

HPM

HPM

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HPM :

HPM

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[8] [2]

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⁴ Homotopy Perturbation Method

⁵ Ji-Huan He

HPM [10]

[11]

HPM Kdv [12]

Kdv k(2,2) [13]

VIM [14]

VIM [15]

HPM [16]

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Maple, Matlab, Mathematica

k m

HPM

:

$A(u) = f(r)$ ()

: () $A(u)$

$A(u) = L(u) + N(u)$ ()

N L

:

$(1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0$ ()

u_0 $p \in [0,1]$ ()

: u HPM

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$$v = v_0 + p v_1 + p^2 v_2 + \dots \quad (1)$$

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots$$

$$u(r) = u_0 + p^2 v_2 + p^1 v_1 + \dots \quad (2)$$

$$L(u_0) = 0$$

HPM

$$\left(\frac{\partial P}{\partial x} \neq 0\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (5)$$

T, P, u, v

$$-\frac{1}{\rho} \frac{\partial P}{\partial x} = U_\infty \frac{dU_\infty}{dx} \quad (6)$$

) 2β

:()

$$U_{\infty}(x) = Cx^m \quad (1)$$

$$m = \frac{\beta}{2\pi - \beta} \quad (2)$$

$$\eta = \frac{y}{\sqrt{x}} \text{Re}_x^{0.5}, \quad f'(\eta) = \frac{u}{U_{\infty}}, \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \quad (3)$$

$$2f''' + (m+1)ff'' + 2m[1 - (f')^2] = 0 \quad (4a)$$

$$2\theta'' + \text{Pr}(m+1)f\theta' = 0 \quad (4b)$$

$$f(0) = 0, f'(0) = 0, \theta(0) = 1, f'(\infty) = 1, \theta(\infty) = 0 \quad (5)$$

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$$2(1-p)(f''' - f_0''') + p(2f''' + (m+1)ff'' + 2m[1 - (f')^2]) = 0 \quad (6a)$$

$$2(1-p)(\theta'' - \theta_0'') + p(2\theta'' + \text{Pr}(m+1)f\theta') = 0 \quad (6b)$$

$$f = f_0 + pf_1 + p^2f_2 + p^3f_3 \quad (7a)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3 \quad (7b)$$

$$f_0''' = \theta_0'' = 0 \quad (8)$$

$$\begin{cases} \frac{d^3 f_0}{d\eta^3} = 0 \\ \frac{d^2 \theta_0}{d\eta^2} = 0 \\ B.C : f_0(0) = 0, f_0'(0) = 0, f_0'(k) = 1, \theta_0(0) = 1, \theta_0(k) = 0 \end{cases} \quad (9a)$$

...

:p

$$\begin{cases} 2 \frac{d^3 f_1}{d\eta^3} + (m+1) f_0(\eta) \frac{d^2 f_0(\eta)}{d\eta^2} + 2m \left[1 - \left(\frac{df_0(\eta)}{d\eta} \right)^2 \right] = 0 \\ 2 \frac{d^2 \theta_1}{d\eta^2} + Pr \times (m+1) f_0(\eta) \frac{d\theta_0(\eta)}{d\eta} = 0 \\ B.C : f_1(0) = 0, f_1'(0) = 0, f_1'(k) = 0, \theta_1(0) = 0, \theta_1(k) = 0 \end{cases}$$

(b)

:p

$$\begin{cases} 2 \frac{d^3 f_2}{d\eta^3} + (m+1) [f_0 f_1'' + f_1 f_0''] - 4m [f_0' f_1'] = 0 \\ 2 \frac{d^2 \theta_2}{d\eta^2} + Pr \times (m+1) [f_0 \theta_1' + f_1 \theta_0'] = 0 \\ B.C : f_2(0) = 0, f_2'(0) = 0, f_2'(k) = 0, \theta_2(0) = 0, \theta_2(k) = 0 \end{cases}$$

(c)

:p

$$\begin{cases} 2 \frac{d^3 f_3}{d\eta^3} + (m+1) [f_0 f_2'' + f_2 f_0'' + f_1 f_1''] - 2m [(f_1')^2 + 2f_0' f_2'] = 0 \\ 2 \frac{d^2 \theta_3}{d\eta^2} + Pr \times (m+1) [f_0 \theta_2' + f_2 \theta_0' + f_1 \theta_1'] = 0 \\ B.C : f_3(0) = 0, f_3'(0) = 0, f_3'(k) = 0, \theta_3(0) = 0, \theta_3(k) = 0 \end{cases}$$

