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[]

()

GPS

M_h^2

GPS

M_g^2

w_0

[]

S_R^2

[]

div grad ()

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$\langle \cdot | \cdot \rangle$

$E\{ \cdot \}$

w

$\| \cdot \|_2$

ω

σ

γ

1. $\text{div grad } w(\mathbf{x}) = 2\omega^2$ (outside the Earth's masses)	$\forall \mathbf{x} \in \mathbb{R}^3 / \mathcal{D} \cup \partial\mathcal{G}_e^+$
2. $\text{div grad } w(\mathbf{x}) = -4\pi G\sigma(\mathbf{x}) + 2\omega^2$ (inside the surface of the Earth)	$\forall \mathbf{x} \in \mathcal{D} \cup \partial\mathcal{G}_e^-$
3. $E\{\ \text{grad } w(\mathbf{x})\ _2\} = \mu_\gamma$ (boundary data of the type modulus of gravity from gravimetry)	$\forall \mathbf{x} \in \partial\mathcal{G}_e = \mathbb{M}_g^2$
4. $w(\mathbf{x}) = w_0$ (Boundary data at the fixed boundary of the type geoid potential)	$\forall \mathbf{x} \in \partial\mathcal{G}_e = \mathbb{M}_g^2$
5. $\lim_{\ \mathbf{x}\ _2 \rightarrow \infty} w(\mathbf{x}) = \frac{1}{2} \omega^2 \ \mathbf{x} - \langle \mathbf{x} \mathbf{e}_\omega \rangle \mathbf{e}_\omega\ _2^2 + \frac{GM}{\ \mathbf{x}\ _2} + \mathcal{O}_w\left(\frac{1}{\ \mathbf{x}\ _2^3}\right)$ (regularity condition at infinity)	

$\gamma = \Gamma + \delta\Gamma$	$w = W + \delta W$
$\omega^2 = \Omega^2 + 2\langle \Omega \delta\Omega \rangle + \delta\Omega^2$	$\sigma = \Sigma + \delta\Sigma$

$\text{div grad } \delta W(\mathbf{x}) = 0$	$\forall \mathbf{x} \in \mathbb{R}^3 / \mathcal{D} \cup \partial\mathcal{G}_g$	} Field Diff. Equ.
$\text{div grad } \delta W(\mathbf{x}) = -4\pi\delta\Sigma(\mathbf{x})$	$\forall \mathbf{x} \in \mathcal{D} \cup \partial\mathcal{G}_g$	
$\delta\Gamma(\mathbf{x}) = \ \nabla_{\mathbf{e}_r} \delta W(\mathbf{x})\ _2$	$\forall \mathbf{x} \in \partial\mathcal{G}_e := \mathbb{M}_g^2$	} Boundary Value
$\delta w_0(\mathbf{x}) = \delta W_0$	$\forall \mathbf{x} \in \partial\mathcal{G}_i := \mathbb{M}_g^2$	
$\lim_{\ \mathbf{x}\ _2 \rightarrow \infty} \delta W(\mathbf{x}) = \mathcal{O}_{\delta\omega}\left(\frac{1}{\ \mathbf{x}\ _2^{L+1}}\right)$		} Regularity condition at Infinity



$\nabla_{\mathbf{e}_\Gamma} ()$

K^L

$[]$

$\{a, b, \varepsilon\}$

$\mathbb{E}_{a,b}^2$

$\delta\Gamma(\mathbf{x})$

$j \max \quad i \max$

J

$\delta W(\mathbf{X})$

$()$

$\mathbb{E}_{a,b}^2$

$\int_J k(\mathbf{s}, \mathbf{t}) f(\mathbf{t}) d\mathbf{t} = g(\mathbf{s}), \mathbf{s} \in I$

$J \quad I$

g

$f \quad K$

$K(f) = g$

$()$

$[-]$

$K : L_2(I) \rightarrow L_2(J)$

$(Kf)(\mathbf{s}) = \langle k(\mathbf{s}, \mathbf{t}) | f \rangle$

$()$

$\langle u | v \rangle = \int_J u(\mathbf{t}) v(\mathbf{t}) d\mathbf{t}, \mathbf{t} \in J$

