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Kelsey

Rendler []

[] Vigness

¹ Hole-Drilling Strain gage method

² Through hole

³ Blind Hole

[] Beaney Procter .

[] Schajer .

[] Flaman .

[] Schajer .

[] ASTM E837

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$$\begin{aligned}
 & \sigma_x \quad P(R, \alpha) \quad () \\
 & \sigma'_R = \frac{\sigma_x}{2} (1 + \cos 2\alpha) \quad () \\
 & \sigma'_\theta = \frac{\sigma_x}{2} (1 - \cos 2\alpha) \quad () \\
 & \tau'_{r\theta} = -\frac{\sigma_x}{2} (\sin 2\alpha) \quad () \\
 & () \quad \sigma_x \quad P(R, \alpha) \quad () \\
 & () \quad ()
 \end{aligned}$$

$$\varphi = \varphi(R, \theta) \quad (1)$$

$$\varphi = f_0(R) + f_2(R) \cos 2\theta \quad (2)$$

$$\nabla^4 \varphi = 0 \quad (3)$$

C_1

$$f_2 \quad f_0 \quad (4) \quad (5)$$

C_8

$$f_0(R) = C_1 R^2 \ln R + C_2 R^2 + C_3 \ln R + C_4 \quad (6)$$

$$f_2(R) = C_5 R^2 + C_6 R^4 + \frac{C_7}{R^2} + C_8 \quad (7)$$

$$(8) \quad (9)$$

$$\varphi = C_1 R^2 \ln R + C_2 R^2 + C_3 \ln R + C_4 + (C_5 R^2 + C_6 R^4 + \frac{C_7}{R^2} + C_8) \cos 2\theta \quad (10)$$

$$(11)$$

$$\sigma_R = \frac{1}{R} \frac{\partial \varphi}{\partial r} + \frac{1}{R^2} \frac{\partial^2 \varphi}{\partial \theta^2} \quad (12)$$

$$\sigma_\theta = \frac{\partial^2 \varphi}{\partial R^2} \quad (13)$$

:

$$\sigma_R = C_1(1 + 2 \ln R) + 2C_2 + \frac{C_3}{R^2} - \left(2C_5 + \frac{6C_7}{R^4} + \frac{4C_8}{R^2} \right) \cos 2\alpha \quad (14)$$

$$\sigma_\theta = C_1(3 + 2 \ln R) + 2C_2 + \frac{C_3}{R^2} - \left(2C_5 + 12C_6 R^2 + \frac{6C_7}{R^2} \right) \cos 2\alpha \quad (15)$$

σ_x

$P(R, \alpha)$

R

R_0

$$\sigma_r'' = \frac{\sigma_x}{2} \left(1 - \frac{1}{r^2} \right) + \frac{\sigma_x}{2} \left(1 + \frac{3}{r^4} - \frac{4}{r^2} \right) \cos 2\alpha \quad (16)$$

$$\sigma_\theta'' = \frac{\sigma_x}{2} \left(1 + \frac{1}{r^2} \right) - \frac{\sigma_x}{2} \left(1 + \frac{3}{r^4} \right) \cos 2\alpha \quad (17)$$

$$r = \frac{R}{R_0 = a} \quad (R \geq R_0)$$

σ_x

$$\Delta\sigma_r = \frac{\sigma_x}{2} \left[-\frac{1}{r^2} + \left(\frac{3}{r^4} - \frac{4}{r^2} \right) \cos 2\alpha \right] \quad ()$$

$$\Delta\sigma_\theta = \frac{\sigma_x}{2} \left[\frac{1}{r^2} - \frac{3}{r^4} \cos 2\alpha \right] \quad ()$$

() ()