(d)

(15-a)

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η k ()

U_∞

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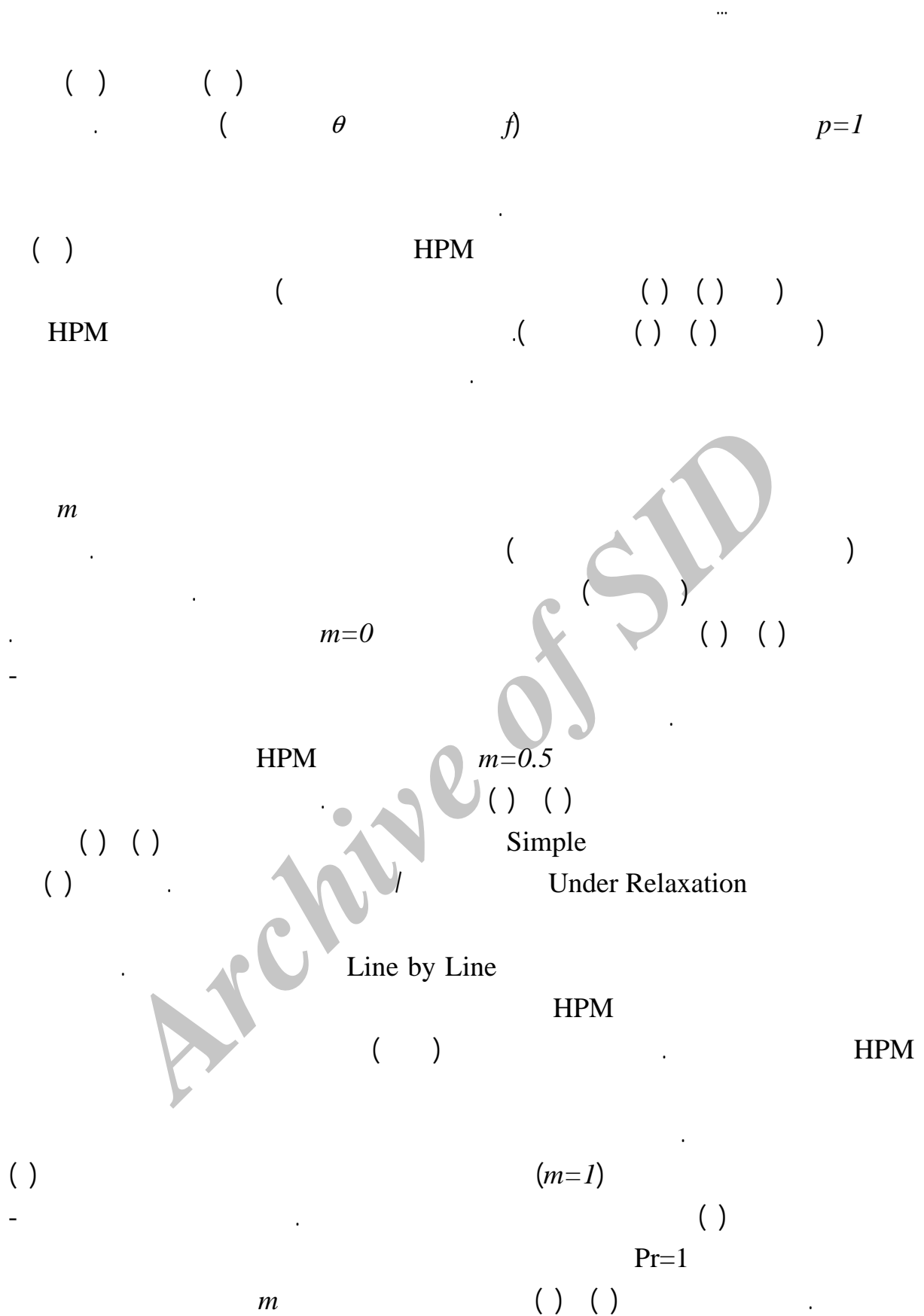
$$\begin{cases} f_0(\eta) = \frac{\eta^2}{2k} \\ \theta_0(\eta) = -\frac{\eta}{k} + 1 \end{cases}$$

