

Using Parabolic Equations to Determine Propagation of Acoustic Waves in the Water and Under Seabed with Variable Shapes

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Abstract

Propagation analysis of the acoustic waves generated by the vessels and submarines nearby the coast, where the seabed shape is not uniform and is a function of distance, needs a method which consider not only acoustic energy transferred from the seabed in shallow waters but can be applied in low frequencies as well. In this article wave equation in cylindrical coordinate is formulated and then by separating functions, parabolic equations are developed. Considering the variable seabed shape, this equation is solved using Finite Difference Method. Solving this equation provides development of a software which can calculate acoustic signal amplitude reached to any point, based on the input data such as frequency, signal amplitude and amplitude and radiation pattern of the source, tilt angle, sound velocity in the water and under the seabed and also environment parameters.

Key words: Wave equation, Parabolic equations, Acoustic waves, Finite difference method, Velocity of propagation, Acoustic energy.

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(FE)⁽¹⁾

(FD)⁽¹⁾

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$$p(r, z) = A(r) \exp(\phi(r, z))$$

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$$p(r, z)e^{-i\omega t}$$

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$$p(r, z) = \Phi(r)\Psi(z)$$

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$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} + k_0^2 n^2 p = 0 \quad ()$$

$$k_0 \quad p(r, z) \quad z \quad r$$

$$C_0 \quad n = C_0 / C(r, z)$$

$$m/s$$

$$p(r, z) = \Psi(r, z) \Phi(r)$$

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- 1- Parabolic Equations
 - 2- Finite Elements
 - 3- Finite Difference

$$p(r, z) = \Psi(r, z) \Phi(r) \quad ()$$

$$()$$

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$$\Phi(r)$$

$$p\Psi = ik_0(Q-1)\Psi \quad ()$$

Q

$()$

$$\Phi(r) = H_0^{(1)}(k_0 r) \approx \sqrt{\frac{2}{\pi k_0 r}} \exp(i(k_0 r - \frac{\pi}{4}))$$

Q

$()$

$$Q = \sqrt{1+q} \quad q = n^2 - 1 + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \quad ()$$

$$\frac{\partial^2 \Psi}{\partial r^2} + 2ik_0 \frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial z^2} + k_0(n^2 - 1)\Psi = 0$$

$()$

θ_0

q []

$()$

$$q = -\sin^2 \theta_0$$

Q

q

Q

(FD)

$$Q = \frac{a_0 + a_1 q}{b_0 + b_1 q} \quad ()$$

$()$

$()$

$$P = \frac{\partial}{\partial r}, \quad Q = \sqrt{n^2 + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \quad ()$$

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$$Error = \left| \sqrt{1+q} - \frac{a_0 + a_1 q}{b_0 + b_1 q} \right| \quad ()$$

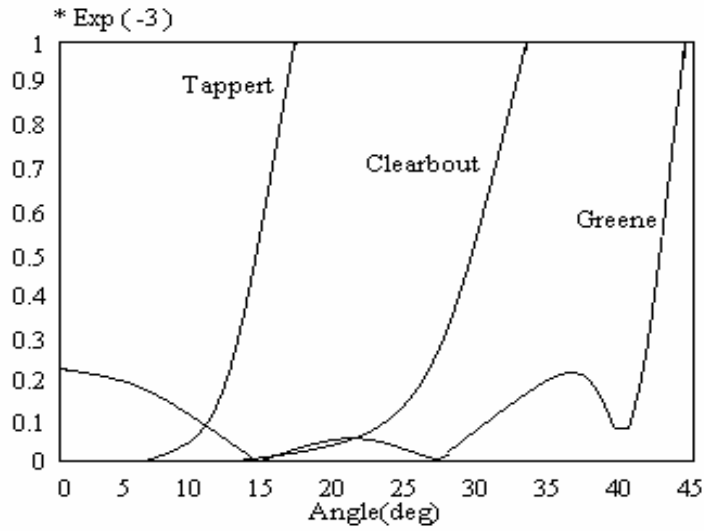
$$(P + ik_0 - ik_0 Q)(P + ik_0 + ik_0 Q)\Psi = 0 \quad ()$$

