $Cn(h)$ and $Cn(k)$. It is well known that this theory of verisimilitude is unsuccessful, though perhaps not as disastrous as it is usually represented to be (see Popper 1976b and also Fine 2018), and in the last 40 years or so there have been many attempts to do better. In one of the early ones, Schurz & Weingartner (1987) proposed to modify Popper's definition by replacing the truth and falsity contents of each statement by proper subsets of these sets, composed of what they called *relevant consequences*. This manoeuvre is noteworthy since it appears to be the first explicit challenge to the appropriateness in general, or the relevance (though not the validity), of the argument from h to $h \vee k$. 'For example,' they write (§3.2.1), 'if a physicist derives from his theory a sentence h which claims the future existence of a solar eclipse, he will

certainly not count sentences like "*h* or the sun will explode tomorrow" etc. as further consequences from his theory — since they are irrelevant'. Schurz & Weingartner do not deny that the argument from *h* to $h \vee k$ is valid, or that in most systems of logic, including most systems of relevance logic, it is authorized by the rule $\vee I$ of disjunction introduction.

Similar opinions, often more strongly worded and seemingly of greater compass, have been articulated by others. Gemes (1994), §2, has noted that if we accept the thesis that the assertoric content of a statement includes the whole of its logical content, 'not only do Relativity theory and Newtonian mechanics share common content but also so do Relativity theory and your favorite crackpot theory, say, Dianetics'. A little later he said that 'the crucial point is that many of the needs of philosophers, especially philosophers of science, are better served by using a non-classical notion of consequence. In particular, we need a notion of consequence that does not automatically count $h \vee k$ as a consequence of *h*.'

Likewise, Mura (2008):28 (echoing Mura 1990:280), says that although¹

the basic dogma that the content of a proposition coincides with the set of its logical consequences sounds *prima facie* very natural from a logical point of view [, it has] some consequences that are, in my view, completely unacceptable (see also Gemes 1994). For example, this view entails that two propositions always share a common content, except when they are logically disjunct, i.e., when their disjunction is a logical truth. In fact, only in that case should two propositions have no factual logical consequence in common. So 'Napoleon was defeated at Waterloo' and 'The population of Chicago amounted in 1990 to 2,783,726 inhabitants' would have a common content. This appears to be completely counterintuitive.

According to Yablo *loc.cit.*, who does not at this point offer any new argument, '*Snow is white* does not in any sense whatsoever share content with *Charlemagne was Holy Roman Emperor*'.

Since one of the main purposes, and main achievements, of scientific and philosophical investigation is to identify, criticize, and correct faulty intuitive judgements, which are usually just dressed-up prejudices, I am little moved by Mura's diagnosis, or by similar complaints of unintuitiveness that appear throughout Gemes *op.cit.* What matters is whether the judgements stand up to criticism. I shall argue that the judgement that $h \vee k$ is not always a part of the content of both *h* and *k* is one intuitive judgement that does not stand up very successfully to criticism.

¹ In (2008) Mura has another argument against the identification of assertoric content and logical content, provoked by the variety of functions that may be used to measure partial deducibility (the converse of the relation that he calls partial entailment), and their connection with measures of relative content. The argument is too technical to be examined here, but I do not doubt that the difficulty that Mura mentions can be satisfactorily resolved.

4.1 Material conditionals

The disjunction $h \vee k$ is logically equivalent, in classical logic (but not in some other systems, such as intuitionistic logic), to the material conditional $\neg h \rightarrow k$, and also of course to $\neg k \rightarrow h$. Does $\neg k \rightarrow h$ belong to the assertoric content of h ; that is, does h assert $\neg k \rightarrow h$? If we understand the material conditional (as I for many years taught students to understand it) as asserting h under the condition $\neg k$ (or h in the circumstance $\neg k$) and h as asserting h under all conditions (or h in all circumstances), the question becomes a special case of the question of whether the rule UI of universal instantiation can add anything to assertoric content; that is, whether there can be an instance of a universal generalization that it does not assert. In the case of the proposal of Schurz & Weingartner *op.cit.*, the answer to this question is negative: in §3.4 they list $\forall xFx \rightarrow Fa$ among what they call the *relevant valid implications*. I do not know if the other authors mentioned are as explicit, but I should be surprised if anyone wishes to exclude the instance Fa from the assertoric content of the universal generalization $\forall xFx$.