(a)

$$\left\{ \begin{aligned} f_1(\eta) &= \frac{(3m-1)}{240k^2} \eta^5 - \frac{m}{6} \eta^3 + \frac{1}{2} \left(\frac{7}{16} mk + \frac{k}{48} \right) \eta^2 \\ \theta_1(\eta) &= \frac{\text{Pr}(m+1)}{48k^2} \eta^4 - \frac{\text{Pr}k}{48} (m+1)\eta \end{aligned} \right. \quad (\text{ b })$$

$$\left\{ \begin{aligned} f_2(\eta) &= \frac{(27m^2 - 42m + 11)}{161280k^3} \eta^8 + \frac{(-320m^2k^2 + 160mk^2)}{57600k^3} \eta^6 \\ &+ \frac{(-90m + 315m^2 - 5)k^3}{28800k^3} \eta^5 + \frac{k^3}{2} \left(-\frac{61m^2}{2688} + \frac{m}{960} + \frac{13}{40320} \right) \eta^2 \\ \theta_2(\eta) &= -\frac{\text{Pr}(m+1)(20\text{Pr}(m+1) - 6m + 2)}{40320} \eta^7 - \frac{4\text{Pr}m(m+1)k^2}{960k^3} \eta^5 \\ &+ \frac{\text{Pr}(m+1)k^3(5\text{Pr}(m+1) + 105m + 5)}{11520} \eta^4 + k^3 \left\{ \frac{\text{Pr}^2}{16128} (m^2 + 1) \right. \\ &\left. - \frac{\text{Pr}}{80640} (442m + 31) + \text{Pr}m \left(-\frac{137m}{26880} + \frac{\text{Pr}}{8064} \right) \right\} \eta \end{aligned} \right. \quad (\text{ c })$$

$$\left\{ \begin{aligned} f_3(\eta) &= \frac{1}{322560k^4} \left\{ \frac{1}{990} (-375 - 1917m^2 + 1671m + 837m^3) \eta^{11} + \frac{k^2}{504} (-10080m + 30464m^2 \right. \\ &- 18592m^3) \eta^9 + \frac{k^3}{336} (7938m + 23814m^3 - 35910m^2 + 462) \eta^8 + \frac{1}{210} (53760m^3k^4 - \\ &26880k^4m^2) \eta^7 + \frac{1}{120} (-94080m^3k^5 + 2240mk^5 + 42560m^2k^5) \eta^6 + \frac{k^6}{60} (-87 + 35325m^3 \\ &- 1377m - 6881m^2) \eta^5 + \frac{k^5}{2} \left(\frac{1}{774144} - \frac{937}{921600} m^2 - \frac{1489}{19353600} m - \frac{5521}{6451200} m^3 \right) \eta^2 \\ \theta_3(\eta) &= \frac{1}{322560k^4} \left\{ \text{Pr}(m+1) \left(\frac{1}{90} (280\text{Pr}^2m^2 + 27m^2 - 252\text{Pr}m^2 + 560\text{Pr}^2m - 42m \right. \right. \\ &- 168\text{Pr}m + 11 + 280\text{Pr}^2 + 84\text{Pr}) \eta^{10} + \dots + \frac{k^5}{20} (-560\text{Pr}m^2 - 560\text{Pr}m) \eta^5 \\ &+ \frac{1}{12} (-10\text{Pr}^2k^6m + 26k^2 - 1830k^6m^2 + 84k^6m + 1212\text{Pr}mk^6 - 5\text{Pr}^2k^6m^2 \\ &+ 1146\text{Pr}m^2k^6 - 5\text{Pr}^2k^6 + 66\text{Pr}k^6) \eta^4 \left. \right\} + \left\{ \frac{k^5}{12} \left(\frac{23}{11612160} \text{Pr}^3m^3 + \frac{631}{1612800} \text{Pr}m^3 \right. \right. \\ &+ \frac{43}{3225600} \text{Pr}^2m^3 + \frac{23}{3870720} \text{Pr}^3m^2 + \frac{53}{138240} \text{Pr}m^2 + \frac{11}{387072} \text{Pr}^2m^2 - \frac{187}{14515200} \text{Pr}m \\ &\left. \left. + \frac{23}{3870720} \text{Pr}^3m + \frac{163}{9676800} \text{Pr}^2m - \frac{73}{14515200} \text{Pr} + \frac{23}{11612160} \text{Pr}^3 + \frac{17}{9676800} \text{Pr}^2 \right) \eta \right\} \end{aligned} \right. \quad (\text{ d })$$



