

CFL

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[] Braza [] Lin Wu .
 [] Fornberg [] Dennis Chang
 [] Franke .
 Re=40 [] Deng . QUICK
 [] Kiris Kwak .
 [] Kumar Mital .
 [] Oh .
 Nithiarasu Zienkiewicz . Roe
 [] Mirzaee . []
 [] Wu Hu .

³ Benchmark

⁴ Flux Difference Splitting

SIMPLE

[]

$$\frac{\partial}{\partial t} \iint_S \mathbf{U} dS + \oint_{\partial S} (\mathbf{F} dy - \mathbf{G} dx) = \oint (\mathbf{R} dy - \mathbf{S} dx) \quad (1)$$

$$\mathbf{U} = \begin{bmatrix} p \\ u \\ v \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \beta^2 u \\ u^2 + p \\ uv \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \beta^2 v \\ uv \\ v^2 + p \end{bmatrix} \quad (2)$$

$$\mathbf{R} = \frac{1}{\text{Re}} \begin{bmatrix} 0 \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix}, \quad \mathbf{S} = \frac{1}{\text{Re}} \begin{bmatrix} 0 \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \end{bmatrix} \quad (3)$$

⁵ Projection
⁶ Fractional Step
⁷ Chorin

$$\frac{1}{\beta^2} \frac{\partial p}{\partial t}$$

β

$l \quad \beta$

O

80×80, 110×110, 130×130

() 110×110

()

$$\oint_{ABCD} (\mathbf{F} dy - \mathbf{G} dx) \approx \sum_{k=1}^4 (\mathbf{F}_k \Delta y_k - \mathbf{G}_k \Delta x_k)$$

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ABCD

F, G

$p \quad v \quad u$

ϕ

: i,j

$$\phi_1 = \phi_{ij} + (x_1 - x_{ij}) \frac{\partial \phi}{\partial x} \Big|_{ij} + (y_1 - y_{ij}) \frac{\partial \phi}{\partial y} \Big|_{ij} + \dots \quad ()$$

()

:

$$\frac{\partial \phi}{\partial x} \Big|_{ij} = \frac{1}{S} \oint \phi dy = \frac{1}{S} \sum_{k=1}^4 (\phi \Delta y)_k \quad ()$$

$$\frac{\partial \phi}{\partial y} \Big|_{ij} = -\frac{1}{S} \oint \phi dx = -\frac{1}{S} \sum_{k=1}^4 (\phi \Delta x)_k$$

: () () . () ABCD S

$$\phi_1 = \phi_{ij} + \frac{1}{S} (x_1 - x_{ij}) [\phi_1 (\Delta y)_1 + \phi_2 (\Delta y)_2 + \phi_3 (\Delta y)_3 + \phi_4 (\Delta y)_4] \quad ()$$

$$- \frac{1}{S} (y_1 - y_{ij}) [\phi_1 (\Delta x)_1 + \phi_2 (\Delta x)_2 + \phi_3 (\Delta x)_3 + \phi_4 (\Delta x)_4]$$

:

$$A_1 \phi_1 + B_1 \phi_2 + C_1 \phi_3 + D_1 \phi_4 = \phi_{ij} \quad ()$$

:

$$A_1 = 1 - \frac{1}{S} (x_1 - x_{ij}) (\Delta y)_1 + \frac{1}{S} (y_1 - y_{ij}) (\Delta x)_1$$

$$B_1 = -\frac{1}{S} (x_1 - x_{ij}) (\Delta y)_2 + \frac{1}{S} (y_1 - y_{ij}) (\Delta x)_2 \quad ()$$

$$C_1 = -\frac{1}{S} (x_1 - x_{ij}) (\Delta y)_3 + \frac{1}{S} (y_1 - y_{ij}) (\Delta x)_3$$

$$D_1 = -\frac{1}{S} (x_1 - x_{ij}) (\Delta y)_4 + \frac{1}{S} (y_1 - y_{ij}) (\Delta x)_4$$