It should be noted, however, that in each of the examples quoted from the writings of Mura (2008) and of Yablo, at least one of the disjuncts in the offending disjunction ('Napoleon was defeated at Waterloo', 'Charlemagne was Holy Roman Emperor') is in the past tense. Now it could be questioned whether, if k is a statement in the past tense, the material conditional $\neg k \rightarrow h$ can fairly be read as h under the condition $\neg k$. Does *If Charlemagne was not Holy Roman Emperor, then snow is white* say the same as *Snow is white under the condition that Charlemagne was not Holy Roman Emperor*? Really, I do not know. Those who like to analyse conditionals in terms of possible worlds may have an easy answer to this question, but for the present discussion it seems to me not to be too important. A restriction to statements formulated in the present and future tenses is enough to show the frailty of the intuitions to which the authors quoted defer.

What I am trying to emphasize here is that no decent argument has been given against the inclusion of the disjunction $h \vee k$ in the assertoric content of both h and k . Intuitions (as Yablo *op.cit.*, §1.3, quoted below, seems to be prepared to admit) are not enough.

4.2 Partial truth

It seems appropriate, almost in conclusion, to call attention to a discussion that unwittingly shows what a linguistic and logical quagmire is entered when it is maintained that the assertoric content of a statement may exclude some consequences that can be obtained by the rule VI. Early in his book (2014), Yablo introduces the problem of partial truth in the following words (*loc.cit.*).

What is it for a hypothesis to be partly true? [footnote] Here is the naivest possible idea about this:

1 A hypothesis is partly true iff it has parts that are wholly true.

Now we must ask what is meant by *part* of a hypothesis. The naivest possible idea about part/whole as a relation on hypotheses is

2 One hypothesis is part of another iff it is implied by the other. [footnote]

The naivest possible idea about partial truth is on the right track, I think ... [b]ut the naivest possible idea about what it takes for h to include k is questionable.

A paradigm of inclusion is the relation that simple conjunctions bear to their conjuncts — the relation *Snow is white and expensive* bears, for example, to *Snow is white*. A paradigm of noninclusion is the relation disjuncts bear to disjunctions. *Snow is white* does not have *Snow is white or expensive* as a part. This is not predicted by (2).

You might say that paradigm case intuitions are a poor base for theory. But the intuitions here are systematic.

In support of this judgement, Yablo lists a number of other contexts in which, it is claimed, our intuitions dictate that a disjunction may not be a part of either of its disjuncts. It seems clear that this latter claim closely resembles the thesis whose truth this paper has been contesting.¹

The thesis that disjunctions are not normally parts of their disjuncts may be supposed — but this may be just my imagination — to apply to material conditionals too, since $h \rightarrow k$ is logically equivalent to $\neg h \vee k$; hence $h \rightarrow k$ is not a part of k , and therefore — but again this may be my imagination — not a part of $h \wedge k$. In any case, it evidently serves to prevent every conjunction of false statements from having a true part, that is, from being partly true. For $h \wedge k$ is logically equivalent to $h \wedge (h \rightarrow k)$, whose second conjunct is true if h and k are both false. (A similar point is made in the discussion in *op.cit.*, §8.3, of what Yablo calls the *horseshoe theory* of relative or surplus content.) The claim of the biconditional $h \leftrightarrow k$ to be a part of the conjunction $h \wedge k$ cannot be dismissed in quite the same way. The logical equivalence of $h \wedge k$ and $h \wedge (h \leftrightarrow k)$ is uncontroversial, and once again the second conjunct of the latter conjunction is true if h and k are both false.

