

Edge Detection Using Combined Isotropic and Directional Wavelet Transform

J. Musevi Niya, A. Aghagolzadeh and M. A. Tinati Faculty of Electrical and Computer Engineering, University of Tabriz

Abstract

In this paper we propose a new approach for solving the edge detection problem using combined isotropic and directional wavelet transform. A primary knowledge about the direction of the gradient of the image, the direction of the probable edges, is extracted by the ordinary separable isotropic wavelet-based edge detector; then based on the computed direction of the probable edges, the adaptive nonseparable directional wavelet-based edge detector is applied on these pixels and the new magnitude of the wavelet transform coefficients is computed and compared to a certain threshold value; then the pixel is classified as edge points or not edge points. This method detects the edges of the image with superior quality and performance comparing to the ordinary wavelet-based edge detection in the presence of noise; and has less computational cost compared to the directional wavelet-based edge detection methods presented before.

Key words: Edge detection, Wavelet transform, Isotropic wavelet, Directional wavelet.

(Fine Scales)

(Coarse Scales)

[] Xu .

()

[]

[] Canny.

[] Hildreth Marr .

[] Haralick

[]

[] Poggio Torre .

(Gaussian Smoothing Function)

()

[] Zhong Mallat

[]

(Isotropic Wavelets)

Mallat .

(Directional Wavelets)

(Wavelet Transform)

[] Zhong

(Multi Scale)

(Multi Resolution)

[]

[]

()

()

()

()

$$f \in L^2(\mathbb{R})$$

$$W_s f(x) = f * \psi_s(x) \quad [\quad] \quad ()$$

$$\psi_s(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x}{s}\right) \quad \psi(x) \in L^2(\mathbb{R})$$

$$L^2(\mathbb{R}) \quad g \quad f$$

$$f * g(x) = \int_{-\infty}^{+\infty} f(u)g(x-u)du$$

$\psi(x)$

(Mother Wavelet Function)

[] Otsu

()

Archive of SID

$$W_s f(x) = f * s^2 \frac{d^2 \theta_s}{d^2 x}(x) = s^2 \frac{d^2}{d^2 x}(f * \theta_s)(x) \quad (1)$$

$$\hat{\psi}(0) = 0 \quad (\text{Admissibility Condition})$$

$$\hat{\psi}(f) = \int_{-\infty}^{+\infty} \psi(x) e^{-j2\pi f x} dx$$

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0$$

(Separable)

$$f(x, y) \in L^2(R^2) \quad (2)$$

$$W_s^1 f(x, y) = f * \psi_s^1(x, y),$$

$$W_s^2 f(x, y) = f * \psi_s^2(x, y) \quad (3)$$

$$\psi_s^2(x, y) \quad \psi_s^1(x, y)$$

$$\hat{\psi}^k(f_x, f_y) \quad \psi^k(x, y) \quad k = 1, 2 \quad \hat{\psi}^k(0, 0) = 0$$

$$\hat{\psi}^k(f_x, f_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi^k(x, y) e^{-j2\pi(f_x x + f_y y)} dx dy \quad (4)$$

$$\psi(x) = \frac{d}{dx} \theta(x) \quad (5)$$

$$W_s f(x) = f * s \frac{d}{dx} \theta_s(x) = s \frac{d}{dx} (f * \theta_s)(x) \quad (6)$$

$$\psi_s(x) = s \frac{d}{dx} \theta_s(x)$$

$$\theta_s(x) = \frac{1}{\sqrt{s}} \theta\left(\frac{x}{s}\right)$$

$$f * \theta_s$$

$$|W_s f(x)|$$

$$\|W_s f(x, y)\| = \sqrt{|W_s^1 f(x, y)|^2 + |W_s^2 f(x, y)|^2}$$

$$\angle W_s f(x, y) = \arctan\left(\frac{W_s^2 f(x, y)}{W_s^1 f(x, y)}\right) \quad ()$$

$$\|W_s f(x, y)\|$$

$$\angle W_s f(x, y) + \frac{\pi}{2}$$

$$E_s(x, y)$$

$$E_s(x, y) = \begin{cases} 0 & \|W_s f(x, y)\| < T \\ 1 & \|W_s f(x, y)\| > T \end{cases} \quad ()$$

[] Bao Zhang [] Xu

[] Xu .

