

## **Circuit Modeling and Measurement of Noise for a Semiconductor Laser Diode**

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### **Abstract**

This paper presents equivalent circuit models for both relative intensity noise (RIN) and phase/frequency noise spectrum (FNS) in a single semiconductor laser diode. The model for the electrical phase noise of a single mode laser is proposed for the first time. These equivalent circuit models are derived from the rate equations including the Langevin noise sources. Then, RIN and FNS are calculated in terms of electrical parameters. Finally, we explain an indirect experimental method used to measure RIN and FNS of a typical optical communication laser diode. Behavior of the experimental results is in agreement with those calculated by circuit models.

**Key words:** Relative intensity noise (RIN), Frequency/ phase noise spectrum (FNS), Equivalent circuit modeling, Semiconductor laser, Mode-Hopping.

$$\frac{dN}{dt} = \frac{I}{q} - \gamma_e N - GP + F_N(t) \quad ( - )$$

$$\frac{dP}{dt} = (G - \gamma)P + R_{sp} + F_P(t) \quad ( - )$$

$$\frac{d\phi}{dt} = -(\omega - \omega_{th}) + \frac{1}{2}\beta_c(G - \gamma) + F_\phi(t) \quad ( - )$$

$R_{sp}$  .  $q$   $I$   $\phi$   $P$   $N$

$F_P(t)$   $F_N(t)$  .  $\omega_{th}$   $\omega$   $\beta_c$   $F_\phi(t)$

$\tau_p$   $\gamma_e = 1/\tau_e$   $\gamma = 1/\tau_p$  .  $G$  .  $\tau_e$

$$G = A(N - N_0)(1 - P/S_p) \quad ( )$$

$$\frac{\bar{I}}{q} - \gamma_e \bar{N} - \bar{G}\bar{P} = 0 \quad ( - )$$

$$(\bar{G} - \gamma)\bar{P} + R_{sp} = 0 \quad ( - )$$

$\bar{G}$   $\bar{I}$   $\bar{P}$   $\bar{N}$

( )

$$\langle F_j(t) \rangle = 0 \quad ( - )$$

$$\langle F_k(t)F_j(t') \rangle = 2D_{kj}\delta(t-t') \quad ( - )$$

(FNS) (RIN)

( ) RIN

FNS

[ ]

RIN

[ ]

FNS

RIN

[ ]

RF

[ ]

FNS RIN

[ - ]

FNS RIN

- 1- Relative Intensity Noise
- 2- Frequency Noise Spectrum
- 3- Mode Hopping
- 4- Langevin rate equations

$$\delta \tilde{\phi} = \left[ \frac{1}{2} A \beta_c \delta \tilde{N} + \tilde{F}_\phi \right] / j \omega \quad ( - ) \quad : \quad D_{kj}$$

$$D = -\omega^2 + j \omega (\Gamma_N + \Gamma_P) + \Gamma_N \Gamma_P + A \bar{P} \bar{G}_l \quad ( - )$$

$$D_{NN} = R_{sp} \bar{P} + \gamma_e \bar{N}; \quad D_{PP} = R_{sp} \bar{P}; \quad ( )$$

$$D_{\phi\phi} = \frac{R_{sp}}{4\bar{P}}; \quad D_{NP} = -R_{sp} \bar{P}; \quad D_{N\phi} = D_{P\phi} = 0$$

$\delta V$

$$\delta V = m V_T \frac{\delta N}{N};$$

$$V_T = kT / q$$

$$m = 2 + \frac{\bar{N}}{\text{Vol} 2\sqrt{2}} \left[ \frac{1}{N_v} + \frac{1}{N_c} \right] \quad ( - )$$