¹ One of the contexts mentioned by Yablo is, or is very similar to, a celebrated conundrum in the logic of imperatives, the question whether the imperative *slip the letter into the letter-box!* implies the disjunction *slip the letter into the letter-box or burn it!* (Ross 1941, pp.61f.; the more common formulation is *post the letter!*). According to Yablo (*loc.cit.*), who does not mention Ross's paradox by name,

[o]rdering Smith to eat pork chops is ordering her to eat pork. Ordering her to eat pork is not ordering her to eat pork or human flesh, though eating pork or human flesh is no less implied by eating pork than is eating pork implied by eating pork chops. One commands (normally) the parts of what one commands, but not its implications more generally.

What, as far as I know, has not been adverted to in the discussions of Ross's paradox is the well known and pervasive ambiguity in everyday English of the particle *or*, which is obliged to perform as the translation of each of the Latin particles *vel* and *aut*. The former is represented in most systems of formal logic by the connective \vee , the *inclusive disjunction* of the items connected, while *aut*, which expresses the *exclusive disjunction* of the items connected, has no established symbolization; Δ , called *symmetric difference*, \leftrightarrow , the negation of the biconditional, and \vee surmounted by the letter *e* (a composite symbol called *eeyore* by Mary Cresswell), have all been used. If the connective *or* is read as the exclusive disjunction *aut*, then it is quite in order that *post the letter or burn it!* is not a consequence of *post the letter!* If *or* is read as the inclusive disjunction *vel*, then *post the letter or burn it!* says no more than does *If you do not burn the letter, then post it!* (Or perhaps more accurately, *bring it about that, in the event that you do not burn the letter, you post it!*). To the extent that this latter command is in idiomatic English, it does appear to be a consequence of *post the letter!* If the argument in the text is correct then, contrary to what Yablo asserts, an order to eat pork chops is, at the same time and in the same action, an order to eat pork or to eat human flesh.

I imagine that Yablo's response here would be to plead that when he says that '[a] paradigm of inclusion is the relation that simple conjuncts bear to their conjuncts', he intends the word 'simple' seriously, to mean something like *primitive*. The conditional *If snow is white, then it is expensive* and the biconditional *Snow is white iff it is expensive* are compound statements and, under the paradigm, do not qualify as parts of *Snow is white and expensive*. It is well known, however, that a language in which *h* and *k* are primitive is equivalent, for every logical purpose, to a language that differs from it only in having *h* and $h \leftrightarrow k$ instead as primitive. This is the simplest instance of the phenomenon of language dependence that was mentioned at the end of §0 above.

To deny the importance of language independence in logical investigations seems to me to be nothing short of a belief in word magic (to use a term of Isaac Levi's). It is discouraging that Chapter 11 of my (2006), which is devoted to rebutting a host of attempts to defend language-dependent theories of verisimilitude, has been so pointedly ignored by the main protagonists in this debate. Only Oddie, in his (2014) article in *The Stanford Encyclopedia of Philosophy*, and in his (2018) article in *The Oxford Handbook of Truth*, even mentions it, but he does not respond to it. It needs to be understood that even in colloquial language there is no sharp distinction between primitive and defined terms, and in science and in mathematics the situation is even starker (for one or two examples, see Miller & Taliga 2008). In other words, we cannot do without the possibility of reformulating in new vocabularies, in other words, and rarely entirely accurately, what it is that we are trying to say. That is how things are, and no amount of semantical sophistication should be allowed to conceal it. The only good answer to the much-posed question *What exactly do you mean?* is *I do not mean anything exactly, but please try to show me where what I said is mistaken*. If this is not obvious, try to say or write something interesting in a foreign language.