[] Xu

()

$$\theta(x, y)$$

y x

[]

$$\psi^1(x, y) = \frac{\partial}{\partial x} \theta(x, y)$$

$$\psi^2(x, y) = \frac{\partial}{\partial y} \theta(x, y) \quad ()$$

$$\theta(x, y)$$

$$\theta(x, y) \rightarrow 0 \quad \int_{R^2} \theta(x, y) dx dy = 1$$

$$\theta(x, y) \quad (x^2 + y^2)$$

y x

$$\theta_s(x, y) = \frac{1}{s} \theta\left(\frac{x}{s}, \frac{y}{s}\right)$$

$$\psi_s^1(x, y) = s \frac{\partial}{\partial x} \theta_s(x, y)$$

$$\psi_s^2(x, y) = s \frac{\partial}{\partial y} \theta_s(x, y) \quad ()$$

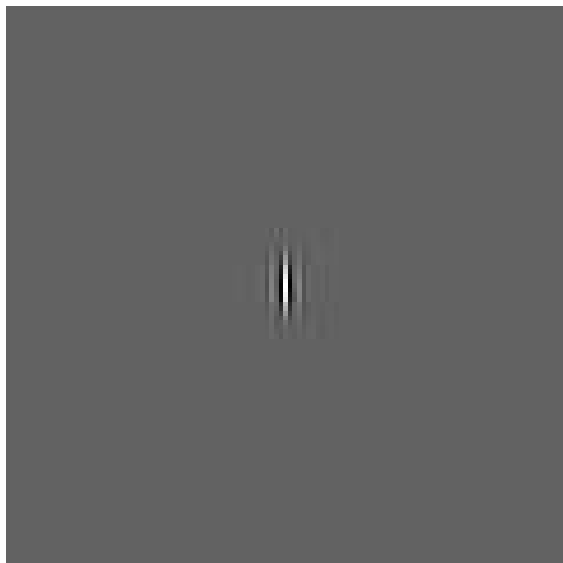
()

$$W_s^1(x, y) = s \frac{\partial}{\partial x} (f * \theta_s)(x, y)$$

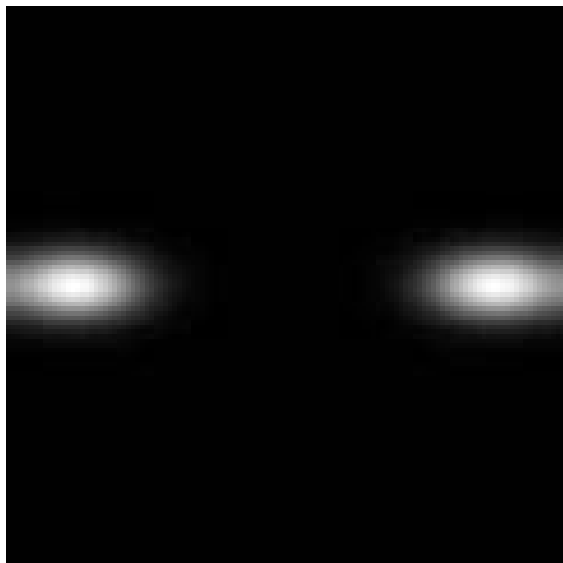
$$W_s^2(x, y) = s \frac{\partial}{\partial y} (f * \theta_s)(x, y) \quad ()$$

$$f * \theta_s$$

$$W_s f(x, y) = [W_s^1 f(x, y) \quad W_s^2 f(x, y)]^T \quad ()$$



()



()

$\sigma_x = 2$

Gabor

- : $\theta = 0^\circ$ $w = 0.375$ $\sigma_y = 4$

Gabor

$\pm W$

f_x

x

y

x

y

) Gabor

Gabor

(Morlet

y x

σ_y σ_x

x W

$$\psi(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\right] \cos(2\pi Wx)$$

$$\hat{\psi}(f_x, f_y) = \frac{1}{4\pi\sigma_{f_x}\sigma_{f_y}} \left(\exp\left\{-\frac{1}{2}\left[\frac{(f_x - W)^2}{\sigma_{f_x}^2} + \frac{f_y^2}{\sigma_{f_y}^2}\right]\right\} + \exp\left\{-\frac{1}{2}\left[\frac{(f_x + W)^2}{\sigma_{f_x}^2} + \frac{f_y^2}{\sigma_{f_y}^2}\right]\right\} \right) \quad ()$$

$$\sigma_{f_y} = \frac{1}{2\pi\sigma_y} \quad \sigma_{f_x} = \frac{1}{2\pi\sigma_x} :$$

