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Email: shmohammadi@gmail.com

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CAPM

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$E[R_i] = R_f + \beta_{im}(E[R_m] - R_f) \quad (1)$$

$$\beta_{im} = \frac{\text{Cov}[R_i, R_m]}{\text{Var}[R_m]} \quad (2)$$

$$R_i \quad R_f \quad R_m$$

$$Z_i \quad :$$

$$Z_i \equiv R_i - R_f$$

¹. Mean- variance efficient portfolio
². Excess return

CAPM

$$E[Z_i] = \beta_{im} E[Z_m] \quad (1)$$

$$\beta_{im} = \frac{\text{Cov}[Z_i, Z_m]}{\text{Var}[Z_m]} \quad (2)$$

$$(1) \quad (2) \quad Z_m$$

$$(1)$$

$$(2)$$

$$(E(Z_m))$$

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$$E[R_i] = E[R_{0m}] + \beta_{im} (E[R_m] - E[R_{0m}]) \quad (3)$$

$$m \quad R_{0m} \quad R_m$$

:

$$) \quad m$$

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β_{im}

¹ Real returns

$$\beta_{im} = \frac{\text{Cov}[R_i, R_m]}{\text{Var}[R_m]} \quad ()$$

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: N $(N \times 1)$ Z_t

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t \quad ()$$

$$E(\varepsilon_t) = 0 \quad ()$$

$$E[\varepsilon_t \varepsilon_t'] = \Sigma \quad ()$$

$$E(Z_{mt}) = \mu_m, \quad E[(Z_{mt} - \mu_m)^2] = \sigma_m^2 \quad ()$$

$$\text{Cov}[Z_{mt}, \varepsilon_t] = 0 \quad ()$$

$$\begin{matrix} Z_{mt} & (N \times 1) & \beta \\ \varepsilon_t & (N \times 1) & \alpha \end{matrix} \quad ()$$

Σ β α

$$\hat{\alpha} = \hat{\mu} - \hat{\beta} \hat{\mu}_m \quad ()$$

$$\hat{\beta} = \frac{\sum_{t=1}^T (Z_t - \hat{\mu})(Z_{mt} - \hat{\mu}_m)}{\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2} \quad ()$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T (Z_t - \hat{\alpha} - \hat{\beta}Z_{mt})(Z_t - \hat{\alpha} - \hat{\beta}Z_{mt})' \quad ()$$

$$(iid) \quad \begin{matrix} Z_{m1} \\ Z_{mt} \dots Z_{m2} \end{matrix}$$

CAPM

CAPM

$$:$$

$$E(R_t) = \gamma + \beta(E[R_{mt}] - \gamma) = (1-\beta)\gamma + \beta E[R_{mt}] \quad ()$$

$$(N \times 1) \quad R_t \quad :$$

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t \quad ()$$

$$E[\varepsilon_t] = 0 \quad ()$$

$$E[\varepsilon_t \varepsilon_t'] = \Sigma \quad ()$$

$$E[R_{mt}] = \mu_m, \quad E[(R_{mt} - \mu_m)^2] = \sigma_m^2 \quad ()$$

$$\text{Cov}[R_{mt}, \varepsilon_t] = 0 \quad ()$$

¹. Identically and Independently distributed

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \quad ()$$

$$S_\beta = \frac{1}{\sqrt{(T-1)}} \times \frac{S_\varepsilon}{S_m} \quad ()$$

¹ Standard error
² Standard deviation
³ Stationarity
⁴ Non-stationarity

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¹. Premiums of factors

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¹. Useless factor

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(OLS)

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¹ Risk aversion
² High-Frequency data
³ Dow Jones Industrial Average

AR()

MA() AR() ARMA ()

ARMA()
ARMA()
/ /

¹. Constant Beta
². Out of Sample

CAPM

CAPM

:

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t \quad ()$$

$$\begin{matrix} N & & (N \times 1) & & \beta & & N & & (N \times 1) & & R_t \\ (N \times 1) & & \varepsilon_t & & \alpha & & t & & R_{mt} \end{matrix}$$