[] Tappert

(r)

(r)

$$\sqrt{1+q} = 1 + 0.5q \quad Tappert \quad ()$$



[] Tappert [] Clearbout [] Greene

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(r, z)

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Δr Δz

$$\frac{\Psi^{m+1} - \Psi^m}{\Delta r} = ik_0 (\sqrt{1+q} - 1) \frac{\Psi^{m+1} + \Psi^m}{2} \quad ()$$

$$\sqrt{1+q} = \frac{1 + 0.75q}{1 + 0.25q} \quad \text{Clearbout} \quad ()$$

$$\sqrt{1+q} = \frac{0.9998 + 0.79624q}{1 + 0.30102q} \quad \text{Greene} \quad ()$$

	m	Δr	m+1	
l-1				خط 1 ρ_1, α
		ψ^m_{l-1}	ψ^{m+1}_{l-1}	
l				
	Δz	ψ^m_l	ψ^{m+1}_l	
l+1				خط 2 ρ_2, α
		ψ^m_{l+1}	ψ^{m+1}_{l+1}	

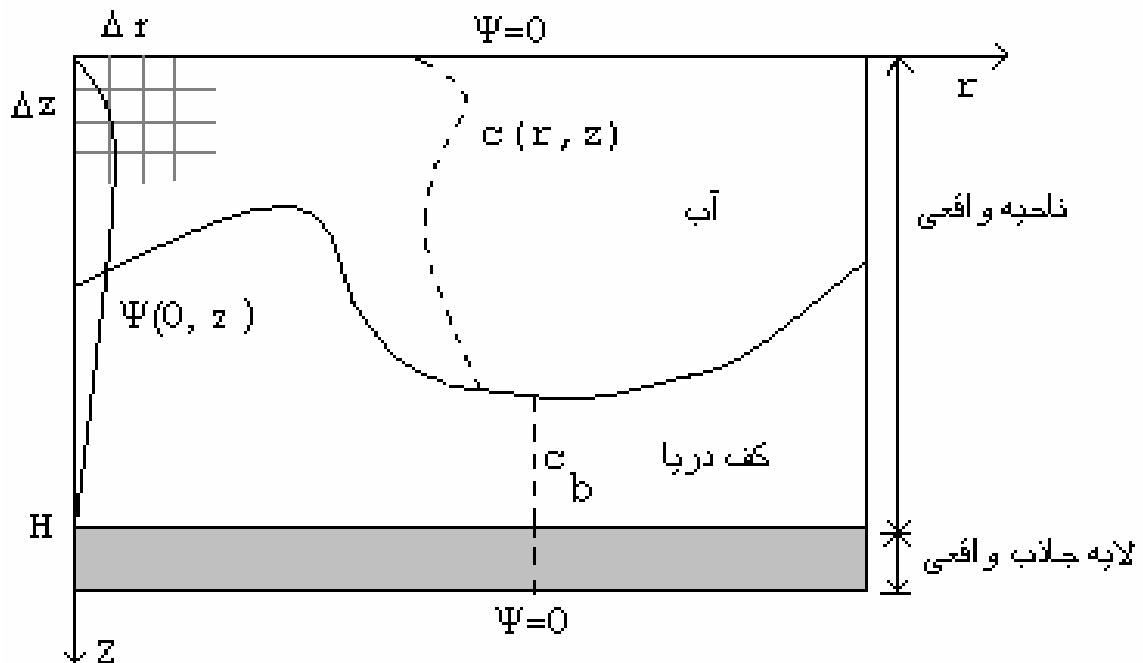
) α

(Nep/m

$$k = \frac{\omega}{c} + i\alpha \quad \alpha > 0 \quad ()$$

$$\alpha_\lambda = -20 \log \left(\frac{e^{-\alpha(r+\lambda)}}{e^{-\alpha r}} \right) = 20 \alpha \lambda \log(e) \quad dB/\lambda \quad ()$$