$\psi(x, y)$

()

Gabor

$\theta = 0$

N M
 W_L $s > 1$ W s

$$\theta_n = \frac{n\pi}{N}$$

$$W_{mn}f(x, y) = (f * \psi_{mn})(x, y) \quad ()$$

()

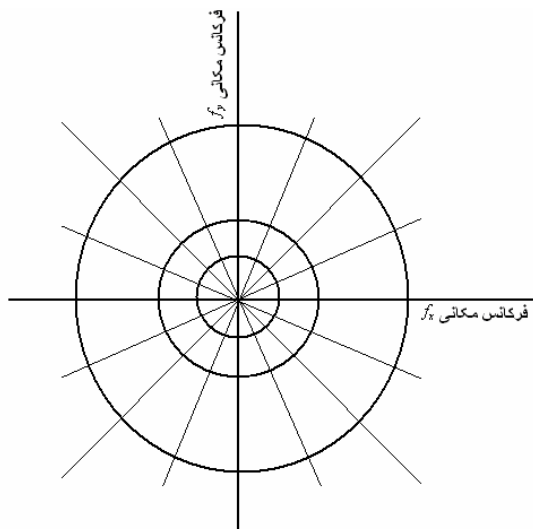
$$\psi_{mn}(x, y)$$

Gabor

Gabor

$$\theta_n = \frac{n\pi}{N}$$

: $\psi(x, y)$



$$\begin{aligned} \psi_{mn}(x, y) &= s^{-m}\psi(x', y'), \\ y' &= s^{-m}(-x \sin \theta_n + y \cos \theta_n), \\ x' &= s^{-m}(x \cos \theta_n + y \sin \theta_n) \end{aligned} \quad ()$$

σ_y σ_x Gabor

$$\theta_n = \frac{n\pi}{N}$$

$$n = 0, 1, 2, \dots, N-1$$

$$m = 0, 1, 2, \dots, M-1 \quad s^m$$

$$W = 0.45$$

Gabor

$$M = 2 \quad N = 8 \quad s = 2$$

(Rosette)

:[]

$$\theta_n = \frac{n\pi}{N}$$

$$s^m \quad n = 0, 1, 2, \dots, N-1$$

$$m = 0, 1, 2, \dots, M-1$$

$$\sigma_{f_x} = \frac{(s-1)W}{(s+1)\sqrt{2 \ln 2}},$$

$$\sigma_{f_y} = \frac{\tan\left(\frac{\pi}{2N}\right) \left[W - \left(\frac{\sigma_{f_x}^2}{W}\right) 2 \ln 2 \right]}{\sqrt{2 \ln 2 - \frac{(2 \ln 2)^2 \sigma_{f_x}^2}{W^2}}},$$

$$\sigma_x = \frac{1}{2\pi\sigma_{f_x}},$$

$$\sigma_y = \frac{1}{2\pi\sigma_{f_y}},$$

$$W_L = \frac{W}{s^{M-1}}, \quad ()$$

$$Mf(x, y) = \text{Max}_n \{ \Pi_{m=1}^2 W_{mn} f(x, y) \} \quad ()$$

$$n \quad \theta_n = \frac{n\pi}{N}$$

$$\theta_s(x, y) = \frac{1}{s} \theta\left(\frac{x}{s}, \frac{y}{s}\right), \quad []$$

$$W_s^1 f(x, y) = s \frac{\partial}{\partial x} (f * \theta_s)(x, y),$$

$$W_s^2 f(x, y) = s \frac{\partial}{\partial y} (f * \theta_s)(x, y), \quad [] \text{ Xu}$$

$$s = a^{-m}, m = 1, 2$$

$$W^1 f(x, y) = \Pi_{m=1}^2 W_s^1 f(x, y),$$

$$W^2 f(x, y) = \Pi_{m=1}^2 W_s^2 f(x, y),$$

$$\theta_e = \angle Wf(x, y) = \arctan\left(\frac{W^2 f(x, y)}{W^1 f(x, y)}\right) \quad ()$$