()

$$() \quad R_t$$

R_{mt}

α

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β

α

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$$\alpha = (1 - \beta)\gamma$$

()

$R_{mt} \quad R_t$

CAPM

OLS

(ML) ()

OLS

$$z_t = \alpha + \beta Z_{mt} + \varepsilon_t \quad (ML)$$

Z_t
:

$$f(Z_t | Z_{mt}) = (2\pi)^{-\frac{T}{2}} |\Sigma|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}(Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})\right] \quad ()$$

$t = 1, 2, 3, \dots, T$

T iid

:

$$f(Z_1, Z_2, \dots, Z_T | Z_{m1}, Z_{m2}, \dots, Z_{mT}) = \prod_{t=1}^T (2\pi)^{-\frac{T}{2}} |\Sigma|^{-\frac{1}{2}} \quad ()$$

$$\times \exp\left[-\frac{1}{2}(Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})\right]$$

$\Sigma \quad \beta \quad \alpha$

$$L(\alpha, \beta, \Sigma) = -\frac{T}{2} \log(2\pi) - \frac{T}{2} \log |\Sigma| \quad ()$$

$$-\frac{1}{2} \sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})$$

$$\left(\begin{array}{c} Z_t \\ \Sigma \end{array} \right) \quad L(\alpha, \beta, \Sigma)$$

$$\frac{\partial L}{\partial \alpha} = \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt}) \right] \quad ()$$

$$\frac{\partial L}{\partial \beta} = \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt}) Z_{mt}' \right] \quad ()$$

$$\frac{\partial L}{\partial \Sigma} = -\frac{T}{2} \Sigma^{-1} + \frac{1}{2} \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt})(Z_t - \alpha - \beta Z_{mt})' \right] \Sigma^{-1} \quad ()$$

$$() () ()$$

$$\hat{\alpha} = \hat{\mu} - \hat{\beta} \hat{\mu}_m \quad ()$$

$$\hat{\beta} = \frac{\sum_{t=1}^T (Z_t - \hat{\mu})(Z_{mt} - \hat{\mu}_m)'}{\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2} \quad ()$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T (Z_t - \hat{\alpha} - \hat{\beta} Z_{mt})(Z_t - \hat{\alpha} - \hat{\beta} Z_{mt})' \quad ()$$

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^T Z_t$$

$$\hat{\mu}_m = \frac{1}{T} \sum_{t=1}^T Z_{mt}$$

OLS

$$\sum e_i^2 / T$$

$$\sum e_i^2 / (T-2)$$

$$\sigma^2$$

$$\sigma^2$$

$$\sigma^2$$

$$\sigma^2$$

CAPM

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OLS

GLS
(BLUE)

¹ Generalized Least Squares
² Best Liner Unbiased Estimator

(GMM) (

v t - student Y_1, Y_2, \dots, Y_T
 $:$ v

$$f(Y_t, v) = \frac{\Gamma[(v+1)/2]}{(\pi v)^{1/2} \Gamma(v/2)} \left[1 + (y_t^2/v)\right]^{-\frac{(v+1)}{2}} \quad ()$$

$$\hat{v} \equiv \arg \max \ln L_T(\theta) = \sum_{t=1}^T \ln f(y_t, v) \quad ()$$

$$\hat{\mu}_{r,T} = \frac{1}{T} \sum_{t=1}^T Y_t^r$$

$$\mu_r = \mu_r(v) \quad \mu_r$$

$$\theta_0 \quad a \times 1 \quad \theta \quad h(\theta, w_t) \quad h \times 1 \quad w_t \quad r$$

$$E[h(\theta_0, w_t)] = 0$$

$$y_t = (w'_1, w'_2, \dots, w'_T)$$

¹. Generalized method of moments

$$h(\theta, w_t) = X_t(Y_t - Z_t'\beta)$$

$$E[h(\theta_0, W_t)] = E[X_t(Y_t - Z_t'\beta_0)] = 0$$

$$g(\theta, y_t) = \frac{1}{T} \sum_{t=1}^T X_t(Y_t - Z_t'\beta) = 0$$

$$g(\theta, y_t) = 0$$

$$\sum_{t=1}^T X_t(Y_t - Z_t'\beta) = 0$$

$$\hat{\beta}_{GMM} = \left[\sum_{t=1}^T X_t Z_t' \right]^{-1} \left[\sum_{t=1}^T X_t Y_t \right] = (X'Z)^{-1} X'Y \quad ()$$