$$n^2 = \left(\frac{k}{k_0} \right)^2 \approx \left(\frac{C_0}{C} \right)^2 \left[1 + i \frac{2\alpha c}{\omega} \right] \approx \left(\frac{C_0}{C} \right)^2 \left[1 + i \frac{\alpha_\lambda}{27.29} \right] \quad ()$$



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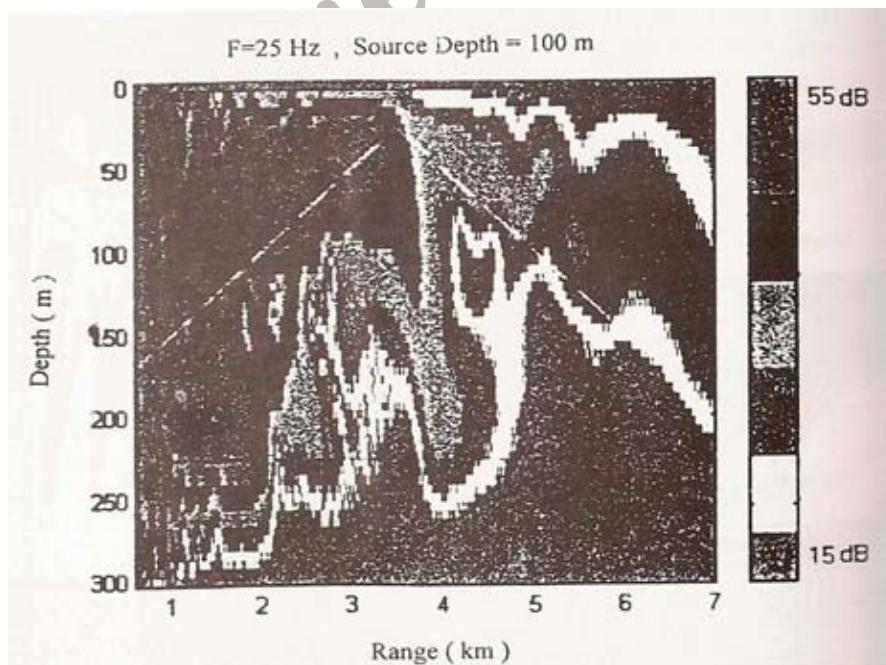
m/s

$1 \text{ dB}/\lambda$

m/s

kg/m^3

()



()