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$$\begin{cases} A_1 \phi_1 + B_1 \phi_2 + C_1 \phi_3 + D_1 \phi_4 = \phi_{ij} \\ A_2 \phi_1 + B_2 \phi_2 + C_2 \phi_3 + D_2 \phi_4 = \phi_{ij} \\ A_3 \phi_1 + B_3 \phi_2 + C_3 \phi_3 + D_3 \phi_4 = \phi_{ij} \\ A_4 \phi_1 + B_4 \phi_2 + C_4 \phi_3 + D_4 \phi_4 = \phi_{ij} \end{cases} \quad ()$$

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$\phi_1, \phi_2, \phi_3, \phi_4$

$$i+1,j \quad \phi_{i+1/2,j}^L \quad ij \quad \phi \quad \phi_{i+1/2,j}^R$$

$$: []$$

$$\left\{ \begin{array}{l} dq_n - \frac{1}{2\beta^2} \left[q_n - \sqrt{q_n^2 + \frac{4\beta^2}{\rho}} \right] dp = 0, \quad \text{along} \quad \frac{dx_n}{dt} = \frac{1}{2} \left(q_n + \sqrt{q_n^2 + \frac{4\beta^2}{\rho}} \right) \\ dq_n - \frac{1}{2\beta^2} \left[q_n + \sqrt{q_n^2 + \frac{4\beta^2}{\rho}} \right] dp = 0, \quad \text{along} \quad \frac{dx_n}{dt} = \frac{1}{2} \left(q_n - \sqrt{q_n^2 + \frac{4\beta^2}{\rho}} \right) \end{array} \right. \quad ()$$

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$$\left\{ \begin{array}{l} (q_n)_{i+1/2,j} - (q_n)_{i+1/2,j}^L - \frac{1}{2\beta^2} \left((q_n)_{i+1/2,j}^L - \sqrt{(q_n)_{i+1/2,j}^L{}^2 + \frac{4\beta^2}{\rho}} \right) (p_{i+1/2,j} - p_{i+1/2,j}^L) = 0 \\ (q_n)_{i+1/2,j} - (q_n)_{i+1/2,j}^R - \frac{1}{2\beta^2} \left((q_n)_{i+1/2,j}^R - \sqrt{(q_n)_{i+1/2,j}^R{}^2 + \frac{4\beta^2}{\rho}} \right) (p_{i+1/2,j} - p_{i+1/2,j}^R) = 0 \end{array} \right. \quad ()$$

$$\phi_{i+1/2,j}^R \quad \phi_{i+1/2,j}^L \quad ()$$

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$$\oint_{ABCD} (\mathbf{R} dy - \mathbf{S} dx) = \oint ([\frac{\partial \phi}{\partial x}] dy - [\frac{\partial \phi}{\partial y}] dx) \quad ()$$

$$= \sum_{k=1}^4 ([\frac{\partial \phi}{\partial x}]_k \Delta y_k - [\frac{\partial \phi}{\partial y}]_k \Delta x_k)$$

ϕ . \phi
 () : ANBM AB

$$[\frac{\partial \phi}{\partial x}]_{AB} = \frac{1}{S'} \iint_{S'} (\frac{\partial \phi}{\partial x}) dS = \frac{1}{S'} \oint_{\partial S'} \phi dy \approx \frac{1}{S'} \sum_{k=1}^4 \phi_k \Delta y_k \quad ()$$

$$= \frac{1}{S'} [0.5(\phi_N + \phi_A) \Delta y_{AN} + 0.5(\phi_N + \phi_B) \Delta y_{NB} + 0.5(\phi_B + \phi_M) \Delta y_{BM} + 0.5(\phi_M + \phi_A) \Delta y_{MA}]$$

: ANBM S'

$$y_N = 0.25(y_{i,j+1} + y_{i,j+2} + y_{i+1,j+2} + y_{i+1,j+1})$$

$$y_M = 0.25(y_{i,j} + y_{i,j+1} + y_{i+1,j+1} + y_{i+1,j}) \quad ()$$

$$\phi_A = 0.25(\phi_{NW} + \phi_N + \phi_M + \phi_W)$$

$$\phi_B = 0.25(\phi_N + \phi_{NE} + \phi_E + \phi_M)$$

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:

$$\mathbf{U}^{(0)} = \mathbf{U}^n$$

$$\mathbf{U}^{(1)} = \mathbf{U}^{(0)} - \frac{\Delta t}{2} \mathbf{Q}^{(0)}$$

$$\mathbf{U}^{(2)} = \mathbf{U}^{(0)} - \frac{\Delta t}{2} \mathbf{Q}^{(1)} \quad ()$$

$$\mathbf{U}^{(3)} = \mathbf{U}^{(0)} - \Delta t \mathbf{Q}^{(2)}$$

$$\mathbf{U}^{(4)} = \mathbf{U}^{(0)} - \frac{\Delta t}{6} (\mathbf{Q}^{(0)} + 2\mathbf{Q}^{(1)} + 2\mathbf{Q}^{(2)} + \mathbf{Q}^{(3)})$$

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[10] Nithiarasu Zienkiewicz

⁸ Normal Momentum

Re=40

von Karman

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CFL=1.7

CFL=1.2

¹ Vortex Shedding

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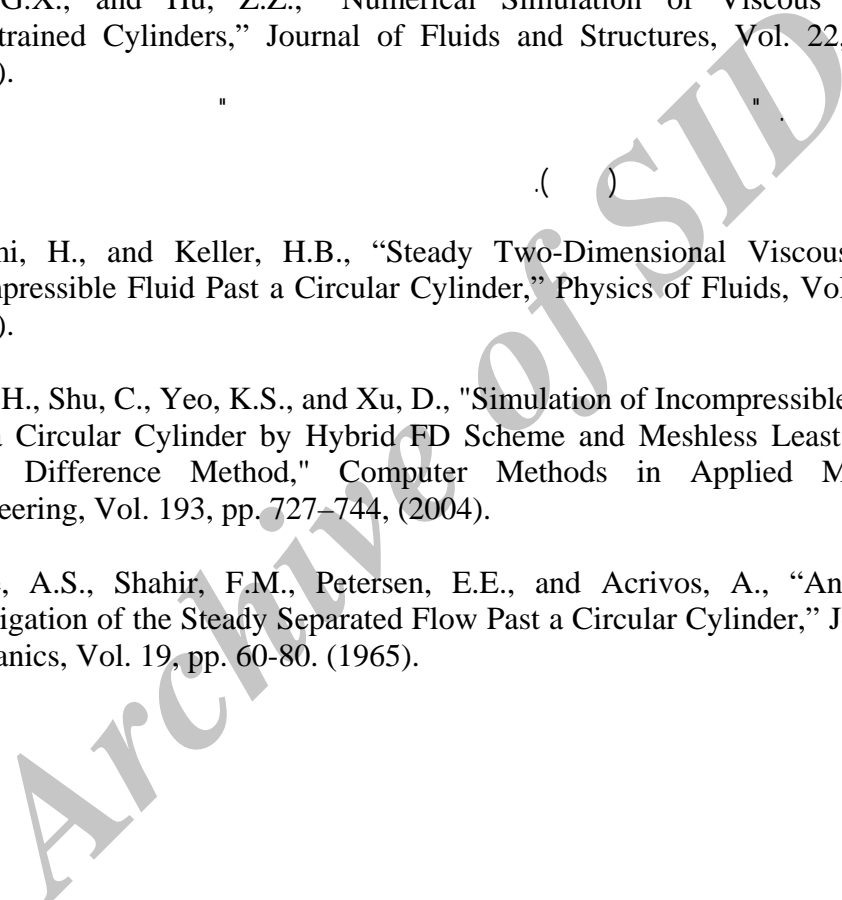
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:CFL
 : C_p
 x : F
 y : G
 : p
 : q_n
 x : R
 y : S
 : S'
 : u, v