 $P(R, \alpha)$

() ()

$$\varepsilon_r = -\frac{\sigma_x(1+\nu)}{2E} \left[\frac{1}{r^2} - \frac{3}{r^4} \cos 2\alpha + \frac{4}{r^2(1+\nu)} \cos 2\alpha \right] \quad ()$$

$$\varepsilon_\theta = -\frac{\sigma_x(1+\nu)}{2E} \left[-\frac{1}{r^2} + \frac{3}{r^4} \cos 2\alpha - \frac{4}{r^2(1+\nu)} \cos 2\alpha \right] \quad ()$$

:

$$\varepsilon_r = \sigma_x (A + B \cos 2\alpha) \quad ()$$

$$\varepsilon_\theta = \sigma_x (-A + C \cos 2\alpha) \quad ()$$

:

$$A = -\frac{1+\nu}{2E} \left(\frac{1}{r^2} \right) \quad ()$$

$$B = -\frac{1+\nu}{2E} \left[\left(\frac{4}{1+\nu} \right) \frac{1}{r^2} - \frac{3}{r^4} \right] \quad ()$$

$$C = -\frac{1+\nu}{2E} \left[-\left(\frac{4}{1+\nu} \right) \frac{1}{r^2} + \frac{3}{r^4} \right] \quad ()$$

 $\sigma_y \quad \sigma_x$

()

$$\varepsilon_r = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

 ε_r

()

$$\varepsilon_1 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

$$\varepsilon_2 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \sin 2\alpha \quad ()$$

$$\varepsilon_3 = A(\sigma_x + \sigma_y) - B(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

() ()

$$\sigma_{x,y} = \frac{\varepsilon_1 + \varepsilon_3}{4A} \pm \frac{\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4B} \quad ()$$

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_2 - \varepsilon_1} \quad ()$$

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$$() \quad \sigma_y \quad \sigma_x \quad ()$$

$$\varepsilon_r = \bar{A}(\sigma_x + \sigma_y) + \bar{B}(\sigma_x - \sigma_y) \cos 2\alpha \quad ()$$

$$\bar{A} = f_A(E, \nu, D_o, Z/D) \quad ()$$

$$\bar{B} = f_B(E, \nu, D_o, Z/D) \quad ()$$

Z D D_o

ν E

()

()

()

$$\sigma_{x,y} = \frac{\varepsilon_1 + \varepsilon_3}{4A} \pm \frac{\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4B} \quad ()$$

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_2 - \varepsilon_1} \quad ()$$

()

σ_c

()

()

$$\varepsilon_r^x(1) = \sigma_x (\bar{A} + \bar{B} \cos 2\alpha_{\alpha=0}) = \sigma_c (\bar{A} + \bar{B}) \quad ()$$

$$\varepsilon_r^x(3) = \sigma_x (\bar{A} + \bar{B} \cos 2\alpha_{\alpha=90}) = \sigma_c (\bar{A} - \bar{B}) \quad ()$$

()

()

$$\bar{A} = \frac{\varepsilon_r^x(1) + \varepsilon_r^x(3)}{2\sigma_c} \quad ()$$

$$\bar{B} = \frac{\varepsilon_r^x(1) - \varepsilon_r^x(3)}{2\sigma_c} \quad ()$$

$\varepsilon_r^x(3) \quad \varepsilon_r^x(1)$

() ()

$$\bar{A} = \frac{(\varepsilon_r^x(1)_{after} - \varepsilon_r^x(1)_{before}) + (\varepsilon_r^x(3)_{after} - \varepsilon_r^x(3)_{before})}{2\sigma_c} \quad ()$$

$$\bar{B} = \frac{(\varepsilon_r^x(1)_{after} - \varepsilon_r^x(1)_{before}) - (\varepsilon_r^x(3)_{after} - \varepsilon_r^x(3)_{before})}{2\sigma_c} \quad ()$$

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[] Manning Flaman

Schajer

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¹ Incremental Starin Method
² Average Stress Method
³ Power Series Method
⁴ Integral Method

()

z

() () () ()

()