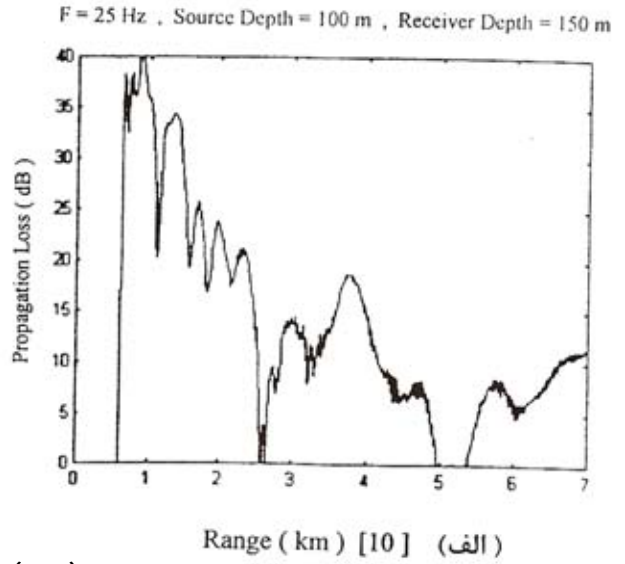
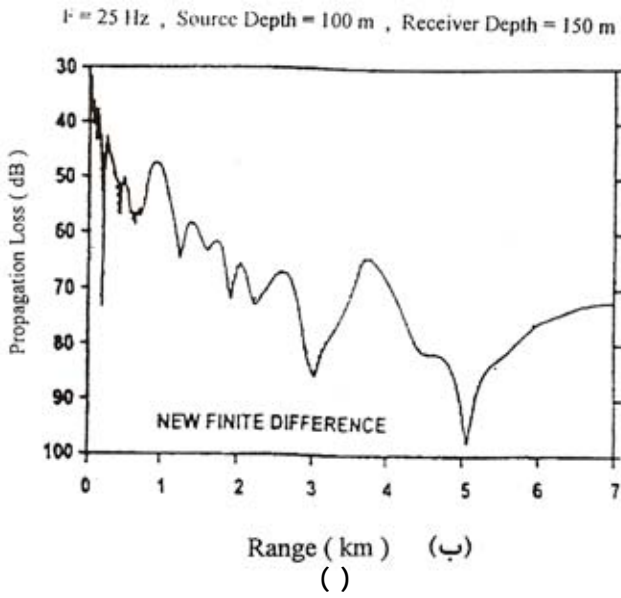
m Hz

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(-)

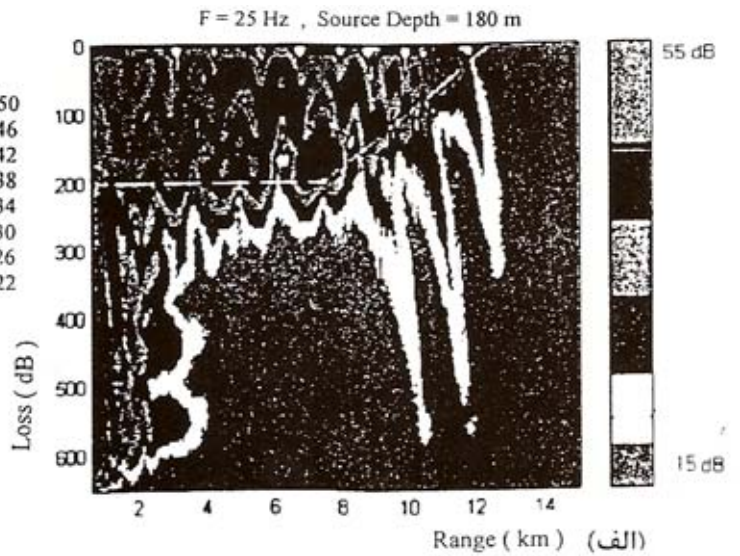
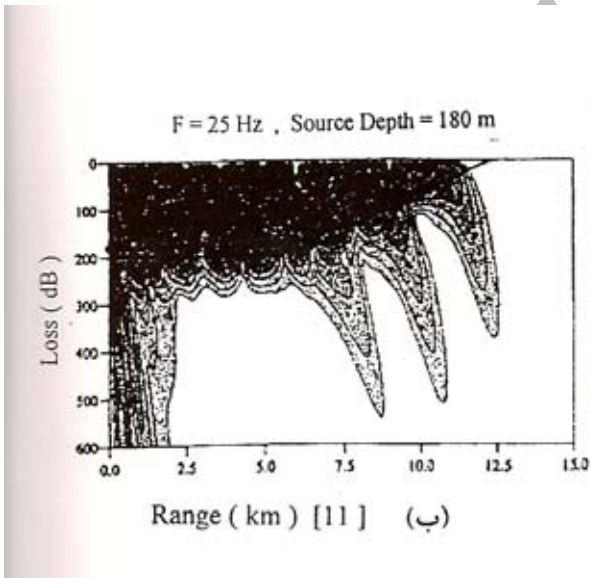
(-) []