$\delta\Gamma(\mathbf{x})$

$f \quad k \quad g$

$() K$

$(k$

$[]$

$()$

$()$

$\{ \Gamma_\lambda, \Gamma_\phi, \Gamma_\eta \}$

Γ

$\mathbf{x} = \{ \lambda, \phi, \eta \}$

$\varpi(\phi')$

$\mathbb{E}_{a,b}^2$

$S_{\mathbb{E}_{a,b}^2}$

$\{ g_{\lambda\lambda}, g_{\phi\phi}, g_{\eta\eta} \}$

K^L

$[-]$

L

$$\begin{aligned}
\delta\Gamma(\mathbf{x}) &= \langle \mathbf{e}_\Gamma | \delta\Gamma(\mathbf{x}) \rangle + \mathcal{O}(\delta\Gamma^2(\mathbf{x})) \\
\delta\Gamma(\mathbf{x}) &= \frac{\Gamma_\lambda}{\|\Gamma\|_2} \delta\Gamma_\lambda + \frac{\Gamma_\phi}{\|\Gamma\|_2} \delta\Gamma_\phi + \frac{\Gamma_\eta}{\|\Gamma\|_2} \delta\Gamma_\eta + \mathcal{O}(\delta\Gamma^2(\mathbf{x})) \\
&= \frac{1}{\sqrt{g_{\lambda\lambda}}} \frac{\Gamma_\lambda}{\|\Gamma\|_2} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \iint_{\mathbb{E}_{a,b}^2} ds' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \lambda} \delta W^L(\lambda', \phi') \\
&\quad + \frac{1}{\sqrt{g_{\phi\phi}}} \frac{\Gamma_\phi}{\|\Gamma\|_2} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \iint_{\mathbb{E}_{a,b}^2} ds' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \phi} \delta W^L(\lambda', \phi') \\
&\quad + \frac{1}{\sqrt{g_{\eta\eta}}} \frac{\Gamma_\eta}{\|\Gamma\|_2} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \iint_{\mathbb{E}_{a,b}^2} ds' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \eta} \delta W^L(\lambda', \phi') + \mathcal{O}(\delta\Gamma^2(\mathbf{x}))
\end{aligned} \tag{ }$$

$$\begin{aligned}
\delta\Gamma(\mathbf{x}) &= \gamma(\mathbf{x}) - \Gamma(\mathbf{x}) = \langle \mathbf{e}_\Gamma | \delta\Gamma(\mathbf{x}) \rangle \\
&= \left(\frac{1}{\sqrt{g_{\lambda\lambda}}} \frac{\Gamma_\lambda}{\|\Gamma\|} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} a \sqrt{b^2 + \varepsilon^2 \sin^2 \phi_{ij}} \cos \phi_{ij} \right. \\
&\quad \times \Delta\lambda' \Delta\phi' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \lambda} \\
&\quad + \frac{1}{\sqrt{g_{\phi\phi}}} \frac{\Gamma_\phi}{\|\Gamma\|} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} a \sqrt{b^2 + \varepsilon^2 \sin^2 \phi_{ij}} \cos \phi_{ij} \\
&\quad \times \Delta\lambda' \Delta\phi' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \phi} \\
&\quad + \left. \frac{1}{\sqrt{g_{\eta\eta}}} \frac{\Gamma_\eta}{\|\Gamma\|} \frac{1}{S_{\mathbb{E}_{a,b}^2}} \sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} a \sqrt{b^2 + \varepsilon^2 \sin^2 \phi_{ij}} \cos \phi_{ij} \right. \\
&\quad \left. \times \Delta\lambda' \Delta\phi' \varpi(\phi') \frac{\partial K^L(\lambda, \phi, \eta; \lambda', \phi', \eta_0)}{\partial \eta} \right) \delta W^L(\lambda', \phi')
\end{aligned} \tag{ }$$

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K :

