



mohyeddin.ali@gmail.com

³ Meshless Local Petrov Galerkin
 ⁴ Functionally Graded Material
 ⁵ Moving Least Squares Approximation



¹ Koizumi ² Meshfree method ³ Atluri and Zhu

www.SID.ir

.



¹ Field variable

² Influence domain

т Р

Ι

• Ω_{Q}

$$\mathbf{V}_{I} = \begin{bmatrix} W_{I,x} & 0 \\ 0 & W_{I,y} \\ W_{I,y} & W_{I,x} \end{bmatrix}$$
()

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} = \mathbf{c} \begin{cases} \frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{cases} \sum_{j}^{N} \boldsymbol{\Phi}_{j} \mathbf{u}_{j} = \mathbf{c} \sum_{j}^{N} \mathbf{B}_{j} \mathbf{u}_{j} \qquad ()$$

•

Ι

$$\boldsymbol{\varepsilon} = \left\{ \varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy} \right\}^{\mathrm{T}}$$

$$\boldsymbol{\varepsilon} = \left\{ \varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy} \right\}^{\mathrm{T}}$$

$$\boldsymbol{\varepsilon} = \frac{E^{*}}{1 - v^{*2}} \begin{bmatrix} 1 & v^{*} & 0 \\ v^{*} & 1 & 0 \\ 0 & 0 & (1 - v^{*})/2 \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \frac{E^{*}}{1 - v^{*2}} \begin{bmatrix} 1 & v^{*} & 0 \\ v^{*} & 1 & 0 \\ 0 & 0 & (1 - v^{*})/2 \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \frac{E^{*}}{1 - v^{*2}} \begin{bmatrix} 1 & v^{*} & 0 \\ v^{*} & 1 & 0 \\ 0 & 0 & (1 - v^{*})/2 \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \frac{E^{*}}{1 - v^{*2}} \begin{bmatrix} 1 & v^{*} & 0 \\ v^{*} & 1 & 0 \\ 0 & 0 & (1 - v^{*})/2 \end{bmatrix}$$

$$E^{*} = E, \quad v^{*} = v$$

$$E^{*} = \frac{E}{1 - v^{2}}, \quad v^{*} = \frac{v}{1 - v}$$
()

n

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$
()

:

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}$$

[] () $w_1(\mathbf{x}) = \sqrt{r} \cos\left(\frac{\theta}{2}\right) w_I(\mathbf{x})$ $w_{2}(\mathbf{x}) = \sqrt{r} \left[1 + \sin\left(\frac{\theta}{2}\right) \right] w_{I}(\mathbf{x})$ () $w_{3}(\mathbf{x}) = \sqrt{r} \left[1 - \sin\left(\frac{\theta}{2}\right) \right] w_{I}(\mathbf{x})$ $w_{I}(\mathbf{x})$ θ r. $(\theta \in [-\pi,\pi])$ $\theta = \pm \pi W_3$ W_2 W_3 () () (\mathbf{x}^n) (e^x) $E = E_1 \exp(\beta V_2), \qquad \beta = \ln\left(\frac{E_2}{E_1}\right)$ () $E_2 \quad E_1$ V_{2}

MLPG

¹ Duflot



:

...

[]

.

$$\frac{\kappa - \kappa_{1}}{\kappa_{2} - \kappa_{1}} = \frac{V_{2}}{1 + (1 - V_{2})(\kappa_{2} - \kappa_{1})/(\kappa_{1} + 4\mu_{1}/3)}$$

$$\frac{\mu - \mu_{1}}{\mu_{2} - \mu_{1}} = \frac{V_{2}}{1 + (1 - V_{2})\frac{\mu_{2} - \mu_{1}}{\mu_{1} + f_{1}}}, \quad f_{1} = \mu_{1}(9\kappa_{1} + 8\mu_{1})/6(\kappa_{1} + 2\mu_{1}) \quad ()$$

.

()

$$V_{2} = \left(0.5 + \frac{x_{2}}{h}\right)^{n}, \quad \left(-h/2 \le x_{2} \le h/2, \quad 0 \le n \le \infty\right) \quad ()$$

$$n \to \infty \quad n = 0$$

$$n$$

$$\cdot$$

.

•

.

•

.