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- 1- Finite Difference
 - 2- Staggered Grid
 - 3- Implicit Scheme

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Matlab Maple)

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(m/s) x : u

(m/s) y : v

(N/m²) : P

(°C) : T

: p

: f

: T_w

: U_∞

(x) : T_∞

: Re

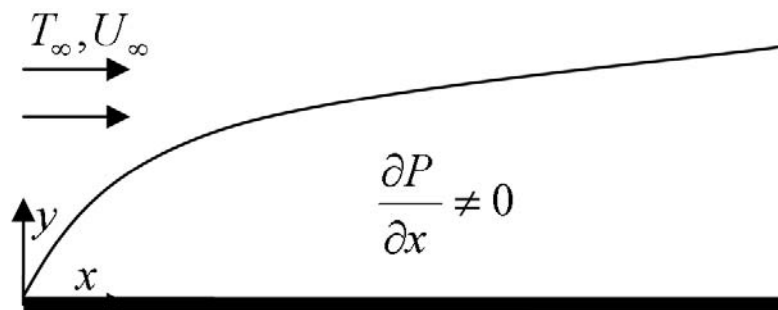
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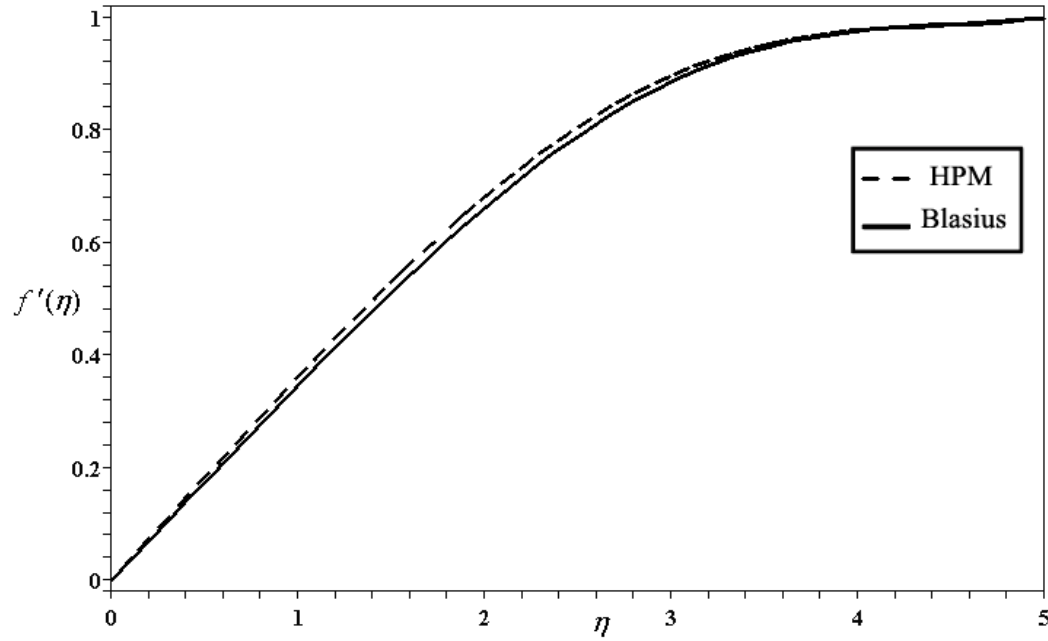
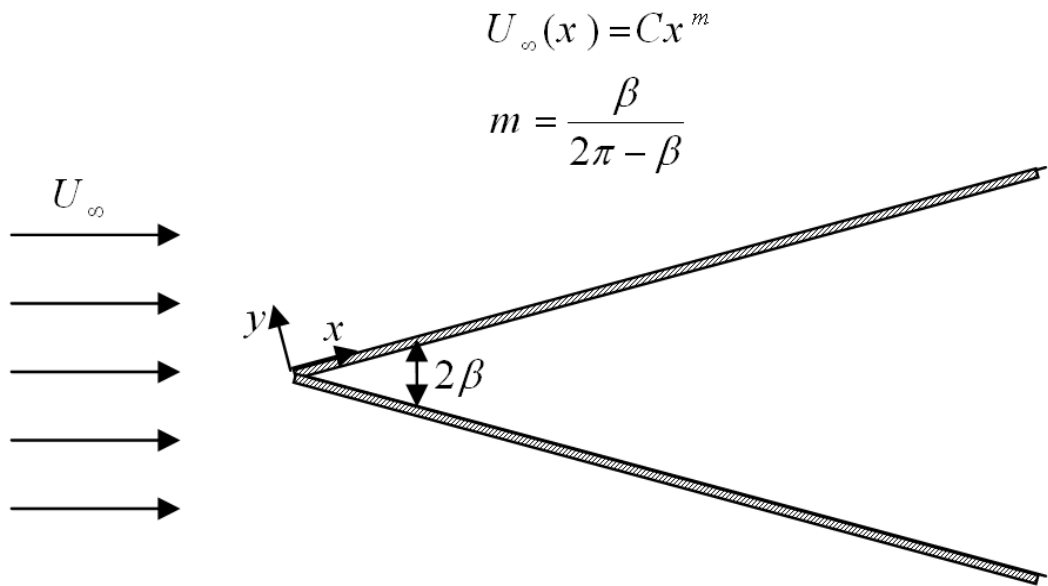
: θ