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K^{-1}

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$$K \quad \sigma_i$$

$$\begin{pmatrix} K^* & K \\ \{\sigma_i^2, \mathbf{v}_i\} & \end{pmatrix} \quad ()$$

$$\begin{pmatrix} \{\sigma_i^2, \mathbf{u}_i\} & \\ & K^* K \end{pmatrix}$$

$$KK^*$$

$$\{\sigma_i, \mathbf{u}_i, \mathbf{v}_i\}$$

$$k$$

$$:$$

$$K(\mathbf{v}_i) = \sigma_i \mathbf{u}_i \quad ()$$

$$K^*(\mathbf{u}_i) = \sigma_i \mathbf{v}_i$$

$$\mathbf{u}_i \quad \mathbf{v}_i$$

$$K^* \quad K \quad \sigma_i$$

$$K \quad K \quad []$$

$$K \quad [-]$$

$$K$$

$$" - " ()$$

$$L_2(I \times J)$$

$$[]$$

$$[]$$

$$(k \in L_2(\mathbb{E}_{\eta_0, \varepsilon}^2 \times \mathbb{E}_{\eta_i, \varepsilon}^2))$$

$$L_2(I \times J) \quad k$$

$$\{\mathbf{v}_i\} \quad \{\mathbf{u}_i\}$$

$$L_2(J) \quad L_2(I)$$

$$:[] \quad k$$

$$k(\mathbf{s}, \mathbf{t}) = \sum_{i=1}^{\infty} \sigma_i \mathbf{u}_i(\mathbf{s}) \mathbf{v}_i(\mathbf{t}) \quad ()$$

$$\{\mathbf{v}_i\} \quad \{\mathbf{u}_i\} \quad ()$$

$$K : L_2(J) \rightarrow L_2(I) \quad :$$

$$\sigma_i$$

$$:$$

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_i \geq \dots \quad ()$$

$$(Kf)(\mathbf{s}) = \int_J k(\mathbf{s}, \mathbf{t}) f(\mathbf{t}) d\mathbf{t} \quad ()$$

$$:$$

$$\mathbf{v}_i \quad \mathbf{u}_i$$

$$K$$

$$k_N(\mathbf{s}, \mathbf{t}) = \sum_{i=1}^N \mathbf{p}_i(\mathbf{s}) \mathbf{q}_i(\mathbf{t}) \quad ()$$

$$\{ \mathbf{p}_i(\mathbf{s}) \} \subset L_2(I)$$

$$\{ \mathbf{q}_i(\mathbf{t}) \} \subset L_2(J)$$

$$K : L_2 \rightarrow L_2$$

$$K(f) = g$$

$$\{ K_N \}$$

$$\{ k_N \}$$

$$K_N : L_2(\mathbb{E}_{\eta_0, \varepsilon}^2) \rightarrow L_2(\mathbb{E}_{\eta, \varepsilon}^2)$$

$$(K_N f)(\lambda, \phi, \eta) = \frac{1}{S} \iint_{\mathbb{E}_{\eta_0, \varepsilon}^2} k_N(\lambda, \phi, \eta, \lambda', \phi', \eta_0) \times f(\lambda', \phi', \eta_0) \omega(\phi') dS \quad ()$$

$$K(f) = g$$

$$k_N(\lambda, \phi, \eta, \lambda', \phi', \eta_0) = \sum_{n=0}^N \sum_{m=-n}^n q_{n|m|} e_{nm}(\lambda', \phi') \times e_{nm}(\lambda, \phi).$$

$$f = \sum_{i=1}^{\infty} \frac{\langle g | \mathbf{u}_i \rangle}{\sigma_i} \mathbf{v}_i \quad ()$$

$$\langle g | \mathbf{u}_i \rangle$$

$$\sigma_i$$

$$=$$

$$\{ K_N \}$$

$$K_{\text{Abel-Poisson}}^*$$

$$N(K_{\text{Abel-Poisson}}^*)^{\perp} = N(K_{\text{Abel-Poisson}})^{\perp} = \{ \mathbf{0} \}^{\perp}$$