()

h

z

$$\varepsilon_r(z) = A(\sigma_x(z) + \sigma_y(z)) + B(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) \quad ()$$

$$A = -\frac{1+\nu}{2E} \times a \quad a = a(z, h) \quad ()$$

$$B = -\frac{1}{2E} \times b \quad b = b(z, h) \quad ()$$

() () ()

$$\varepsilon_r(z) = \frac{1+\nu}{2E} \times a(z, h) \times (\sigma_x(z) + \sigma_y(z)) + \frac{1}{2E} \times b(z, h) \times (\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) \quad ()$$

z

h

$$\varepsilon_r(h) = \int_0^h \varepsilon_r(z) dz \quad ()$$

$$\varepsilon_r(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z) dz \quad ()$$

$$\alpha \quad \sigma_y(z) \quad \sigma_x(z)$$

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: ()

$$\varepsilon_1(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z)) \cos 2\alpha(z) dz \quad ()$$

$$\varepsilon_2(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z)) dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z)) \sin 2\alpha(z) dz \quad ()$$

$$\varepsilon_3(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{2E} \int_0^h b_h(z)(\sigma_x(z) - \sigma_y(z))\cos 2\alpha(z)dz \quad ()$$

$$\sigma_2 \sigma_1 \quad () \quad \sigma_y \sigma_x \quad \tau_{13}$$

$$\sigma_x(z) + \sigma_y(z) = \sigma_1(z) + \sigma_3(z) \quad ()$$

$$(\sigma_x(z) - \sigma_y(z))\cos 2\alpha = \sigma_1(z) - \sigma_3(z) \quad ()$$

:

$$\varepsilon_1(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_1(z) + \sigma_3(z))dz + \frac{1}{2E} \int_0^h b_h(z)(\sigma_1(z) - \sigma_3(z))dz \quad ()$$

$$\varepsilon_2(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{E} \int_0^h b_h(z)\tau_{13} dz \quad ()$$

$$\varepsilon_3(h) = \frac{1+\nu}{2E} \int_0^h a_h(z)(\sigma_x(z) + \sigma_y(z))dz - \frac{1}{2E} \int_0^h b_h(z)(\sigma_1(z) - \sigma_3(z))dz \quad ()$$

: ()

$$\varepsilon_3(h) + \varepsilon_1(h) = \frac{1+\nu}{E} \int_0^h a_h(z)(\sigma_3(z) + \sigma_1(z))dz \quad ()$$

$$\varepsilon_3(h) - \varepsilon_1(h) = \frac{1}{E} \int_0^h b_h(z)(\sigma_3(z) - \sigma_1(z))dz \quad ()$$

$$\varepsilon_3(h) + \varepsilon_1(h) - 2\varepsilon_2(h) = \frac{2}{E} \int_0^h b_h(z)\tau_{13} dz \quad ()$$

)

: (n h

$$\varepsilon_1(h) + \varepsilon_3(h) = \frac{1+\nu}{E} \sum_0^n a_h(j)(\sigma_x(j) + \sigma_y(j)) \quad ()$$

$$\varepsilon_1(h) - \varepsilon_3(h) = \frac{1}{E} \sum_0^n b_h(j)(\sigma_x(j) - \sigma_y(j)) \quad ()$$

$$\varepsilon_3(h) + \varepsilon_1(h) - 2\varepsilon_2(h) = \frac{2}{E} \sum_0^n b_h(j)\tau_{13}(j) \quad ()$$

