

*

(SPI)

SPI

(1959

() Shiau

(MSD)

Loukas () Conceleiere and Salas ,(۲۰۰۳)Gonzalez
- () Salas et al. () and Vasiliades

and () Shiau and Shen

Mishra and () Hong Wu and Wilhite
()Desaia

() Salas et al. () Gonzalez Valdez

SPI

Sklar,) (Copula)

()

$$SPI = \frac{\sigma_n}{\mu_n} \quad (D)$$

Loukas and)

(Vasiliades, 2004; Shiau, 2006

$$S = - \sum_{i=1}^D SPI_i \quad ()$$

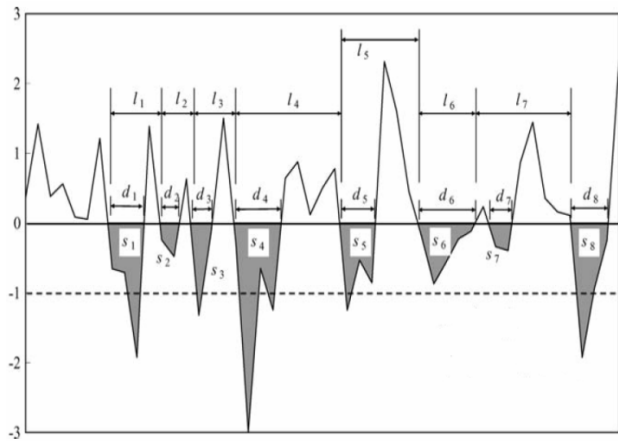
$$d_i = s_i - l_i \quad (i+1)$$

(Sheskin, 2000)

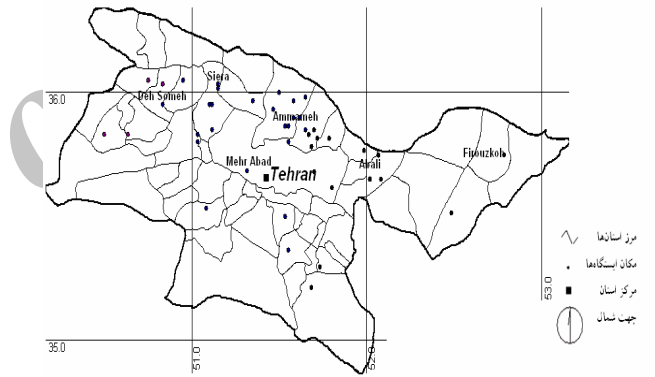
Pilon et al.,)

(1989

(Midment, 1992)



(Shiau, 2006)



SPI

SPI

$$SPI_n = \frac{[P_0 + \sum(P_{-i}) - \mu_n]}{\sigma_n} \quad ()$$

/ / / / / /

P_{-i}

n

P_0

$$f_{X,Y}(x,y) = c(F_X(y), F_Y(y))f_X(x)f_Y(y) \quad (1)$$

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}, 0 < u < 1, 0 < v < 1 \quad (2) \quad [0,1]$$

(3) Joe, (4) Nelsen, (5) Sklar, (6) Marginal Distributions

$H(x,y) = C(F_X(x), G_Y(y))$

$$\rho_s = 12 \int_0^1 \int_0^1 [C(u,v) - uv] dudv$$

$$\frac{[1+(\theta-1)(u+v)] - \sqrt{[1+(\theta-1)(u+v)]^2 - 4uv\theta(\theta-1)}}{2(\theta-1)}$$

$$1 - [(1-u)^\theta + (1-v)^\theta - ((1-u)(1-v))^\theta]^{\frac{1}{\theta}}$$

$$\frac{(u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}}{\theta} \ln \left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right]$$

$$uv \exp\{ [(-\ln u)^{-\theta} + (-\ln v)^{-\theta}]^{\frac{-1}{\theta}} \}$$

$$uv \exp(-\theta \ln u \ln v)$$

$$\exp\{ -[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}} \}$$

$$\frac{uv}{1 - \theta(1-u)(1-v)}$$

SPI

/	/	/	/	/	/	λ
/	/	/	/	/	/	α
/	/	/	/	/	/	β

$$\lambda e^{-\lambda d}, d \geq 0$$

$$\frac{s^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{s}{\beta}}, s \geq 0$$