$$N$$

$$A_1, A_2, A_3, \dots$$

$$N$$

$$A$$

$$M$$

$$(\lim_{n \rightarrow \infty} A_n = A)$$

$$\{ K_N \}$$

$$(L_2(\mathbb{E}_{\eta_0, \varepsilon}^2), L_2(\mathbb{E}_{\eta, \varepsilon}^2))$$

$$K_{\text{Abel-Poisson}}(\delta W_{\mathbb{E}_{\eta_0, \varepsilon}^2}^L) = \delta W^L \quad () \quad ()$$

$$\delta W_{\mathbb{E}_{\eta_0, \varepsilon}^2}^L$$

$$E_{\eta_0, \varepsilon}^2$$

$$E_{\eta_0, \varepsilon}^2$$

$$\left| \frac{\langle \delta W^L(\lambda, \phi, \eta) | e_{nm}(\lambda, \phi) \rangle}{q_{n|m|}} \right|^2 < \infty \quad ()$$

$$\iint_{\mathbb{E}_{\eta_0, \varepsilon}^2} |\delta W^L(\lambda, \phi, \eta)|^2 \omega(\phi) dS < \infty$$

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0 \quad ()$$

$$\mathbf{A} \quad \Sigma$$

$$\mathbf{A}\mathbf{A}^T = \mathbf{U}\Sigma\Sigma^T\mathbf{U}^T \quad \mathbf{A}^T\mathbf{A} = \mathbf{V}\Sigma^T\Sigma\mathbf{V}^T$$

$$\mathbf{A}\mathbf{A}^T \quad \mathbf{A}^T\mathbf{A}$$

$$()$$

$$\mathbf{A}\mathbf{x} = \mathbf{b} \quad \mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{x} \in \mathbb{R}^n, \mathbf{b} \in \mathbb{R}^m \quad ()$$

$$\mathbf{x} \quad ()$$

[]

$$\mathbf{A}$$

$$\begin{cases} t = j + (i-1) \times jMax, 1 \leq i \leq iMax, 1 \leq j \leq jMax \\ x_t = \delta W_{\mathbb{E}^2}^L(\lambda_{ij}, \phi_{ij}) \\ \mathbf{x} = [x_1 \ x_2 \ \dots \ x_t \ \dots \ x_n]^T, \quad n = iMax \times jMax \end{cases}$$

$$i \quad \sigma_i \quad ()$$

$$i \quad \mathbf{v}_i, \mathbf{u}_i$$

$$\mathbf{b} \quad (\quad m)$$

$$() \quad ()$$

$$\mathbf{b} = [b_1 \ b_2 \ \dots \ b_s \ \dots \ b_m]^T, 1 \leq s \leq m, \quad ()$$

$$b_s = \delta \Gamma^L(\lambda_s, \phi_s, \eta_s)$$

$$\begin{cases} \mathbf{A}\mathbf{v}_i = \sigma_i \mathbf{u}_i & \|\mathbf{A}\mathbf{v}_i\|_2 = \sigma_i \\ \mathbf{A}^T \mathbf{u}_i = \sigma_i \mathbf{v}_i & \|\mathbf{A}^T \mathbf{u}_i\|_2 = \sigma_i \end{cases} \quad i = 1, 2, \dots, n \quad ()$$

$$\mathbf{A}$$

$$\mathbf{A}_{mm}$$

$$m \geq n$$

$$\mathbf{A}_{mm} = \mathbf{U}_{mm} \Sigma_{mm} \mathbf{V}_{mm}^T = \sum_{i=1}^n \mathbf{u}_i \sigma_i \mathbf{v}_i^T \quad ()$$

$$\mathbf{U}_{mm} = (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_m)$$

$$\mathbf{V}_{mm} = (\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n)$$

$$\mathbf{V}^T \mathbf{V} = \mathbf{I}_{mm} \quad ()$$

$$\mathbf{U}^T \mathbf{U} = \mathbf{I}_{mm}$$

$$\mathbf{A}$$

$$\mathbf{V} \quad \mathbf{U}$$

$$\mathbf{A}$$

$$[]$$

$$\mathbf{A}$$

$$()$$

$$1 \leq i \leq m$$

$$\delta_{ij}$$

$$\Sigma_{mm} = [\delta_{ij} \sigma_i]$$

$$1 \leq j \leq n$$

eps
 (eps = 2.22 × 10⁻¹⁶ "MATLAB-7")