References

- Bar-Hillel, Y. & Carnap, R. (1954). Semantic Information. *The British Journal for the Philosophy of Science*, Vol. 4, pp.145–157.
- Bunge, M. A. (1987). Seven Desiderata for Rationality. In J. Agassi & I. C. Jarvie, editors, pp.5–15. *Rationality: The Critical View*. Dordrecht: Martinus Nijhoff Publishers.
- Carnap, R. & Bar-Hillel, Y. (1952). An Outline of a Theory of Semantic Information. Technical Report #247. Cambridge MA: Research Laboratory of Electronics, Massachusetts Institute of Technology. Reprinted in Part IV of Y. Bar-Hillel (1964). *Language and Information. Selected Essays on Their Theory and Application*. Reading MA: Addison-Wesley.
- Church, A. (1942). *Introduction to Mathematical Logic* I. Princeton University Press.
- Church, A. (1932). A Set of Postulates for the Foundation of Logic. *Annals of Mathematics* 33(2), 346–366.
- Fine, K. (2018). Some Remarks on Popper's Qualitative Criterion of Verisimilitude. *Erkenntnis*, 87(1), 213–236.
- Frederick, D. (2011). Deduction and Novelty. *The Reasoner*, 5(4), 56b–57b. Slightly expanded version made available by the author.
- Frederick, D. (2014). Deduction and Novelty Again. *The Reasoner*, 8(5), 51b–52b.
- Frederick, D. (2020). *Against the Philosophical Tide. Essays in Popperian Critical Rationalism*. Critias Publishing.
- Gemes, K. (1994). A New Theory of Content I: Basic Content. *Journal of Philosophical Logic* 23(6), 595–620.

- Hamilton, W. (1859–1860). *Lectures on Metaphysics and Logic*, Vol. III. Edited by H. L. Mansel & J. Veitch. W. Blackwood and Sons. <https://babel.hathitrust.org/cgi/pt?id=uc2.ark:/13960/t3707z04z;view=lup;seq=62>
- Kaye, R. W. (2014). Circularity in Soundness and Completeness. *The Bulletin of Symbolic Logic*, 20(1), 24–38.
- Kleene, S. C. & Rosser, J. B. (1935). The Inconsistency of Certain Formal Logics. *Annals of Mathematics*, 36(3), 630–636.
- Marans, D. (2018). *Logic Gallery*. 8th edition. Morrisville NC: Lulu Press, Inc. Electronic edition continually updated on line at <https://archive.org/details/logic-gallery>
- Meredith, C. A. (1953). Single axioms for the systems (C, N), (C, 0), and (A, N) of the two-valued propositional calculus. *Journal of Computing Systems*, 1(3), 155–164.
- Mill, J. S. (1843). *A System of Logic, Ratiocinative and Inductive*. Longmans, Green & Co. Ltd. 8th ed. 1882.
- Miller, D. W. (2005). Do We Reason When We Think We Reason, or Do We Think? *Learning for Democracy*, 1(3), 57–71. <http://www.warwick.ac.uk/go/dwmiller/lfd-.pdf>
- Miller, D. W. & Taliga, M. (2008). Through A Glass Darkly. A Final Rejoinder to Raclavský. *Organon F* (Bratislava) 15(4) 473–476. <http://www.klemens.sav.sk/fiusav/doc/organon/2008/4/473-476.pdf>
- Miller, D. W. (1994). *Critical Rationalism. A Restatement and Defense*. Open Court Publishing Company. Electronic edition 2015.
- Miller, D. W. (1995). À quoi sert la logique?. *Hermès*, 15, pp.291–300. French translation by A. Boyer of 'What Is Logic For?' (Unpublished).
- Miller, D. W. (1998). Is Scientific Knowledge an Inexhaustible Economic Resource? *The Critical Rationalist* 3, 1, 17. iv. <http://www.eeng.dcu.ie/~tkpw/tcr/volume-03/number-01>
- Miller, D. W. (2006). *Out of Error. Further Essays on Critical Rationalism*. Ashgate Publishing Company. Paperback and electronic editions (2017). Routledge.
- Miller, D. W. (2008). Overcoming the Justificationist Addiction. *Iranian Journal of Philosophical Investigations* 4(11), 167–182. <http://wuwr.pl/spwr/article/view/5234/4917>. Persian translation by A. Paya. `Ghalabeh bar E'tiyād beh Movajjah Sāzī'. *Baharestan* ([Iranian Academy of Science] 5, 2019, pp. 145–158 [the journal has now been renamed *Nameh Farhangestan-e Ulum*]).
- Miller, D. W. (2014). Some Hard Questions for Critical Rationalism. *Discusiones Filosóficas*, 15(24) 15–40. <http://www.scielo.org.co/pdf/difil/v15n24/v15n24a02.pdf>
- Miller, D. W. (2015). Errando el blanco. La desdichada historia de las críticas al falsacionismo. In R. López O., C. Verdugo S. & B. Bevensee B., editors, pp. 89–125. *El legado de Karl Popper (1902–1994). Reflexiones a 20 años de su muerte*. Instituto de Filosofía, Universidad de Valparaíso. Updated English translation at: <http://www.warwick.ac.uk/go/dwmiller/uvtext.pdf>
- Minto, W. (1893). *Logic, Inductive and Deductive*. John Murray.
- Montague, R. M. & Henkin, L. A. (1956). On the Definition of “Formal Deduction”. *The Journal of Symbolic Logic*, 21(2), 129–136.
- Mura, A. M. (1990). When probabilistic support is inductive. *Philosophy of Science*, 57(2), 278–289.
- Mura, A. M. (2008). Can Logical Probability Be Viewed as a Measure of Degrees of Partial Entailment?. *Logic & Philosophy of Science*, VI(1), 25–33. http://www.academia.edu/26156970/Can_Logical_Probability_Be_Viewed_as_a_Measure_of_Partial_Entailment?
- Oddie, G. J. (2014). Truthlikeness. In E. N. Zalta, editor (2016). *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition). <https://plato.stanford.edu/archives/win2016/entries/truthlikeness/>
- Oddie, G. J. (2018). Truth and Truthlikeness. In M. Glanzberg, ed., pp. 625–647. *The Oxford Handbook of Truth*, Oxford University Press.
- Petroski, H. (2010). *The Essential Engineer. Why Science Alone Will Not Solve Our Global Problems*. Alfred A. Knopf.

