



Reproducing Kernel Particle Method <sup>£</sup>

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3 - Moving Least Squares

 <sup>1 -</sup> Smoothed Particle Hydrodynamics
 2 - Element Free Galerkin Method

RKPM

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$$u^{h}(\mathbf{X}) = \int_{\Omega_{\mathbf{Y}}} K(\mathbf{X}, \mathbf{Y}) u(\mathbf{Y}) d\Omega_{\mathbf{Y}} = \int_{\Omega_{\mathbf{Y}}} C(\mathbf{X}, \mathbf{X} - \mathbf{Y}) w(\mathbf{X} - \mathbf{Y}) u(\mathbf{Y}) d\Omega_{\mathbf{Y}}$$
()

$$C(\mathbf{X}, \mathbf{Y}) = \mathbf{P}^{\mathsf{T}}(\mathbf{Y} - \mathbf{X})\mathbf{a}(\mathbf{X})$$

$$\xrightarrow{d=1}{} C(x, y) = a_{0}(x) + a_{1}(x)(y - x) + a_{2}(x)(y - x)^{2} + \dots + a_{n}(x)(y - x)^{n} \qquad ()$$

$$d = 1 \qquad \mathbf{P} \qquad ()$$

$$u(\mathbf{Y}) = \sum_{|\mathbf{a}|=0}^{\infty} \frac{(\mathbf{Y} - \mathbf{X})^{a}}{|\mathbf{a}|!} D^{a}u(\mathbf{X}), \qquad ()$$

$$\xrightarrow{d=1}{} u(y) = u(x) + u'(x)(y - x) + \frac{1}{2!} u''(x)(y - x)^{2} + \dots + \frac{1}{n!} u^{(n)}(x)(y - x)^{n} + \dots$$

$$\vdots \qquad ()$$

$$u^{h}(\mathbf{X}) = \int_{\Omega_{Y}} \left[ \mathbf{P}^{T}(\mathbf{Y} - \mathbf{X})\mathbf{a}(\mathbf{X})w(\mathbf{X} - \mathbf{Y})\sum_{|\mathbf{a}|=0}^{\infty} \frac{(\mathbf{Y} - \mathbf{X})^{a}}{|\mathbf{a}|!} D^{a}u(\mathbf{X}) \right] d\Omega_{Y}. \qquad ()$$

$$\int_{\Omega_{Y}} w(\mathbf{X} - \mathbf{Y})\mathbf{P}(\mathbf{Y} - \mathbf{X})\mathbf{P}^{T}(\mathbf{Y} - \mathbf{X})\mathbf{a}(\mathbf{X}) d\Omega_{Y} = \begin{cases} 1\\ 0\\ 0\\ 0 \end{cases} = \mathbf{P}(0). \qquad ()$$

$$() \qquad \mathbf{A}(\mathbf{X}) = () \qquad \mathbf{A}(\mathbf{X}) = () \qquad \mathbf{A}(\mathbf{X}) = ()$$

$$u^{h}(X) = \boldsymbol{P}^{T}(X) \left[ \int_{\Omega_{Y}} w(X - Y) \boldsymbol{P}(Y) \boldsymbol{P}^{T}(Y) d\Omega_{Y} \right]^{-1} \int_{\Omega_{Y}} w(X - Y) \boldsymbol{P}(Y) u(Y) d\Omega_{Y} \quad ()$$

$$u^{h}(X) = \int_{\Omega_{Y}} C(X, Y) w(X - Y) u(Y) d\Omega_{Y}$$
  

$$= \sum_{i=1}^{N} C(X, X_{i}) w(X - X_{i}) u_{i} \Delta V_{i}$$
  

$$= \mathbf{P}^{T}(X) [\mathbf{M}(X)]^{-1} \sum_{i=1}^{N} \mathbf{P}(X_{i}) w(X - X_{i}) u_{i} \Delta V_{i}.$$
  
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$$M = \sum_{i=1}^{N} w(X - X_i) P(X_i) P^T(X_i) \Delta V_i \qquad ()$$

$$RKPM$$

$$MLS \qquad MLS \qquad \Delta V_i = 1$$

$$\Delta V_i = 1 \qquad RKM$$

$$\Delta V_i = 1 \qquad RKM$$

$$\Delta V_i = 1 \qquad RKM$$

$$\Delta V_i = 1 \qquad AV_i \neq 1$$

$$AV_i \neq 1 \qquad AV_i = c, c \in R$$

$$(MLS RKPM)$$

$$\vdots$$

$$u^h(\mathbf{X}) = \sum_{i=1}^{N} \Phi_i(\mathbf{X}) u_i \qquad ()$$

$$N \qquad u_i \qquad \Phi_i$$

$$() () () ()$$

$$\Phi_i(\mathbf{X}) = P^T(\mathbf{X}) [M(\mathbf{X})]^{-1} w(\mathbf{X} - \mathbf{X}_i) P(\mathbf{X}_i) \qquad ()$$