$$\{\mathbf{u}_i^T \mathbf{b}\} \quad (\varepsilon_A < \sigma_i) \quad \varepsilon_A \quad [\quad]$$

$$\{\sigma_i \mid \sigma_i > \varepsilon_A\}$$

$$\mathbf{x} = \sum_{\sigma_i > \varepsilon_A} \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad ()$$

\mathbf{A}

\mathbf{b}

\mathbf{b}

\mathbf{A}

گسسته

\mathbf{b}

\mathbf{b} \mathbf{A}

$$\mathbf{x} = \mathbf{A}^\dagger \mathbf{b} \quad ()$$

\mathbf{A}

\mathbf{A}^\dagger

$$\|\mathbf{b} - \mathbf{b}^{\text{exact}}\|_2 / \|\mathbf{b}^{\text{exact}}\|_2$$

(

$\mathbf{b}^{\text{exact}}$

$$\|\mathbf{A} - \mathbf{A}^{\text{exact}}\|_2 / \|\mathbf{A}^{\text{exact}}\|_2$$

[]

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \Rightarrow \mathbf{A}^\dagger = \mathbf{V}\mathbf{\Sigma}^\dagger\mathbf{U}^T \quad ()$$

()

$$\mathbf{e} = \mathbf{b} - \mathbf{b}^{\text{exact}} \quad ()$$

$$\mathbf{b} = \mathbf{b}^{\text{exact}} + \mathbf{e}$$

$$\mathbf{x} = \sum_{i=1}^n \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad ()$$

()

$$\mathbf{b}^{\text{exact}} \quad ()$$

$\mathcal{R}(\mathbf{A})$

\mathbf{e}

(\mathbf{A}

گسسته

\mathbf{b}

$\mathbf{Ax} = \mathbf{b}$

\mathbf{b}

ε_A) ε_A

{ σ_i }

\mathbf{e}

$$\sigma_0^2$$

U

$$(\mathbf{u}_i^T \mathbf{e})$$

$$\sigma_0^2$$

$$\mathbf{u}_i^T \mathbf{b} / \sigma_i$$

:[]

Error! Reference source not found.

$$E(|\mathbf{u}_i^T \mathbf{e}|) = \sqrt{\frac{2}{\pi}} \sigma_0 = \sqrt{\frac{2}{\pi}} m^{-\frac{1}{2}} \varepsilon \quad ()$$

ε

$$m \sigma_0^2$$

$$|\mathbf{u}_i^T \mathbf{e}|$$

$$\left(\sqrt{\frac{2}{\pi}} \sigma_0\right)$$

α

:

$$\sigma_i = \mathcal{O}(i^{-\alpha})$$

$\{\sigma_i\}$

c

$\{\sigma_i\}$

$$\sigma_i \leq ci^{-\alpha}$$

α

i

$(\alpha \leq 1)$

$(\alpha > 1)$

$(\{\sigma_i\})$

$(\{|\mathbf{u}_i^T \mathbf{e}|\})$

$(\alpha \gg 1)$

[]

σ_i

$$\mathbf{u}_i^T \mathbf{b}$$

$$\mathbf{u}_i^T \mathbf{b} / \sigma_i$$

$$(23.5 \leq \phi \leq 40 \quad 43.5 \leq \lambda \leq 64)$$

[]

WGD2000

) Error! Reference source not found.