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$$\ln L(d,s;\lambda,\alpha,\beta,\theta) = \ln L_c(F_D(d), F_S(s); \theta) + \ln L_D(d; \lambda) + \ln L_S(s; \alpha, \beta) \quad ()$$

MSD	θ	
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/	/	/
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$$F_D(d) \quad F_S(s)$$

$$\ln L_c(.)$$

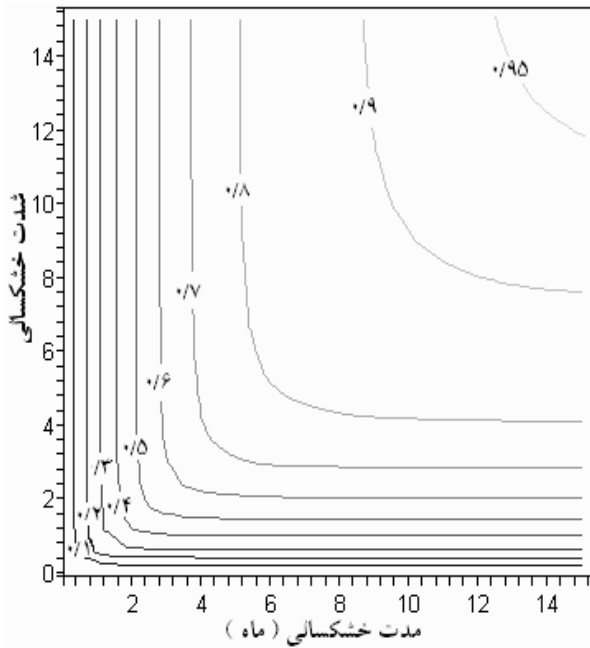
$$() \quad \lambda \quad \beta \quad \alpha$$

$$\theta$$

$$\theta$$

MSD

MSD



Contour)

S D

(plot

$$P(D \geq d, S \geq s) = 1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s)) \quad ()$$

θ

/

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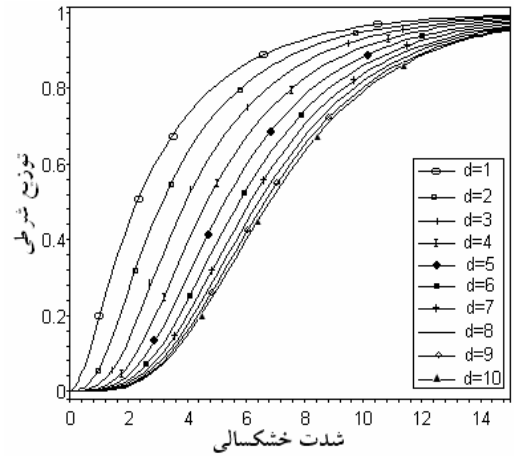
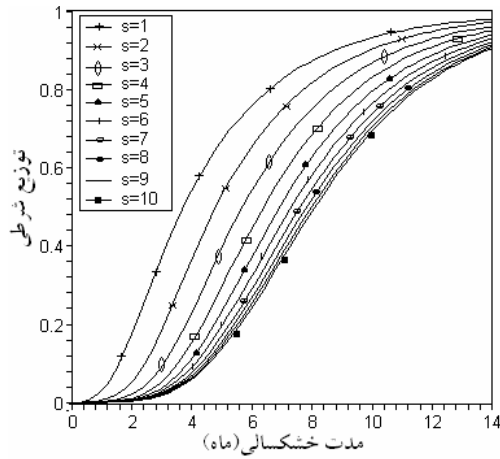
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$$P(S \leq s | D \geq d) = \frac{F_S(s) - C(F_D(d), F_S(s))}{1 - F_D(d)} \quad ()$$

$$P(D \leq d | S \geq s) = \frac{F_D(d) - C(F_D(d), F_S(s))}{1 - F_S(s)} \quad ()$$



$E(L)$

()

$X \leq x_t$

() Shiau and Shen

$$T_D = \frac{E(L)}{1 - F_S(s)}, \quad T_S = \frac{E(L)}{1 - F_D(d)} \quad ()$$

$E(L)$

$E(L)$

() Tr

$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \quad ()$$

() (SPI

$$T'_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \quad ()$$

()

$d = 3$

() Shiau .