:

$$p = \frac{\varepsilon_3 + \varepsilon_1}{2}, q = \frac{\varepsilon_3 - \varepsilon_1}{2}, t = \frac{\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2}{2} \quad ()$$

$$P = \frac{\sigma_3 + \sigma_1}{2}, Q = \frac{\sigma_3 - \sigma_1}{2}, T = \tau_{13} \quad ()$$

$$Q \quad q \quad P \quad p$$

() ()

i z

: ()

$$p_i = \frac{1+\nu}{E} \sum_0^n a_{ij} P_j = \frac{1+\nu}{E} (a_{i1}P_1 + a_{i2}P_2 + a_{i3}P_3 + a_{i4}P_4 + a_{i5}P_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

$$q_i = \frac{1}{E} \sum_0^n b_{ij} Q_j = \frac{1}{E} (b_{i1}Q_1 + b_{i2}Q_2 + b_{i3}Q_3 + b_{i4}Q_4 + b_{i5}Q_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

$$t_i = \frac{1}{E} \sum_0^n b_{ij} T_j = \frac{1}{E} (b_{i1}T_1 + b_{i2}T_2 + b_{i3}T_3 + b_{i4}T_4 + b_{i5}T_5 + \dots) \quad 1 \leq j \leq i \leq n \quad ()$$

h . h n

: $n=4$. Q_i P_i

$$p_i = \frac{1+\nu}{E} \sum_0^n a_{ij} P_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

$$q_i = \frac{1}{E} \sum_0^n b_{ij} Q_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

$$t_i = \frac{1}{E} \sum_0^n b_{ij} T_j \quad 1 \leq j \leq i \leq 4 \quad ()$$

:

$$[\bar{a}][P] = \frac{E}{1+\nu} [p] \quad ()$$

$$[\bar{b}][Q] = E [q] \quad ()$$

$$[\bar{b}][T] = E [t] \quad ()$$

τ_{13} σ_2 σ_1

$$\sigma_{j \max}, \sigma_{j \min} = P_j \pm \sqrt{Q_j^2 + T_j^2} = E \left[\frac{p_i}{\bar{a}_{i,j}(1+\nu)} + \frac{\sqrt{q_i^2 + t_i^2}}{\bar{b}_{i,j}} \right] \quad ()$$

$$\beta_j = \frac{1}{2} \tan^{-1} \left(\frac{T_j}{Q_j} \right) = \frac{1}{2} \tan^{-1} \left(\frac{t_j}{q_j} \right) \quad ()$$

z h . () () ()

b_{ij} a_{ij}

a_{ij} $()$ $: h$

$$\sigma_x(z) = \sigma_y(z) = \sigma(z) \quad ()$$

$$\sigma_x(z) - \sigma_y(z) = 0 \quad ()$$

$$\varepsilon_r(h) = \frac{1+\nu}{E} \int_0^h a_h(z) \sigma(z) dz \quad ()$$

 $)$ $\varepsilon_{ir}(h)$ $(n \quad h$ a_{ij} i

$$\varepsilon_{ir}(h) = \frac{1+\nu}{E} \sum_0^n a_{ij} \sigma_j = \frac{1+\nu}{E} (a_{i1} \sigma_1 + a_{i2} \sigma_2 + a_{i3} \sigma_3 + \dots) \quad ()$$

 a_{ij} $:$

$$\sigma_{j \neq m} = 0 \quad ()$$

 $() \quad ()$ σ_j

$$a_{ij} = \frac{E}{1+\nu} \frac{\varepsilon_{ir}}{\sigma_{ij}} \quad 1 \leq j \leq i \quad ()$$