(

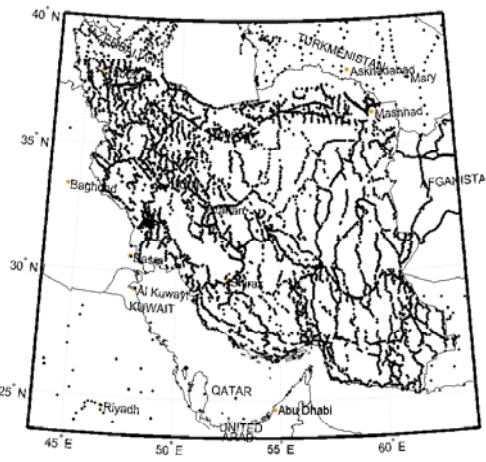
BGI

()

$(\|A\|_2 = \sigma_{Max} :$
 $(Cond(A))$ (σ_{Min}) $[]$ EGM96
 (α) (α) $1^{km} \times 1^{km}$
 $[]$ NIMA
 GTM3AR

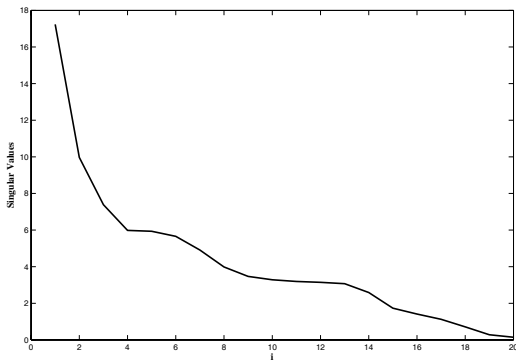
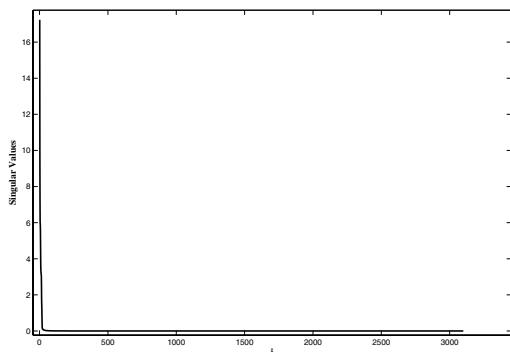
$r_\epsilon(A)$	σ_{Max}	σ_{Min}	$Cond(A)$	α
2572	17.24	1.55×10^{-14}	1.11×10^{15}	0.62

[]

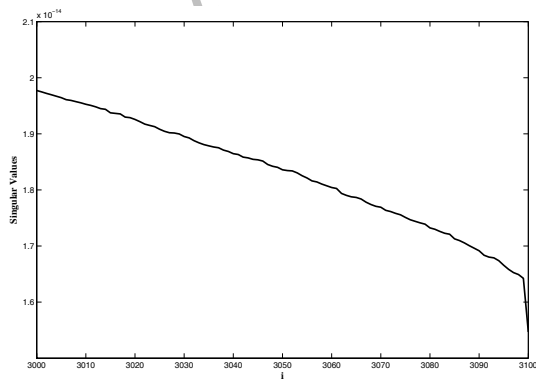


BGI

() () ()



BGI



[]

"

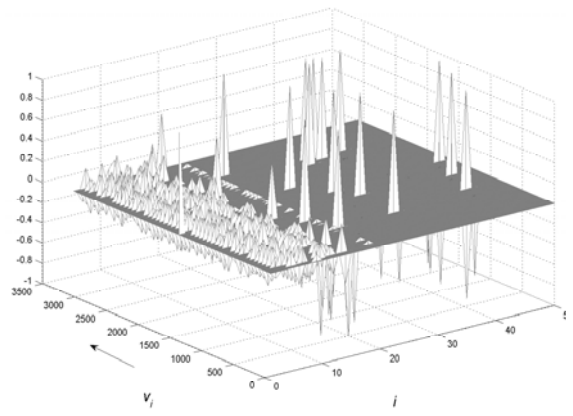
$20' \times 20'$ ()
 ()

() گسسته شده

رتبه :