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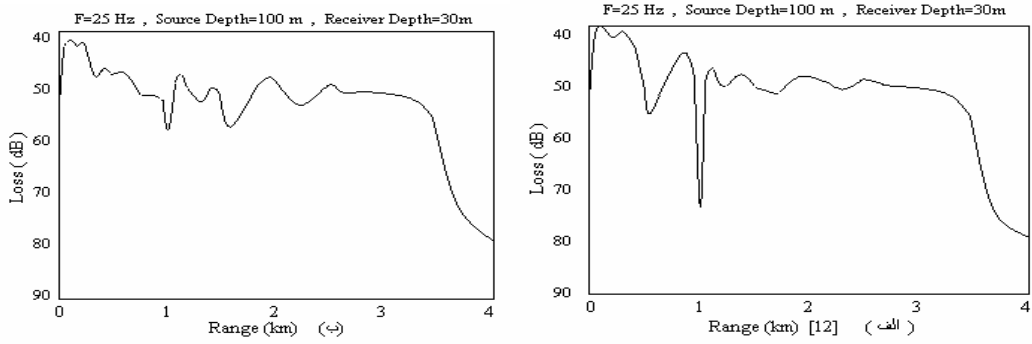
m

Hz

km

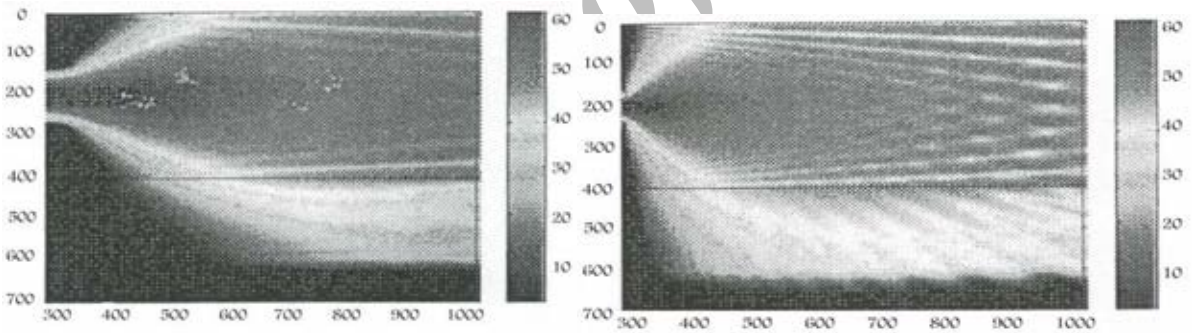
m

m



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Hz

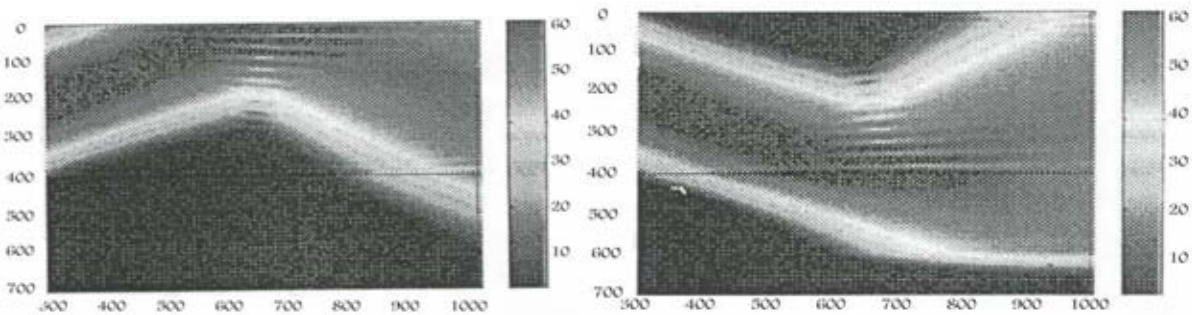


(+30°)

5° 60°

(-30°)

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" (5°)
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