 $()$ b_{ij} $: h$

$$\sigma_x(z) = -\sigma_y(z) = \sigma(z) \quad ()$$

$$\sigma_x(z) + \sigma_y(z) = 0 \quad ()$$

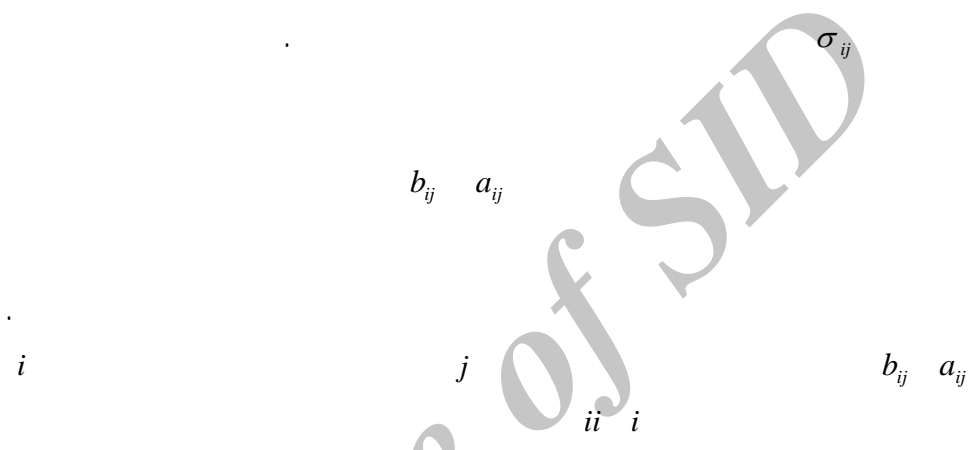
$$\varepsilon_r(h) = \frac{1}{E} \int_0^h b_h(z) \sigma(z) \cos 2\alpha(z) dz \quad ()$$

h $\alpha = 0$ b_{ij} $()$ $(n$

$$\varepsilon_{ir}(h) = \frac{1}{E} \sum_{j=1}^n b_{ij} \sigma_j = \frac{1}{E} (b_{i1} \sigma_1 + b_{i2} \sigma_2 + b_{i3} \sigma_3 + \dots) \quad ()$$

$\alpha = 0$ $\varepsilon_{ir}(h)$ $()$

$$b_{ij} = E \frac{\varepsilon_{ir}}{\sigma_{ij}} \quad 1 \leq j \leq i \quad ()$$



$()$ $\sigma_x(z) = \sigma_y(z) = P$ a_{ij}

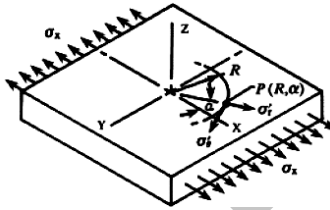
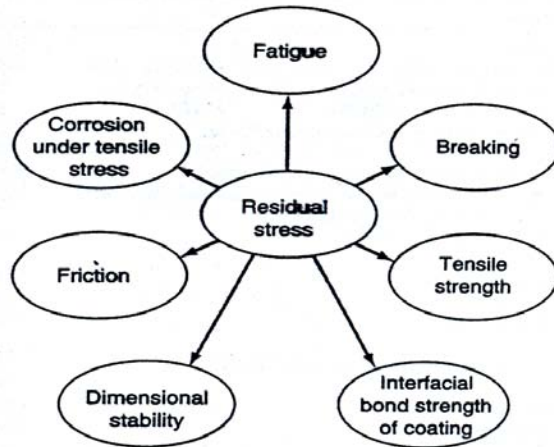
$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = P \quad ()$$