() $(r_\epsilon(A))$

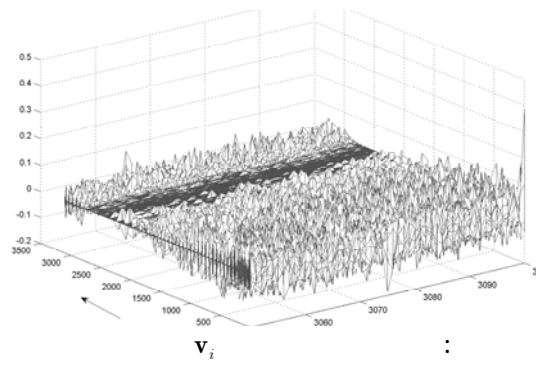
() ()



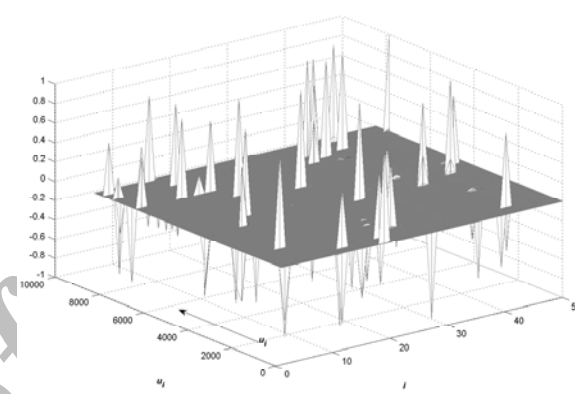
v_i :

() ()

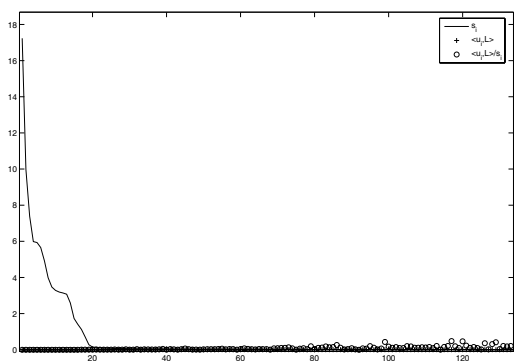
()



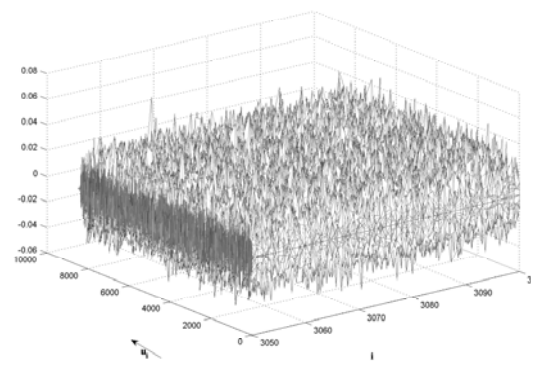
v_i :



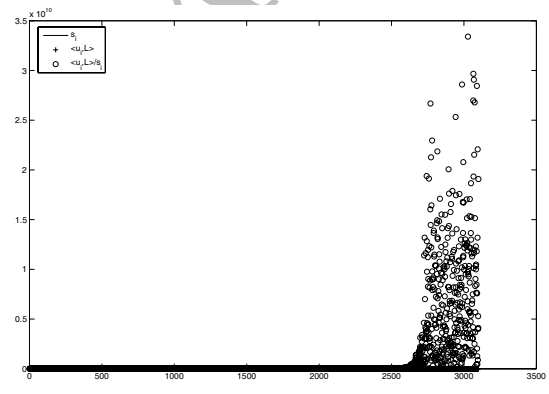
u_i :



:



u_i :



:

205

$$(|\mathbf{u}_i^T \mathbf{b}|)$$

$$\sigma_i$$

()

()

()

σ_{Max}

$$\sigma_{Max} \gg \sigma_{Min}$$

σ_{Min}

$$(1.11 \times 10^{15} \gg 1)$$

$$.(r_\epsilon(\mathbf{A}) = n = 3100)$$

0.62

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- 1 - Molodensky
2 - Quasi Geoid
3 - Normal height Systems
4 - Oblique
5 - Ill-posed
6 - Singular Value Decomposition
7 - Range
8 - Mildly Ill-posed
9 - Moderately Ill-posed
10 - Severely Ill-posed