•

.

$$\begin{split} -1 < v(\mathbf{x}) \leq 1/2 \quad E(\mathbf{x}) \geq 0 \\ (\) \\ s_{ij} = \frac{1 + v^{*}(\mathbf{x})}{E^{*}(\mathbf{x})} \sigma_{ij} + \frac{v^{*}(\mathbf{x})}{E^{*}(\mathbf{x})} \sigma_{kk} \delta_{ij} , \quad i, j = 1, 2, 3 \qquad (\) \\ (\) \\ v^{*}(\mathbf{x}) \quad E^{*}(\mathbf{x}) \\ \varphi \\ \hline \\ \nabla^{2} \left(\frac{\nabla^{2} \varphi}{E^{*}(\mathbf{x})} \right) - \frac{\partial^{2}}{\partial x_{2}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})} \right) \frac{\partial^{2} \varphi}{\partial x_{1}^{2}} - \frac{\partial^{2}}{\partial x_{1}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{E^{*}(\mathbf{x})} \right) \frac{\partial^{2} \varphi}{\partial x_{1}^{2}} + 2 \frac{\partial^{2}}{\partial x_{1} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{E^{*}(\mathbf{x})} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} = 0 \quad (\) \\ \nabla^{2} \left(\frac{\nabla^{2} \varphi}{E^{*}(\mathbf{x})} \right) - \frac{\partial^{2}}{\partial x_{2}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})} \right) \frac{\partial^{2} \varphi}{\partial x_{1}^{2}} - \frac{\partial^{2}}{\partial x_{1}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{E^{*}(\mathbf{x})} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} = 0 \quad (\) \\ \nabla^{2} \left(\frac{\nabla^{2} \varphi}{E^{*}(\mathbf{x})} \right) - \frac{\partial^{2}}{\partial x_{2}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1}^{2}} - \frac{\partial^{2}}{\partial x_{1}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} = 0 \quad (\) \\ \nabla^{2} \left(\frac{1}{E^{*}(\mathbf{x})} \right) - \frac{\partial^{2}}{\partial x_{1}^{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1}^{2}} + 2 \frac{\partial^{2}}{\partial x_{1} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2} \partial x_{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2} \partial x_{1}^{2} \partial x_{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2} \partial x_{1} \partial x_{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2} \partial x_{1} \partial x_{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1}^{2} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1}^{2} \partial x_{1}^{2} \partial x_{2}} \left(\frac{1 + v^{*}(\mathbf{x})}{D^{*}(\mathbf{x})^{2} \partial x_{1} \partial x_{2}} \right) \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{1} \partial x_{1} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{1} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{1} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{1} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{2}} \frac{\partial^{2} \varphi}{\partial x_{1} \partial x_{1} \partial x_{2} \partial x_{2} \partial x_{2} \partial x_{2} \partial x_{1} \partial x_{2} \partial x_{2} \partial$$

¹ Jin and Noda ² Eischen

 $J_{1}^{*} = \frac{J_{k}^{*}}{E_{\text{tip}}^{*}}$ $J_{2}^{*} = \frac{-2K_{1}K_{II}}{E_{\text{tip}}^{*}}$

()

...

()()

() ()

()

()

)

 $E_{
m tip}^{\,*}$

[]

.

:

.

[]

$$G(\theta) = \frac{4}{E_{\rm tip}^*} \left(\frac{1}{3 + \cos(\theta)}\right)^2 \left(\frac{1 - \pi/\theta}{1 + \pi/\theta}\right)^{\theta/\pi} \left[\left(1 + 3\cos(\theta)\right)K_{\rm I}^2\right]$$
()

$$+8\sin(\theta)\cos(\theta)K_{I}K_{II} + (9-5\cos(\theta))K_{II}^{2}$$

$$: \qquad ()$$

$$\frac{\partial G}{\partial \theta} = 0, \quad \frac{\partial^{2}G}{\partial \theta^{2}} < 0 \implies \theta = \theta_{0}$$

$$()$$

$$\frac{G}{\theta} = 0 , \quad \frac{\partial^2 G}{\partial \theta^2} < 0 \implies \theta = \theta_0 \tag{()}$$