$V_T$

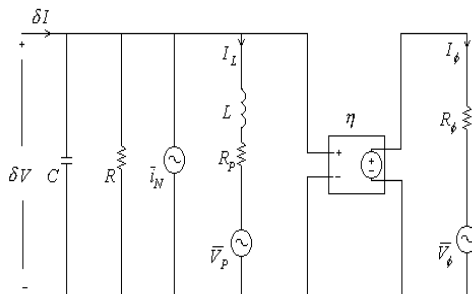
$N_c$   $N_v$   
 $\text{Vol}$

$$i_L = q G_l \delta P \quad ( )$$

$$i_\phi = \omega \delta \phi \quad ( )$$

FNS

[ ] [ ] RIN



$$\frac{d \delta N(t)}{dt} = \frac{\delta I(t)}{q} - \Gamma_N \delta N(t) - \bar{G}_l \delta P(t) + F_N(t) \quad ( - )$$

$$\frac{d \delta P(t)}{dt} = A \bar{P} \delta N(t) - \Gamma_P \delta P(t) + F_P(t) \quad ( - )$$

$$\frac{d \delta \phi(t)}{dt} = \frac{1}{2} A \beta_c \delta N(t) - \frac{1}{2} \beta_c \frac{\bar{G}_{nl}}{\bar{P}} \delta P(t) + F_\phi(t) \quad ( - )$$

$$\Gamma_N = \gamma_e + A \bar{P} \quad ( - )$$

$$\Gamma_P = R_{sp} / \bar{P} + 2 \bar{G}_{nl} \quad ( - )$$

: ( )

$$\delta \tilde{N} = [(j\omega + \Gamma_P) \tilde{F}_N - \bar{G}_l \tilde{F}_P] / D \quad ( - )$$

$$\delta \tilde{P} = [A \bar{P} \tilde{F}_N + (j\omega + \Gamma_N) \tilde{F}_P] / D \quad ( - )$$

$$S_p(\omega) \quad : \quad ( )$$

( )

$$f \ll f_r$$

$$f_r$$

RIN

$$C \frac{d \delta V}{dt} = \delta I - \frac{\delta V}{R} - i_L + i_N \quad ( - )$$

$$L \frac{d i_L}{dt} = \delta V - R_p i_L - V_p \quad ( - )$$

$$R_\phi i_\phi = \eta \delta V - V_\phi \quad ( - )$$

1.3 μm	λ	
250 μm	L	
2 μm	W	
0.2 μm	d	
0.1 ps	τ	
10 <sup>8</sup>	N <sub>0</sub>	
3.3×10 <sup>9</sup>	S <sub>p</sub>	
1.7×10 <sup>12</sup> s <sup>-1</sup>	R <sub>sp</sub>	
1.6 ps	τ <sub>p</sub>	
2.2 ns	τ <sub>e</sub>	
5625 s <sup>-1</sup>	A	
5	β <sub>c</sub>	

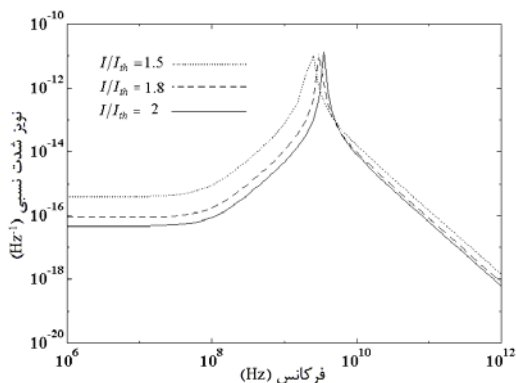
$$C = \frac{q \bar{N}}{m V_T}; \quad R = \frac{1}{C \Gamma_N}; \quad L = \frac{1}{C A \bar{G}_l \bar{P}};$$

$$R_p = L \Gamma_p; \quad R_\phi = \frac{2 q \eta}{A C \beta_c}; \quad \eta = A^3 \quad ( )$$

$$S_N = \frac{\bar{i}_N^2}{\Delta f} = 2 q^2 (\gamma_e \bar{N} + R_{sp} \bar{P}) \quad ( - )$$

$$S_p = \frac{\bar{V}_p^2}{\Delta f} = 2 (q L \bar{G}_l)^2 R_{sp} \bar{P} \quad ( - )$$

$$S_\phi = \frac{\bar{V}_\phi^2}{\Delta f} = R_\phi^2 \frac{R_{sp}}{2 \bar{P}} \quad ( - )$$