: η

: w

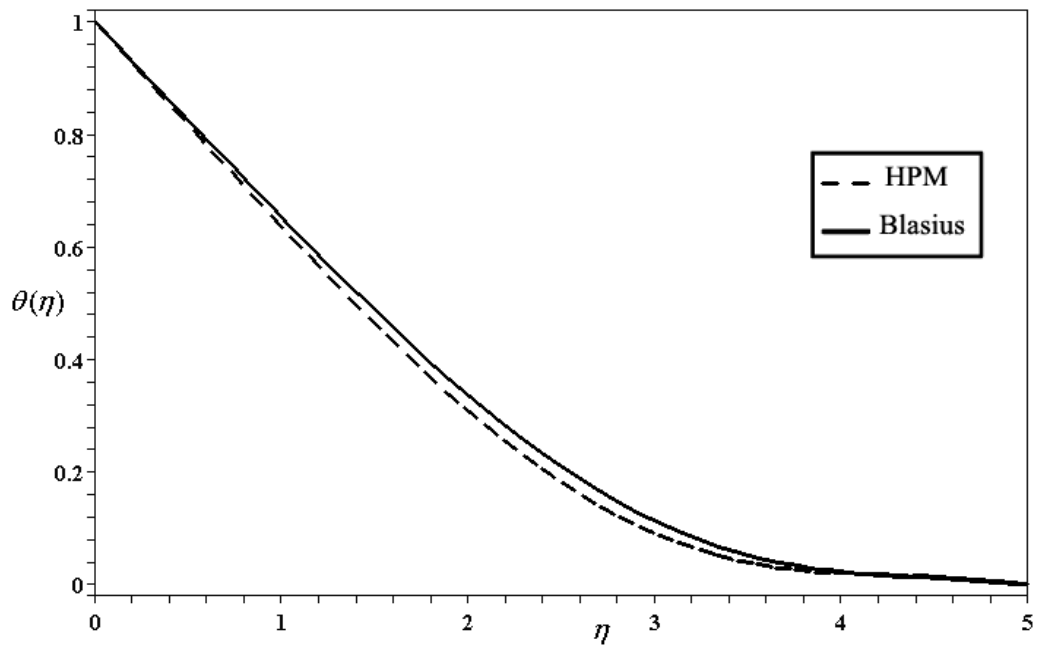
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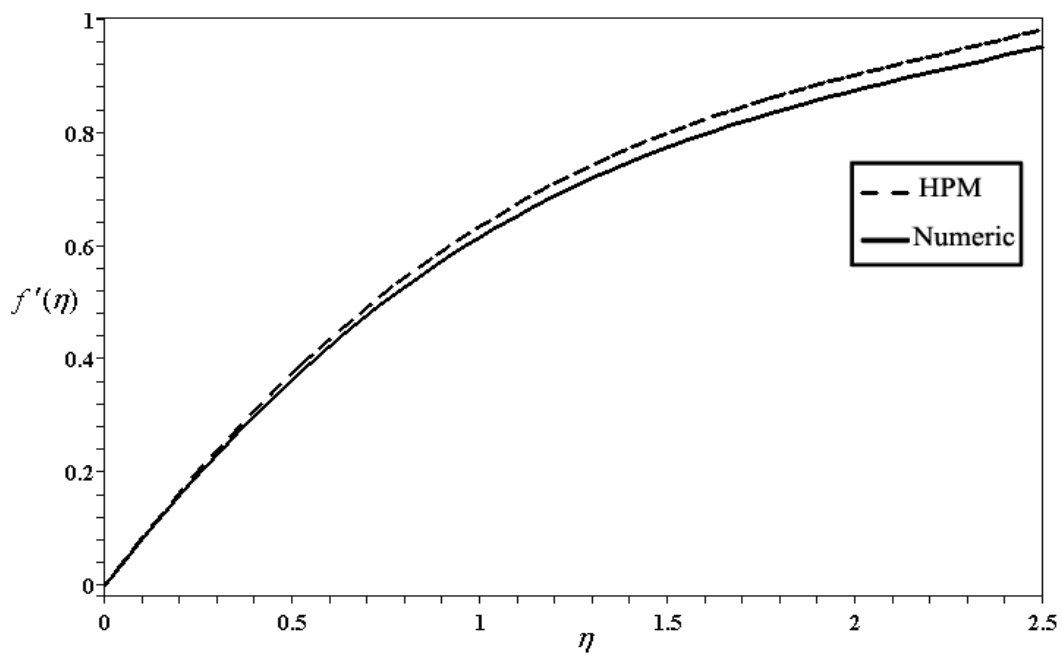
$m=0$