$$G\left(\theta_{0}\right) = G_{cr}\left(\mathbf{x}_{tip}\right) \tag{()}$$



MLPG

MLPG

 $n \qquad 0.2 \le x / L \le 0.6$





- [1] Ching, H.K., and Yen, S.C., "Meshless Local Petrov-Galerkin Analysis for 2D Functionnally Graded Elastic Solids under Mechanical and Thermal Loads", Composites: Part B, Vol. 36, pp. 223-240, (2005).
- [2] Erdogan F., "Fracture Mechanics of Functionally Graded Materials", Composites Eng., Vol. 5, pp. 753-770, (1995).
- [3] Atluri, S.N., and Zhu, T.L., "A New Meshless Local Petrov-Galerkin (MLPG) Approach in Computational Mechanics", Comp. Mech., No. 22, pp. 117-127, (1998).
- [4] Duflot, M., "A Meshless Method with Enriched Weight Functions for Three-dimensional Crack Propagation", International Journal for Numerical Methods in Engineering, Vol. 65, pp. 1970–2006, (2006).
- [5] Mori, T., and Tanaka, K., "Average Stress in Matrix and Average Elastic Energy of Materials with Misfitting Inclusions", Acta Metallurgica, Vol. 21, pp. 571–574, (1973).
- [6] Eischen, J.W., "Fracture of Nonhomogeneous Materials", Int. J. Fract. Vol. 34, pp. 3–22, (1987).
- [7] Jin, Z.H., and Noda, N., "Crack-tip Singular Fields in Nonhomogeneous Materials", J. Appl. Mech. Vol. 61, pp. 738-740, (1994).
- [8] Rice, J.R., "A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks", J. Appl. Mech. Vol. 35, pp. 379–386, (1968).
- [9] Gu, P., and Asaro, R.J., "Crack Deflection in Functionally Graded Materials", Int. J. Solids. Struct., Vol. 34, pp. 3085-3098, (1997).

- [10] Hussain, M.A., Pu, S.L., and Underwood, J., "Strain Energy Release Rate for a Crack under Combined Mode I and Mode II, "Fracture Analysis, ASTM STP 560, American Society for Testing and Materials, Philadelphia, pp. 2-28, (1974).
- [11] Anderson, T.L., "Fracture Mechanics: Fundamentals and Application", CRC Press, (1995).

Ι : **u**₁ : u^h w_I 1 : **f**, : c : **K**, W K_{I} K_{II} : G $G_{\rm cr}(\mathbf{x}_{\rm tip})$: *φ* : Ø, Ι :μ : κ $: \nabla^2$ $: \theta_0$

(GPa)E	V	$(GPa\sqrt{m}) K_{Ic}$
360	0.2	4
200	0.33	100

		<i>E</i> ₂ / <i>E</i> ₁	$\frac{K_I}{\sigma_0 \sqrt{\pi a}}$					
			a/w = 0.5	a/w = 0.4	a/w = 0.3	a/w = 0.2	a/w = 0.6	
		0.1	1.3034	1.8522	2.5534	3.5576	5.1205	
		0.2	1.3880	1.8290	2.4339	3.3333	4.7729	
		1	1.3742	1.6688	2.0894	2.8087	4.0482	
		5	1.1390	1.3818	1.7676	2.3878	3.4836	
		10	1.0192	1.2507	1.6172	2.2097	3.2702	
[]	0.1	1.2965	1.8581	2.5699	3.5701	5.1880	
		0.2	1.3956	1.8395	2.4436	3.3266	4.7614	
		1	1.3734	1.6628	2.1066	2.8298	4.0302	
		5	1.1318	1.3697	1.7483	2.3656	3.4454	
		10	1.0019	1.2291	1.5884	2.1762	3.2124	





()

n





a = 0.2 W

www.SID.ir

n

n



n

п

п





n



п



Abstract

This paper presents crack analysis in a finite Functionally Graded plate using one of meshfree methods called the Meshless Local Petrove-Galerkin (MLPG). Shape functions are obtained by Moving Least Squares approximation method. In order to solve fracture problems without need to any additional nodes, the weight function is modified. The path independent *J* integrals developed for non-homogeneous materials are used to calculate the stress intensity factors (SIFs) in mixed-mode problems. Assuming that the crack is parallel to the material gradient, an example including an edge-cracked plate is presented to evaluate the accuracy of numerical method. A good agreement is obtained between the results of the presented method and the reference solutions. Then the effect of direction and intensity of material gradient on SIFs and energy release rate is considered by solving some examples. The results reveal the inverse relation between the intensity of elastic modulus gradient and SIF in mode-I fracture condition. Also the energy release rate and the crack initiation angle increase with increasing the intensity of compliancy gradient normal to crack direction.