θ_e

Gabor

$$x' = s^{-m} (x \cos \theta_e + y \sin \theta_e),$$

$$y' = s^{-m} (-x \sin \theta_e + y \cos \theta_e),$$

$$\psi_m(x, y) = s^{-m} \psi(x', y'),$$

$$W_m f(x, y) = (f * \psi_m)(x, y),$$

$$Mf(x, y) = \Pi_{m=1}^2 W_m f(x, y) \quad ()$$

$$\theta_n = \frac{n\pi}{N}$$

N

N

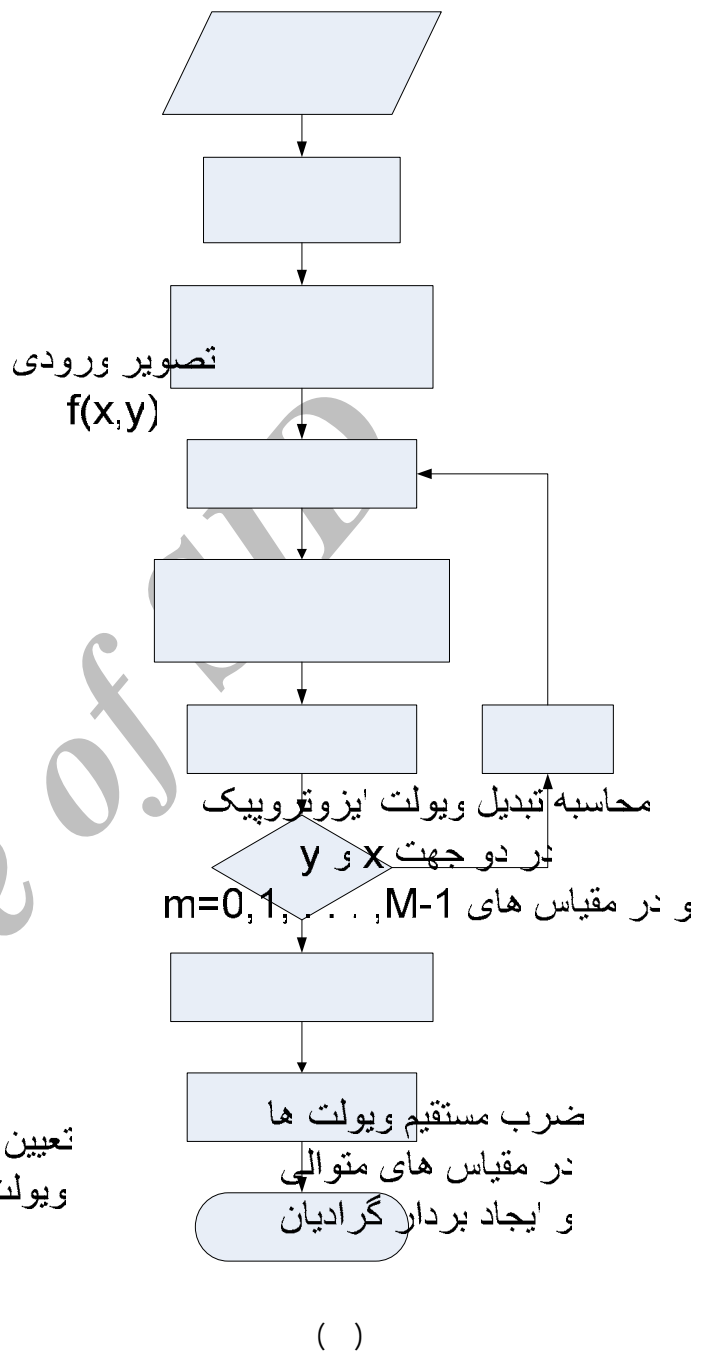
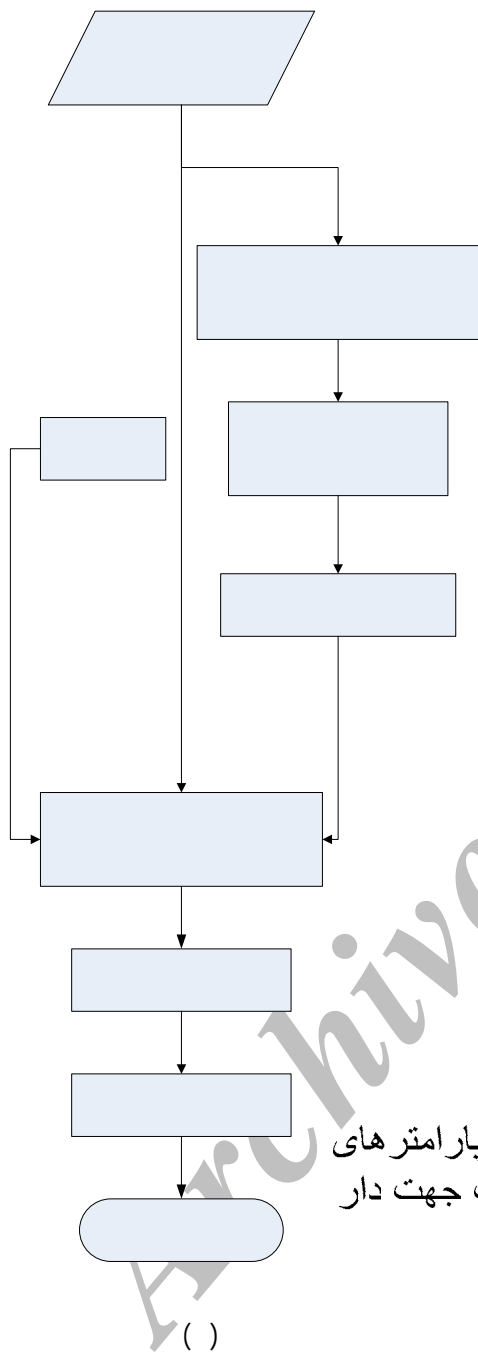
[]