-
- Popper, K. R. & Miller, D. W. (1983). A Proof of the Impossibility of Inductive Probability. *Nature* 302, 21.iv.1983, pp. 687f.
- Popper, K. R. (1934). *Logik der Forschung*. Julius Springer Verlag. English translation, Popper (1959). *The Logic of Scientific Discovery*. Hutchinson.
- Popper, K. R. (1957). Philosophy of Science: A Personal Report. In C. A. Mace, ed. (1957), pp. 155–191. *British Philosophy in the Mid-Century*. George Allen & Unwin Ltd. Reprinted as Chapter 1 of Popper (1963).
- Popper, K. R. (1962). Die Logik der Sozialwissenschaften. *Kölner Zeitschrift für Soziologie und Sozialpsychologie* 14(2), 233–248. English translation: Popper (1994), Chapter 5.
- Popper, K. R. (1963). *Conjectures and Refutations*. 5th edition 1989. Routledge & Kegan Paul.
- Popper, K. R. (1972). *Objective Knowledge*. 2nd edition 1979, Clarendon Press.
- Popper, K. R. (1976a). *Unended Quest. An Intellectual Autobiography*. Routledge Classics edition. Routledge (2002). Originally published in P. A. Schilpp, ed. (1974), pp.1–181. *The Philosophy of Karl Popper*. Open Court Publishing Company.
- Popper, K. R. (1976b). A Note on Verisimilitude. *The British Journal for the Philosophy of Science*, 27(2), 147–159.
- Popper, K. R. (1994). *In Search of a Better World*. Routledge.
- Ross, A. (1941). Imperatives and Logic. *Theoria*, 7(1), 53–71.
- Schurz, G. & Weingartner, P. (1987). Verisimilitude Defined by Relevant Consequence-Elements. In T. A. F. Kuipers, ed. (1987), pp. 47–77, *What Is Closer-to-the-Truth?* Rodopi.
- Williamson, T. (2012). Logic and Neutrality. *Opinionator. The New York Times*. The Stone, 13. <https://opinionator.blogs.nytimes.com/2012/05/13/logic-and-neutrality/>
- Yablo, S. (2014). *Aboutness*. Princeton University Press.