:U

: x_n

: β

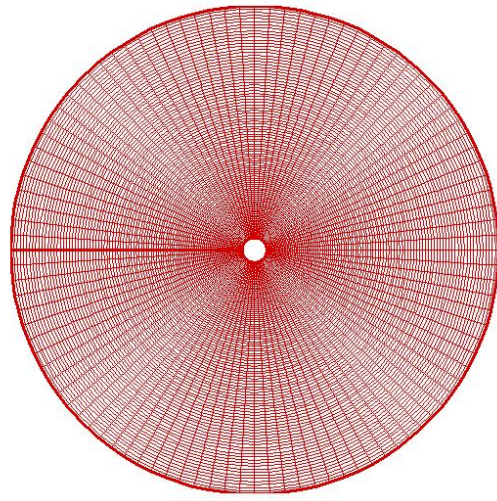
: Δt

: ϕ

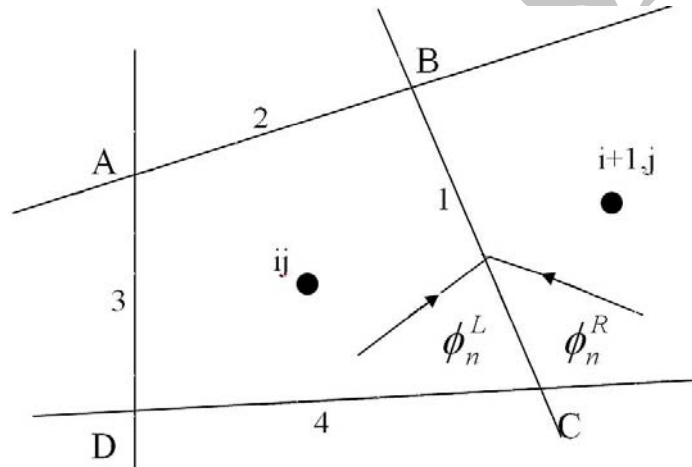
: ρ

Reynolds number	4	10	20	40	60	80	100	140	200
[14] Takami Keller	-	-	2	1.54	1.42		1.32	-	-
[1] Dennis Chang	-	2.8	2.05	1.52	1.37	1.25	1.06	-	-
[2] Fornberg	-	-	2	1.5	-	-	1.06	-	0.83
[4] Braza	-	-	2.18	1.61	-	1.35	1.15	1.08	-
[15] Ding	-	3.07	2.18	1.71	-	-	-	-	-
[10] Nithiarasu	-	2.85	2.06	1.56	-	-	-	-	-
	4.62	2.79	2.02	1.51	-	1.31	1.13	1.08	1.06

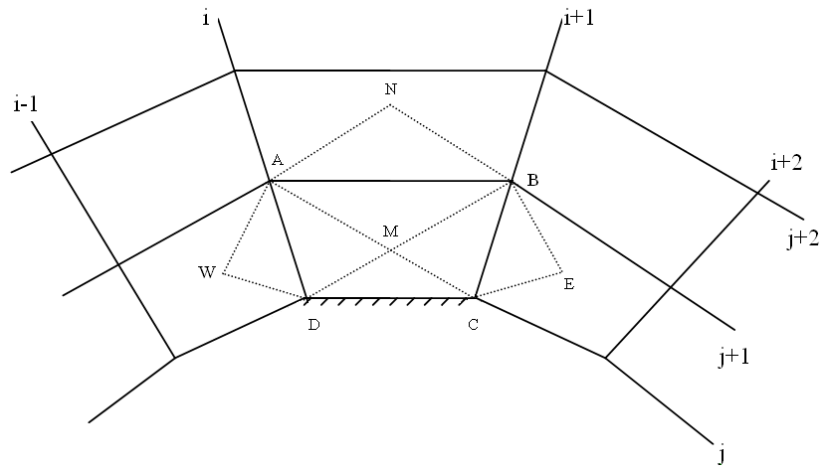
Reynolds Number	4	10	20	40	60	80	100
[14] Takami Keller	-	-	0.631	0.57	0.548	-	-
[1] Dennis Chang	-	-	0.634	0.572	-	-	0.53
[2] Fornberg	1	0.718	0.638	0.576	-	-	0.533
	0.9	0.628	0.57	0.541	0.533	0.531	0.529