- [1] *Hand book of Residual Stresses and Distortion*, ASM International, Materials Park, Ohio, USA, (2002).
- [2] Masubuchi, K., *Analysis of Welded Structures*, Pergamon Press, New York, USA, (1980).
- [3] *Welding Handbook: Welding Technology*, Vol. 1, American Welding Society, Ohio, USA, (2002).
- [4] Kandil, F.A., Lord, J.D., Fry, A.T., and Grant, P.V., "Measurement of Residual Stress in Components, A Review of Residual Stress Measurement Methods," NPL Report, UK, (2001).
- [5] Measurement of Residual Stresses by the Hole-Drilling Strain Gage Method, Technical Note, TN-503, Vishay Measurements Group, Carolina, USA, (1993).
- [6] Kelsey, R. A., "Measurement of Residual Stresses by Hole-Drilling Method," Proc., SESA XIV, 1, pp. 181-194, (1956).
- [7] Rendler, N.J., and Vigness, I., "Hole-Drilling Strain-Gage Method of Measuring Residual Stresses," Experimental Mechanics, Vol. 6, pp. 577–586, (1966).
- [8] Beaney, E.M., and Procter, E., "A Critical Evaluation of the Center Hole Drilling Technique for Measurement of Residual Stresses," Strain, Vol. 10, pp. 7-14, (1974).
- [9] Schajer, G.S., "Application of Finite Element Calculations to Residual Stress Measurements," Transactions of ASME, Journal of Engineering Materials and Technology, Vol. 103, pp. 157–163, (1981).
- [10] Flaman, M.T., "Brief Investigation of Induced Drilling Stresses in the Centre-Hole Method of Residual Stress Measurement," Experimental Mechanics, Vol. 22, pp. 26–30, (1982).
- [11] Schajer, G.S., "Judgment of Residual Stress Field Uniformity when using the Hole-Drilling Strain Gage Method," Proceeding of ICRS II, France, pp. 71-77, (1988).
- [12] ASTM E 837 "Standard Test Method for Determining Residual Stresses by the Hole-Drilling Strain Gage Method," ASTM Standards, Section 3, Vol. 03.01, Ohio, USA, (1986).
- [13] *Handbook of Measurement of Residual Stress*, Society for Experimental Mechanics, 1th Edition, pp. 2-7, SEM Press, Ohio, USA, (1996).
- [14] Flaman, M.T., and Manning, B.H., " Determination of Residual Stress Variation with Depth by Hole Drilling Method," Experimental Mechanic, Vol. 25, pp. 205-207, (1985).
- [15] Schajer, G.S., "Measurement of Non-Uniform Residual Stresses using the Hole Drilling Method," Journal of Strain Analysis, Vol. 28, pp. 19–22, (1988).
- [16] Schajer, G.S., "Non-Uniform Residual Stresses Measurement by the Hole Drilling Method: Part I - Stress Calculation Procedures," Transactions of ASME, Journal of Engineering Materials and Technology, Vol. 110, pp. 338–343, (1988).
- [17] Schajer, G.S., "Measurement of Non-Uniform Residual Stresses using the Hole Drilling Method: Part II - Practical Application of the Integral Method," Transactions of ASME, Journal of Engineering Materials and Technology, Vol. 110, pp. 344–349, (1992).

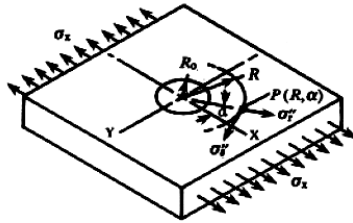
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:C B A
: \bar{B} \bar{A}
:b a
: b_{ij} a_{ij}
: C_{1-9}
:D
: R_0 D_0
:E
:h
:P p
:Q q
:R
: r
:Z

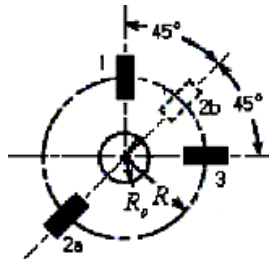
: α
: σ_Y σ_X
: σ'_θ σ'_r
: σ''_θ σ''_r
: ε_r
: φ
:v