$f'(\eta)$



$m=0$ $Pr=1$

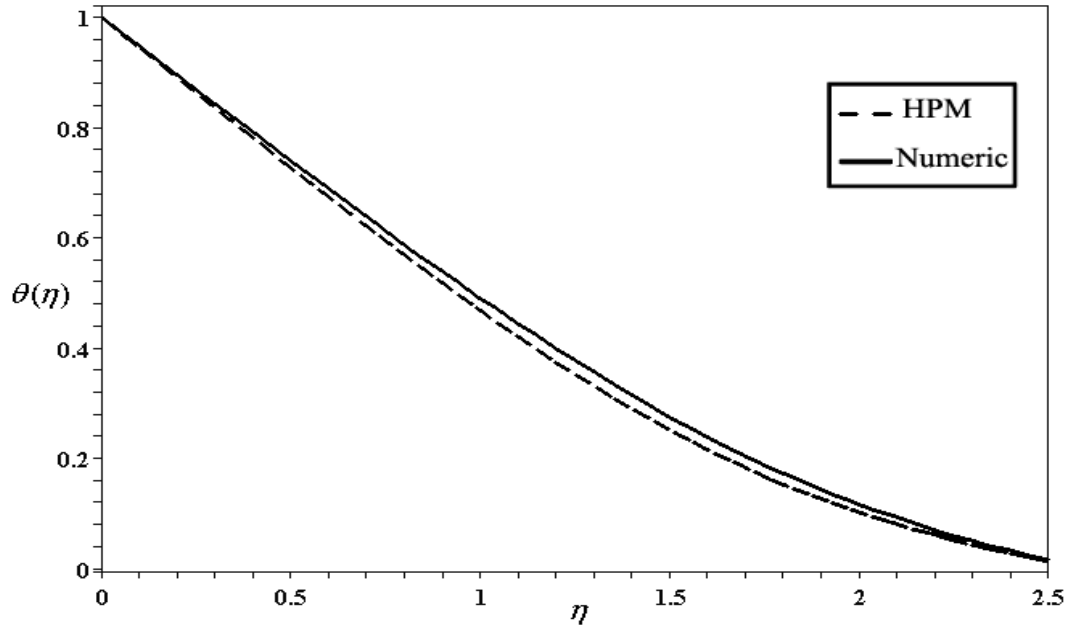
$\theta(\eta)$