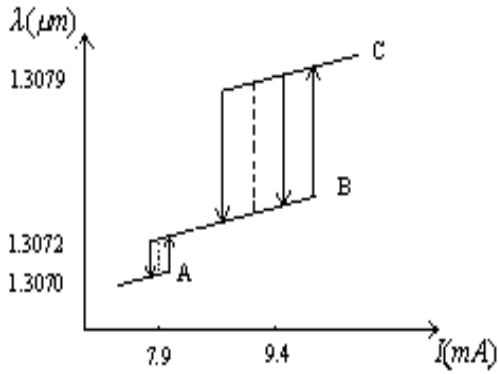


$$RIN(\omega) = \frac{S_p(\omega)}{\bar{P}^2} = \frac{\bar{i}_L^2(\omega)}{(I - I_{th})^2} \quad ( )$$

MHN

C B

RIN



.FLD3C5LK

(C B )

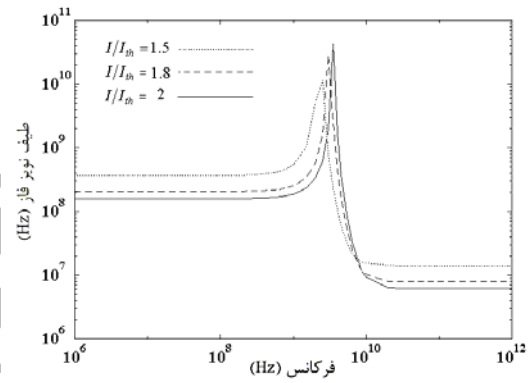
(A )

MHN

$$S_{\phi}(\omega) = \left\langle \left| \omega \delta \tilde{\phi}(\omega) \right|^2 \right\rangle = \bar{i}_{\phi}^{-2} \quad ( )$$

FNS ( )

RIN FNS



FNS RIN

MHN

$N_0 \beta_c A$

FNS RIN (MHN)

(FLD3C5LK)

( )

$\Delta f$

MHN / mA

$f_0$

$P_{out}$

[ ] [ ]

FNS RIN

[ ]

MHN

( )

FLD3C5LK

[ ]

$\omega = 0$

(C B ) / mA (A ) / mA

/ mA

$$\bar{P} = \frac{R_{sp}}{4\pi\Delta f} (1 + \beta_c^2) \quad ( )$$

1- Mode Hopping Noise

$$f_r = \omega_r / 2\pi; \quad \omega_r = \left( \frac{A(I - I_{th})}{q} \right)^{1/2} \quad ( )$$

$$\Gamma_R = \frac{1}{2}(\Gamma_N + \Gamma_P), \quad ( )$$

$$\bar{P} = \frac{I/q + AN_0 \bar{P}}{\gamma_e + A \bar{P}} \quad ( )$$

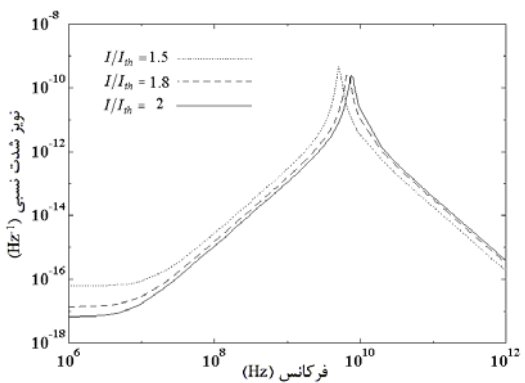
$$RIN = 2R_{sp} \times \left\{ \frac{(\Gamma_N^2 + \omega^2) + A^2 \bar{P}^2 (1 + \gamma_e \bar{N} / R_{sp} \bar{P}) - 2A\Gamma_N \bar{P}}{\bar{P}[(\omega_r - \omega)^2 + \Gamma_R^2][(\omega_r + \omega)^2 + \Gamma_R^2]} \right\} \quad ( )$$