N

[]

()

$$\theta(x, y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2\sigma^2}(x^2 + y^2)\right],$$



)

(

- محاسبه تابع فاز برای
بردار گرادینان

$y \ x$

N

پیکسل مورد بررسی
امتداد لبه احتمالی در

SNR

N ()

M

()

Gabor

$$\frac{\pi}{N}$$

N

$$W = 0.375$$

M

Xu

$$\sigma_x = 1.5$$

$$M = 2$$

$$N = 12$$

$$s = 2$$

$$\sigma_y = 4$$

()

$$512 \times 512$$

N

$$N/3$$

$$\sigma_x = 1.5 \quad M = 2 \quad s = 2$$

$$\sigma_y = 1.5$$

N

$$N = 12 \quad s = 2 \quad W = 0.375$$

$$\sigma_y = 4 \quad \sigma_x = 1.5 \quad M = 2$$

()

$$\sigma_y = 1.5 \quad \sigma_x = 1.5 \quad M = 2 \quad s = 2$$

$$s = 2 \quad W = 0.375$$

$$\sigma_y = 4 \quad \sigma_x = 1.5 \quad M = 2 \quad N = 12$$

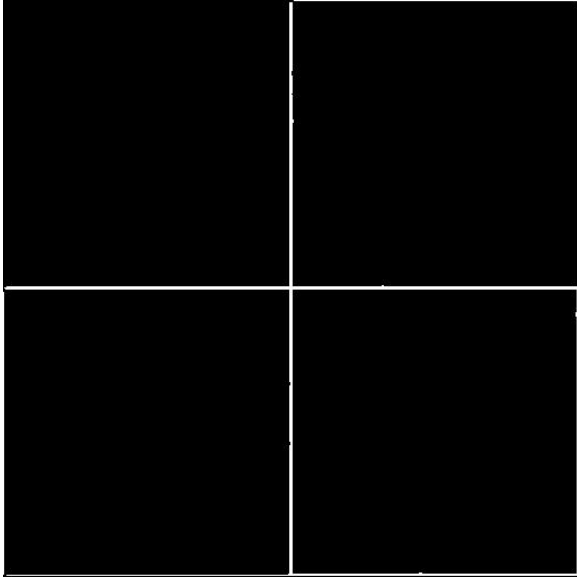
[] Otsu

$$512 \times 512$$

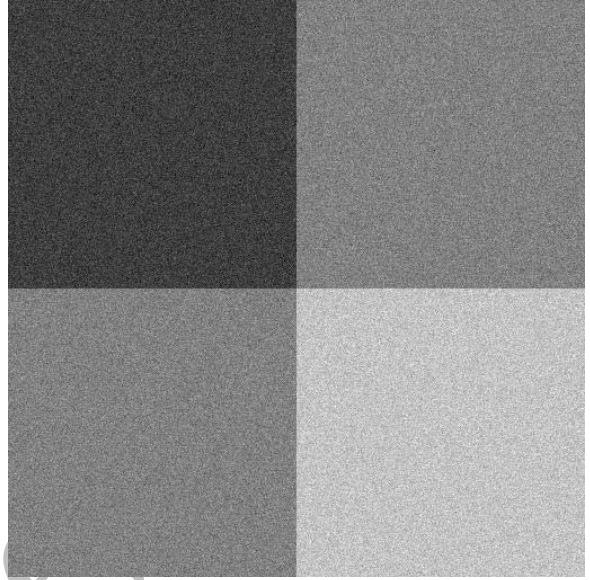
()

$$512 \times 512$$

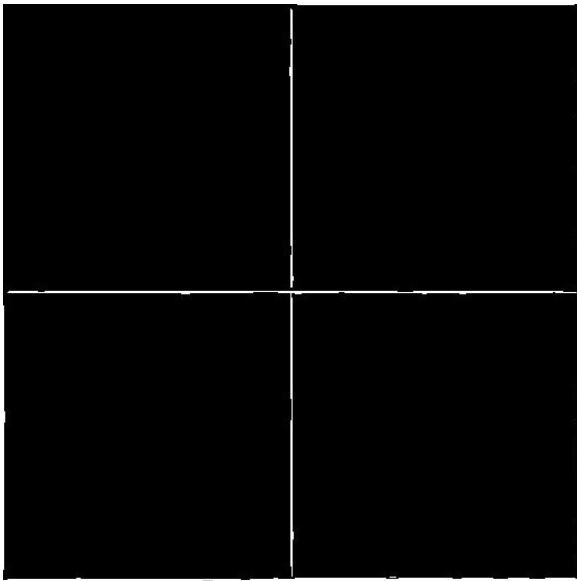
()



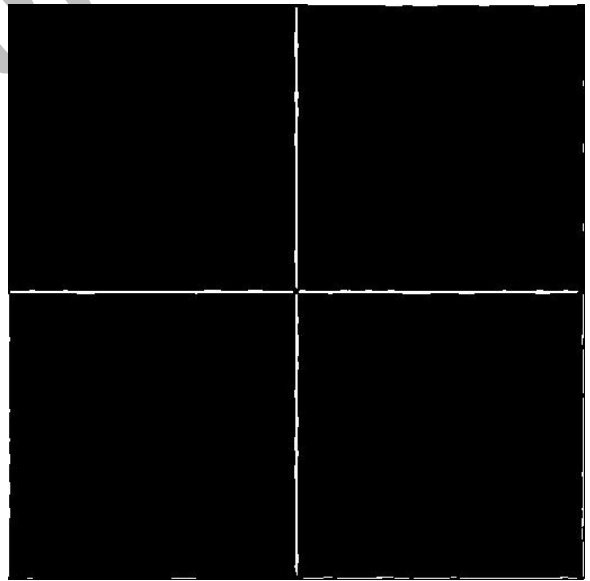
()



()



()



()