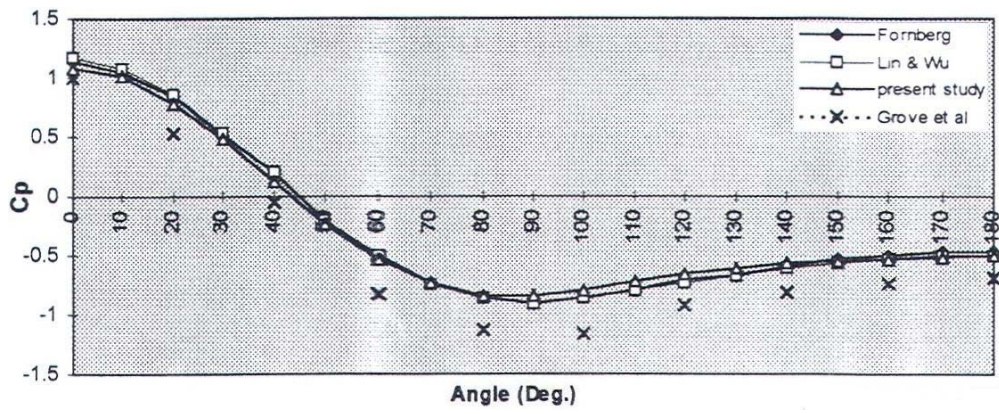


O

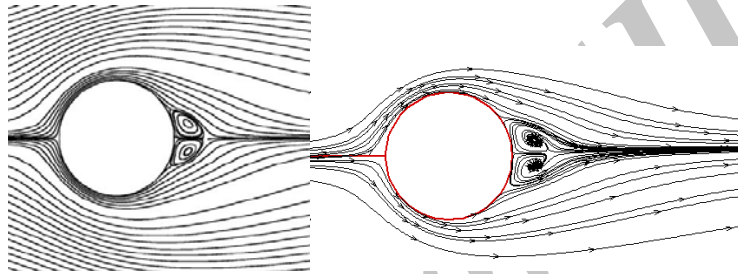


Arc

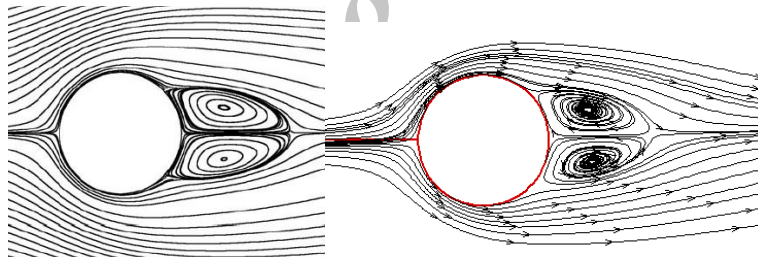




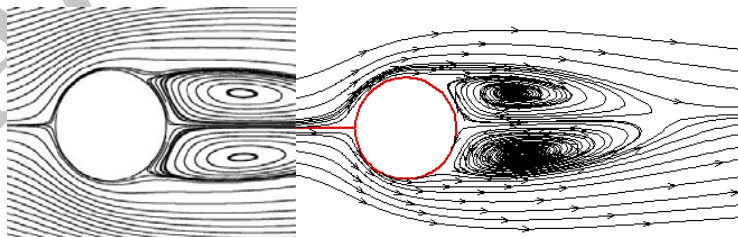
Re=40 [16 3 2]