GMM

ML

GMM

(LAD)

(

()

$$\min_{\beta \in R^k} \sum_{t=1}^T |Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}| \quad ()$$

:

$$\min_{\beta \in R^k} \sum_{t=1}^T \rho_\tau(Y_t - \xi(x_t, \beta))$$

¹. Least Absolute Deviation
². Quantile regression

$$\xi_t = Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}$$

$$\rho_\tau(\cdot) \quad \xi = f(x_{1t}, x_{2t}, \dots, x_{kt}; \beta_1, \beta_2, \dots, \beta_k)$$

$$\rho_\tau(u) = u(\tau - 1(u < 0))$$

$$Q_\tau = \min_{\xi \in R} \left\{ \sum_{t: Y_t \geq \xi} \tau |Y_t - \xi| + \sum_{t: Y_t < \xi} (1 - \tau) |Y_t - \xi| \right\} \quad ()$$

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(NP)

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Eviews

Stata9

². Non- Parametric

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T

T(T+1)/2

$$\min \sum_{t=1}^T [\text{rank}(Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}) - T(T+1)/2] p_t \quad ()$$

[]

$$(\text{rank}(e) - (T+1)/2)' e \quad ()$$

$$e = y - x\hat{\beta}$$

$$x = [x_2, x_3, \dots, x_k]; \hat{\beta}' = [\hat{\beta}_2, \dots, \hat{\beta}_k]$$

e

¹. Histogram
². Kernel
³. Rank Regression
⁴. Nadaraia- Watson

$$f(\hat{\beta}) = (\text{rank}(y - x\hat{\beta}) - (T+1)/2)'(y - x\hat{\beta}) \quad (49)$$

β

$$\hat{\beta}^i = \hat{\beta}^0 + pd$$

$$\delta \quad \hat{\beta}^i - \hat{\beta}^{i-1} < \delta$$

: d p 1e-10 1e-6

$$z = y - x\hat{\beta}^0; u^0 = \text{rank}(z) - 0.5(T+1); x_c = x - \bar{x}$$

$$d = (x_c' x_c)^{-1} x_c' u^0; w = xd$$

$$\frac{p}{|w_i - w_j| / \sum |w_i - w_j|} \quad \frac{w}{(z_i - z_j) / (w_i - w_j)}$$

LAD NP, GLS, ML, GMM

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LAD NP, GLS, GLS GMM

LAD NP, GLS, GLS GMM

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GLS					GLS				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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ML					ML				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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GMM					GMM				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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NP					NP				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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LAD					LAD				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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GLS					GLS				
RSS	R-squared	t-statistic	Beta		RSS	R-squared	t-statistic	Beta	
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ML

GLS

GLS

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¹. Low Risk
². High Risk

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/		GLS	
/		ML	
/		GMM	
/		NP	
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/		Black	
/		Sharp	
/		GLS	
/		ML	
/		GMM	
/		NP	
/		LAD	
/		Black	
/		Sharp	

H_0

$(\sum e_i^2)$

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/		GLS	black
/		ML	
/		GMM	
/		NP	
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$$\sum e_i^2$$

NP

$$\sum e_i^2$$

ML

NP

LAD

ML

GLS NP

ML

GMM LAD

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/		Sharp	
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R

CAPM
CAPM

$\sum e_i^2$ R
CAPM

LAD NP ML GMM GLS

OLS

NP

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LAD

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GMM

GLS

GMM

NP

 $\sum e_i^2$

NP

 $\sum e_i^2$

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