$$S_{\phi}(\omega) \cong \frac{R_{sp}}{2\bar{P}} \times \left\{ 1 + \left\{ \beta_c^2 \omega_r^4 / [(\omega_r - \omega)^2 + (2\omega\Gamma_R)^2] \right\} \right\} \quad ( )$$

$$R_{sp} = \beta \bar{N} / \tau_e = \beta \bar{N} \gamma_e \quad ( - )$$

$$\beta = \frac{\zeta k A (\bar{N} - N_0)}{\gamma_e \bar{N} (E_f - h f_0)} T \quad ( - )$$

$$E_f \quad \zeta \sim 1.4 \quad 10^{-5} < \beta < 10^{-3} \quad ( )$$



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FNS RIN

FNS RIN

[3] T. T. Bich-Ha and J. Mollier, "Noise Equivalent Circuit of a Two-Mode Semiconductor Laser with the Contribution of Both the Linear and the Nonlinear Gain," IEEE J. Selected Topics in Quantum Electronics, vol. 3, No. 2, pp. 304-308, 1997.

[4] G. E. Obarski, and P. D. Hale, "How to Measure Relative Intensity Noise in Lasers," Laser Focus World, May 1992.

[5] Yuh-Jen Cheng, Paul L. ,and E. Siegman, "Measurement of Laser Quantum Frequency Fluctuations Using a Pound-Driver Stabilization System," IEEE J. Quantum Electron., vol. 30, No. 6, pp. 1498-1504, 1994.

[6] Y. Yamamoto, T. Mukai, and S. Saito, "Quantum Phase Noise and Line width of a semiconductor Laser," Electronic Letters, vol. 17, No. 9, pp. 327-329, 1981.

[7] B. Daino, P. Spano, M. Tamburrini, and S. Piazzolla, "Phase Noise and Spectral Line shape in semiconductor Lasers," IEEE J. Quantum Electron., vol. QE-19, No. 3, pp. 266-270, 1983.

[8] Ady Arie and Moshe Tur., "Noise Measurement in Two-Beam Interferometers Excited by Semiconductor lasers with Non-Lorentzian Line Shapes," IEEE J. Quantum Electron., vol. 26, No. 11, pp. 1999-2005, 1990.

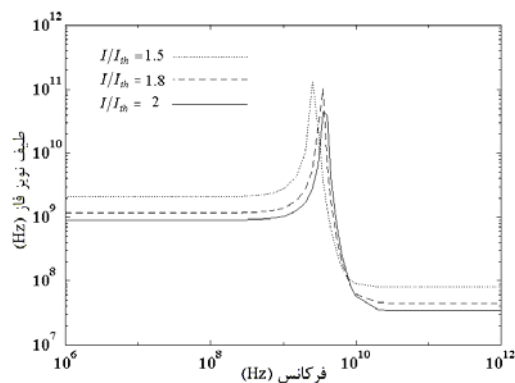
[9] G. P. Agrawal and N. K. Dutta, Semiconductor Lasers, 2<sup>nd</sup> ed., New York: Van Nostrand Reinhold, 1993.

[10] D. E. Dodds, and M. J. Sieben, "Fabry-Perot Laser Diode Modeling," IEEE Photonics Technology letters, vol. 7, No. 3, pp. 254-257, 1995.

[11] M. Ohtsu, and Y. Teramachi, "Analyses of Mode Partition and Mode Hopping in Semiconductor Lasers," IEEE J. Quantum Electron., vol. 25, No. 1, pp. 31-38, 1989.

[12] M. R. Alalusi, and R. B. Darling, "Effects of Nonlinear Gain on Mode-Hopping in Semiconductor Laser Diodes," IEEE J. Quantum Electron., vol. 31, No. 7, pp. 1181-1192, 1995.

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[1] K. Petermann, Laser Diode Modulation and Noise, Dordrecht, The Netherlands: Kluwer Academic, 1988.

[2] C. Harder, J. Katz, S. Margalit, J. Shacham, and A. Yariv, "Noise equivalent Circuit of a Semiconductor Laser Diode," IEEE J. Quantum Electron., vol. QE-18, No. 3, pp. 333-337, 1982.