$s = 3$

$T'_{DS} = / \quad T_{DS} = /$

$P(D \geq d, S \geq s) \quad P(D \geq d, S \geq s)$

()

s

()

$$T_{DS \geq s} = \frac{T_s}{P(D \geq d, S \geq s)} = \frac{E(L)}{[1 - F_S(s)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]}$$

d

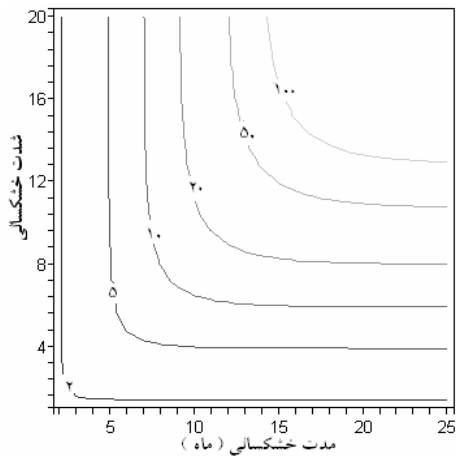
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$$T_{SD \geq d} = \frac{T_s}{P(D \geq d, S \geq s)} =$$

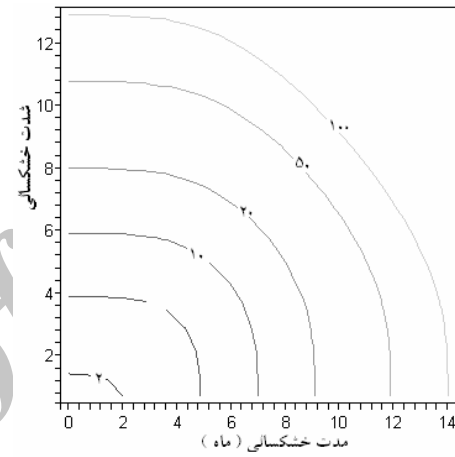
E(L)

$$[1 - F_D(d)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]$$

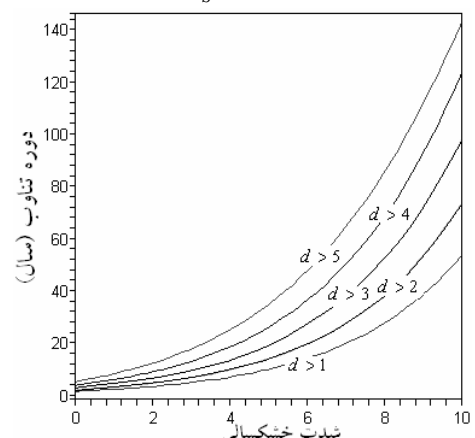
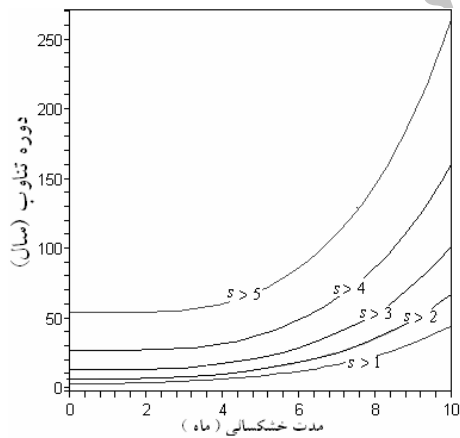
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T_{DS}



T_{DS}



() Shiau

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