Re=10



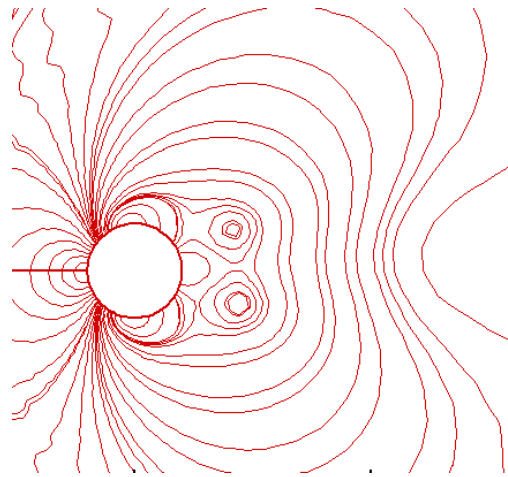
Re=20



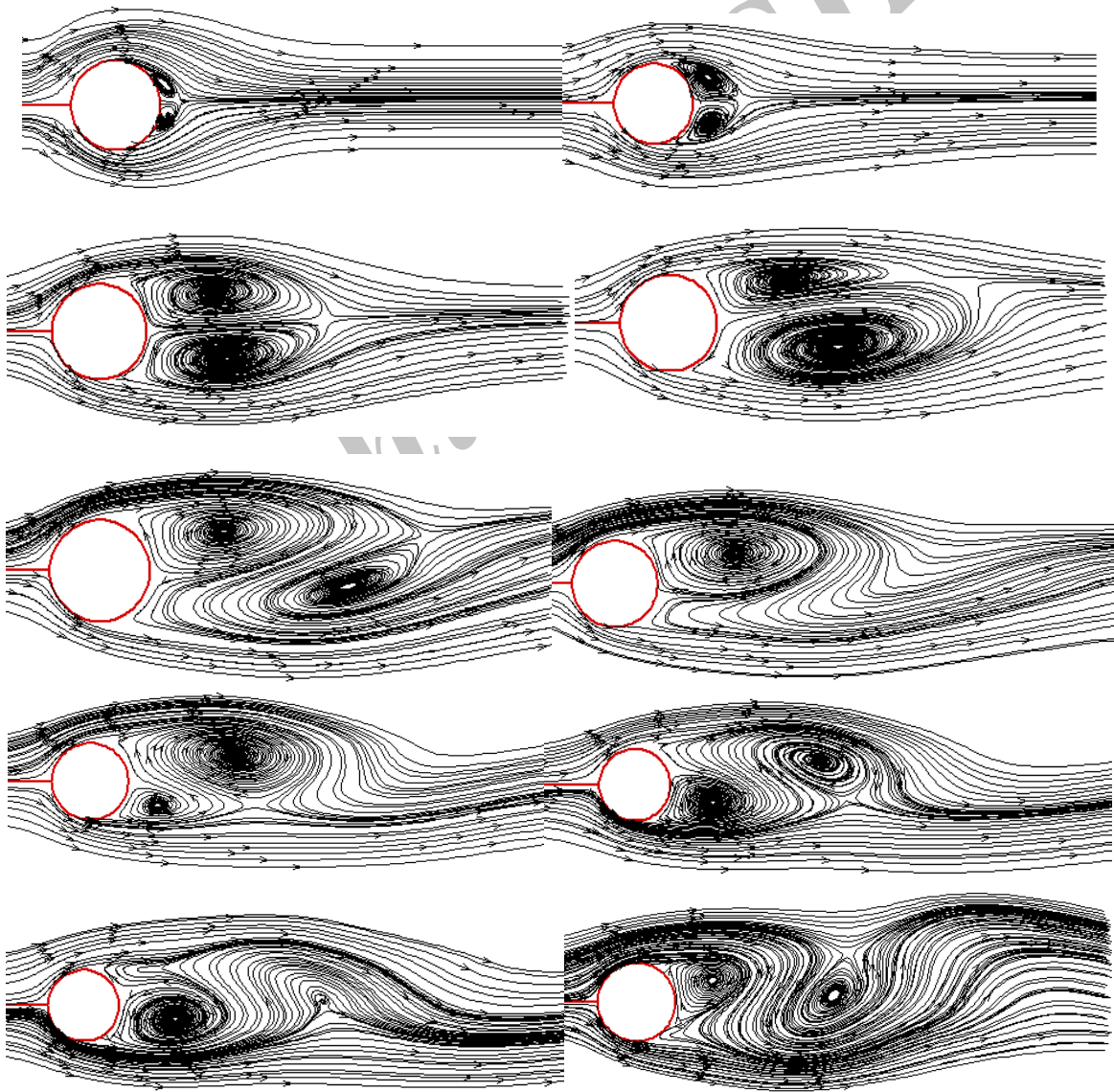
Re=40

Nithiarasu Zienkiewicz ()

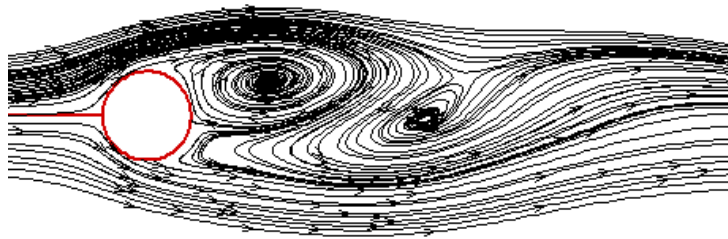
() [10]