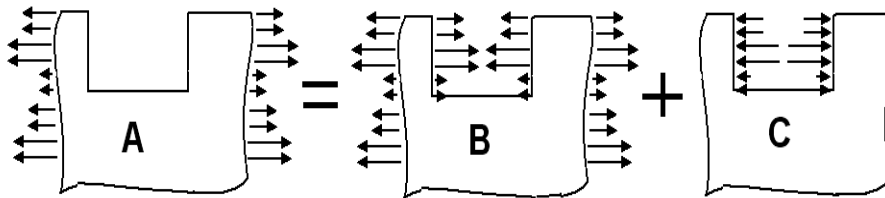
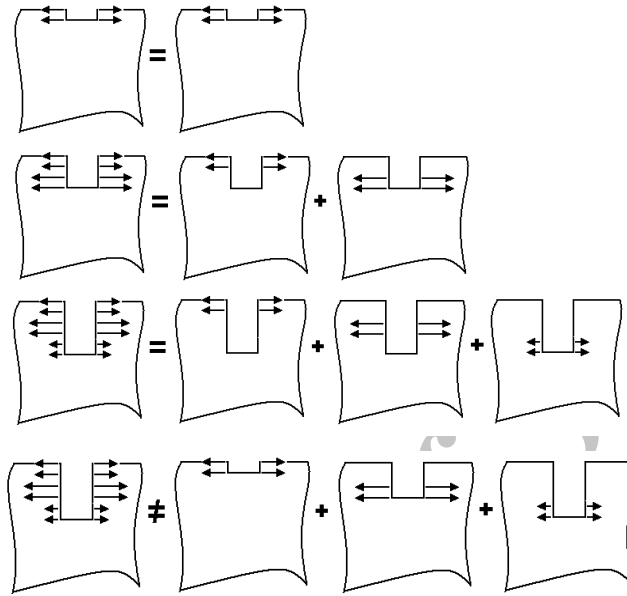
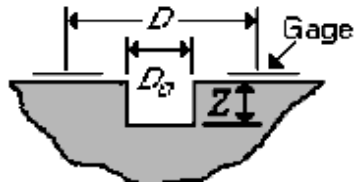


$P(R, \alpha)$

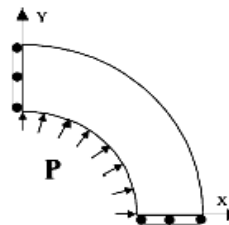
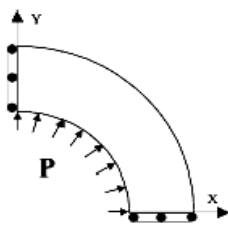


$P(R, \alpha)$





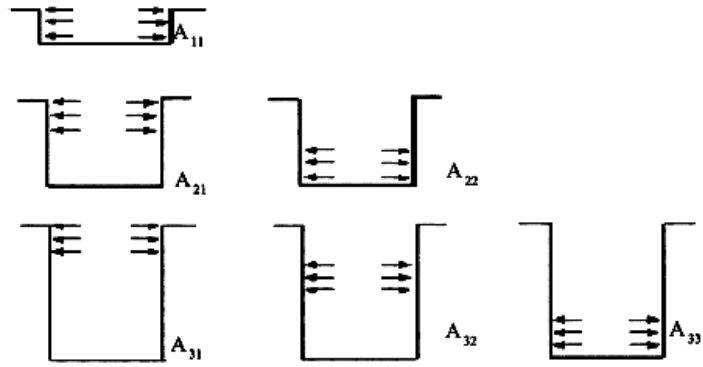
A



$$a_{ij} = \frac{E}{1+\nu} \frac{\epsilon_{ir}^i}{P_{ij}} \quad 1 \leq j \leq i$$

$$b_{ij} = E \frac{\epsilon_{ir}}{P_{ij}} \quad 1 \leq j \leq i$$

$$b_{ij} \quad a_{ij}$$



a_{ij}

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Abstract

Non-uniform residual stresses arise from most mechanical or thermal operations, performed in processing engineering materials, like welding and machining. They may enhance occurrence of brittle fracture, fatigue, structural buckling and stress-cracking-corrosion. Therefore, estimation of their magnitude and distribution are of great importance in integrity assessments of load bearing structures. The hole-drilling strain gauge method (HDSG), described in ASTM E837, is the common method used in evaluation of through thickness uniform residual stresses. However, according to this standard, the use of this method is limited for uniform stress distribution through the thickness of the body. In this paper, the capability of the HDSG method in evaluation of non-uniform stresses is studied by using the integral technique. Here, assuming linear elastic materials, the basic relations of the method are derived. Also, a procedure for determination of the required coefficients is proposed, using by the integral method, were presented.

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