1 -Cubic Bspline

$$u^{h}(x_{i}) = \sum_{j=1}^{N} \Phi_{j}(x_{i})u_{j} \qquad ( )$$

$$\vdots$$

$$\hat{u}_{i} = [\Phi_{1}(x_{j}), \Phi_{2}(x_{i}), \dots, \Phi_{N}(x_{j})] \begin{bmatrix} u_{1} \\ u_{2} \\ \vdots \\ u_{N} \end{bmatrix} \qquad ( )$$

$$i \qquad u^{h} T \qquad i = 1, 2, \dots, N$$

$$\begin{bmatrix} \hat{u}_{i} \\ \hat{u}_{j} \\ \vdots \\ \hat{u}_{N} \end{bmatrix} = \begin{bmatrix} \Phi_{1}(x_{i}) & \Phi_{2}(x_{i}) & \cdots & \Phi_{N}(x_{N}) \\ \Phi_{1}(x_{j}) & \Phi_{2}(x_{j}) & \cdots & \Phi_{N}(x_{N}) \\ \vdots \\ \vdots \\ \hat{u}_{N} \end{bmatrix} \begin{bmatrix} u_{i} \\ u_{i} \\ \vdots \\ u_{N} \end{bmatrix} \qquad ( )$$

$$\hat{u}_{i} = T u \longrightarrow u = T^{-1} \hat{u} \qquad ( )$$

$$\hat{u}^{h}(X) = \sum_{i=1}^{N} \hat{\Phi}_{i}(X) \hat{u}_{i} \qquad ( )$$

$$\hat{u}^{h}(X) = \sum_{i=1}^{N} \hat{\Phi}_{i}(X) \hat{u}_{i} \qquad ( )$$

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 $d_{\text{max}} = 2$ 

$$u(x) = (1 - x)[\tan^{-1}(\alpha(x - x_0)) + \tan^{-1}(\alpha x_0)]$$

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 $\rho$ 

$$\alpha = 25, x_0 = .25 \quad \rho = 2 \cdot \Delta x \qquad P = [1, x]$$

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$$u$$

$$RKPM$$

$$\begin{cases}
-\nabla^{2}u = f(x, y) = [110x^{9} - \pi^{2}(1 - x^{11})]\cosh(\pi y) & in \quad \Omega = [-1,1] \times [-1,1] \\
-\frac{\partial u}{\partial n} = g(x) = -\pi \sinh(\pi)(1 - x^{11}) & on \quad \Gamma_{N} \\
-\frac{\partial u}{\partial n} = (u - u_{x}) = (u - 13\cosh(\pi y)) & on \quad \Gamma_{C} \\
u = 0 & on \quad \Gamma_{D} \\
f(x, y) & g(x) & u_{x} \\
u = (1 - x^{11})\cosh(\pi y) & u_{x} \\
u = (1 - x^{11})\cosh(\pi y) & u_{x} \\
15 \times 15 \\
n_{q} \times n_{q} = 3 \times 3 \quad 12 \times 12 \quad () \\
= 1.5 \cdot \Delta x \\
(y \times \Delta x) \\
RKPM \\
\vdots \\
n_{x} \times N_{y} = 40 \times 40 \quad :
\end{cases}$$



$$\nabla .\boldsymbol{\sigma} + \boldsymbol{b} = 0 \quad in \quad \Omega \\ \boldsymbol{\sigma} .\boldsymbol{n} = \bar{\boldsymbol{t}} \quad on \quad \Gamma_t \\ \boldsymbol{u} = \bar{\boldsymbol{u}} \quad on \quad \Gamma_u \end{cases}$$

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$$\int_{\Omega} \delta(\nabla_{s} v^{T}) : \sigma d\Omega - \int_{\Omega} \delta v^{T} \cdot b d\Omega - \int_{\Gamma_{t}} \delta v^{T} \cdot \bar{t} d\Gamma = 0$$

$$\nabla v^{T} . ( ) \nabla_{s} v^{T} . \delta v(x) u(x)$$

$$K . u = f$$

$$\vdots$$

$$K . u = f$$

$$f_{t} = \int_{\Omega} B_{t}^{T} DB_{t} d\Omega$$

$$f_{t} = \int_{\Gamma_{t}} \Phi_{t} \bar{t} d\Gamma + \int_{\Omega} \Phi_{t} b d\Omega$$

$$\vdots$$

$$B_{t} = \begin{bmatrix} \Phi_{T,x} & 0 \\ 0 & \Phi_{t,y} \\ \Phi_{t,y} & \Phi_{t,x} \end{bmatrix}$$

$$D$$

$$\vdots$$

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RKPM  $N_y \times N_x = 9 \times 33$  :  $n_q \times n_q = 4 \times 4$   $8 \times 32$  :  $\mathbf{P}^{\mathsf{T}}(\mathbf{X}) = \mathbf{P}^{\mathsf{T}}(x, y) = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3].$ 



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## Abstract

In this paper RKPM method is used for simulation of one and two dimensional linear boundary value problems. Due to the loss of kronecker delta properties in the mesh less shape functions, the imposition of essential boundary conditions is the main problem in mesh free computations. In this work transformation method is used for imposition of essential boundary conditions. Several linear boundary value problems with various type of boundary conditions are simulated and Results obtained from these simulations are compared with exact solutions.

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