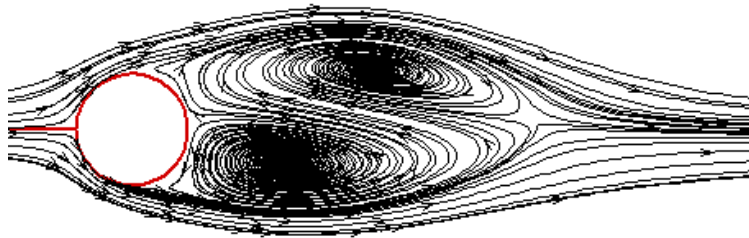
Re=40



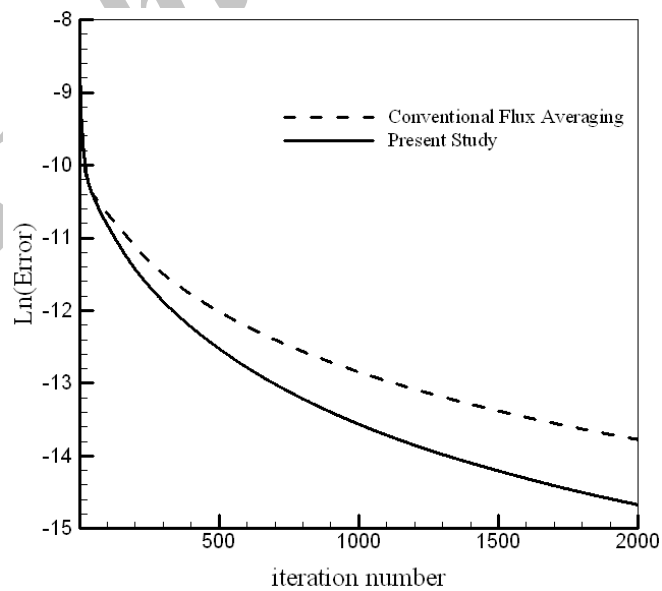
() Re=80 CFL=1.4



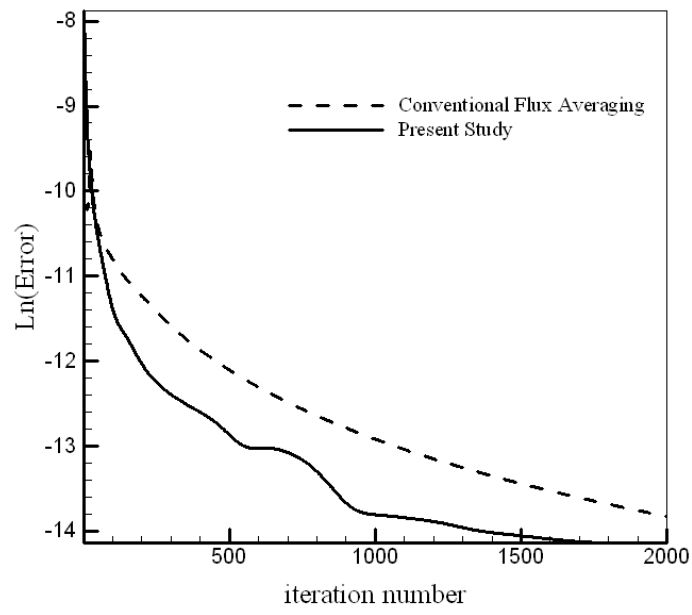
CFL=1.4 Re=100



CFL=1.4 Re=150



CFL=1.0 Re=20



CFL=1.0 Re=40

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Abstract

A new finite-volume method has been developed for solving the viscous incompressible flows. Artificial compressibility technique along with time marching is applied. The method consists of convective flux computation based on the two variable Taylor series expansions and characteristic of artificial compressibility equations. The flow primitive variables are determined in cell faces by solving a linear system of equations and then discrete forms of characteristic relations between two cells are used for calculation of convective fluxes. The proposed flux calculation method is employed for solving the cross flow over a circular cylinder in steady and unsteady regimes. For this test case, the proposed method can noticeably increase the numerical stability region by allowing the elevated CFL numbers. Also, the method was led to reduced iteration steps during the convergence process.

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