(

(

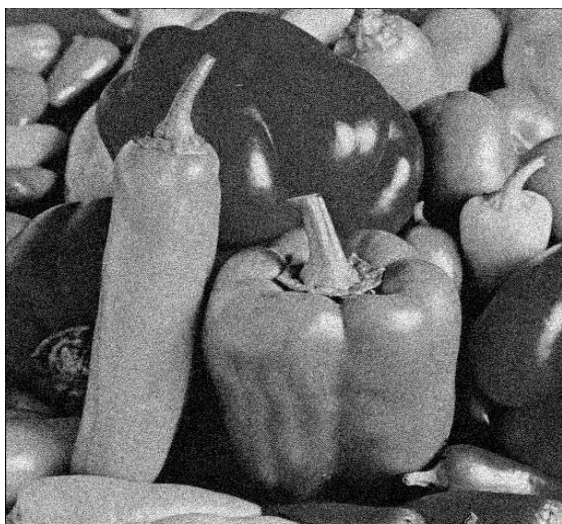
(:

(

-



()



()



()



()

Archives

[1] J. Canny, "A Computational approach to edge detection", IEEE Trans. Pattern Anal. Machine Intel., Vol. 8, pp. 639-643, Nov. 1986.

[2] D. Marr and E. Hildreth, "Theory of edge detection", Proc. Royal Soc., London, Vol. 207, pp. 187-217, 1980.

[3] R. M. Haralick, "Digital step edges from zero crossing of second directional derivative", IEEE Trans. Pattern Anal. Machine Intel., Vol. 6, pp. 58-68, 1984.

- [12] C.K. Chui, (Ed.), *Wavelet: A Tutorial in Theory and Application*, Academic Press, New York, 1992.
- [13] V. Torre and T. Poggio, "On edge detection", *IEEE Trans. Pattern Anal. Machine Intel.*, Vol. 8, pp. 147-163, 1986.
- [14] K. Takaya et al, "Multiresolution 2-dimensional edge analysis using wavelets", *IEEE Wescanex 93, Communications, Power, and Computing*, Saskatoon, May 1993.
- [15] R.M. Rangayyan et al, "Directional analysis of images with Gabor wavelets", *Proc. SIBGRAPI, XIII Brazilian Symp. Computer Graph. And Image Proc.*, pp. 170-177, 2000.
- [16] R.J. Ferrari et al, "Analysis of Asymmetry in Mammograms via Directional Filtering With Gabor Wavelets", *IEEE Trans. Medical Imaging*, Vol. 20, No. 9, pp. 953-964, Sep. 2001.
- [17] J. M. Niya and A. Aghagolzadeh, "Edge Detection Using Combined Isotropic and Directional Wavelet Transform", *Proc. NEU-CEE*, pp. 246-251, Nicosia, March 2004.
- [18] N. Otsu, "A Threshold Selection Method from Gray Level Histograms", *IEEE Trans. System, Man and Cybernetics*, Vol. 9, pp. 62-66, Jan. 1979.
- [4] R. Sundaram, "Algorithms for Adaptive Transform Edge Detection", *IEEE Trans. Signal Processing*, Vol. 47, No. 8, pp. 2313-2317, Aug. 1999.
- [5] S. Mallat and S. Zhong, "Characterization of signals from multiscale edges", *IEEE Trans. Pattern Anal. Machine Intel.*, Vol. 14, pp. 710-732, 1992.
- [6] S. Mallat and W. Hwang, "Singularity detection and processing with wavelets", *IEEE Trans. Inform Theory*, Vol. 38, pp. 617-637, Mar. 1992.
- [7] Y. Xu et al, "Wavelet transform domain filters: a spatially selective noise filtration technique", *IEEE Trans. Image Processing*, Vol. 3, pp. 747-758, 1994.
- [8] L. Zhang and P. Bao, "Edge detection by scale multiplication in wavelet domain", *Pattern Recognition Letters*, Vol. 23, No. 14, December 2002, pp. 1771-1784.
- [9] L. Zhang and P. Bao, "A wavelet-based edge detection method by scale multiplication", *Proc. IEEE Int. Conf. on Pattern Recognition*, pp. 501-504, Québec, Canada, Aug. 2002.
- [10] S. Mallat, "A theory for multi-resolution signal decomposition", *IEEE Trans. Pattern Anal. Machine Intell.*, Vol. 11, pp. 674-693, July 1989.
- [11] S. Mallat, *A Wavelet Tour of Signal Processing*. New York: Academic Press, 1998.

Archive of SID

Archive of SID