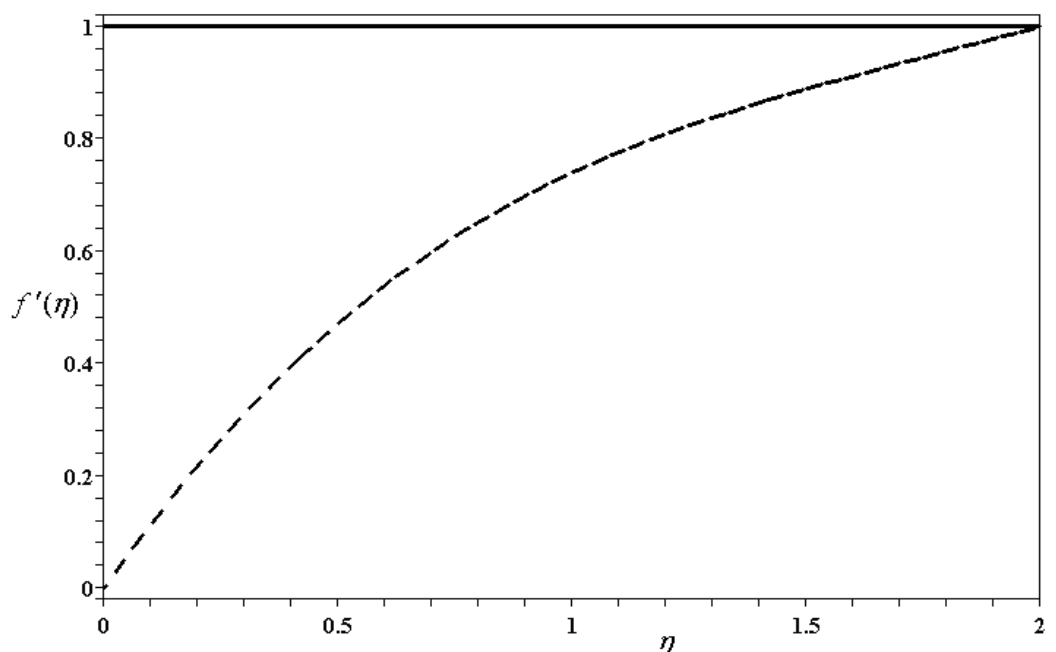
$m=0.5$ η

$f'(\eta)$

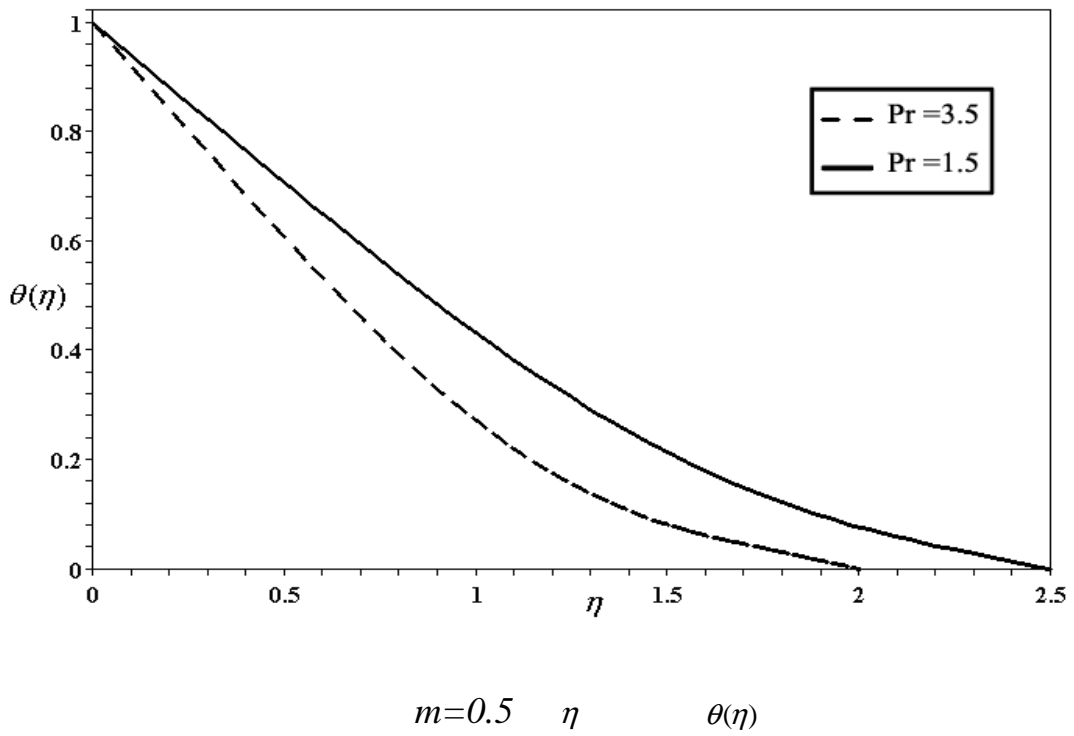
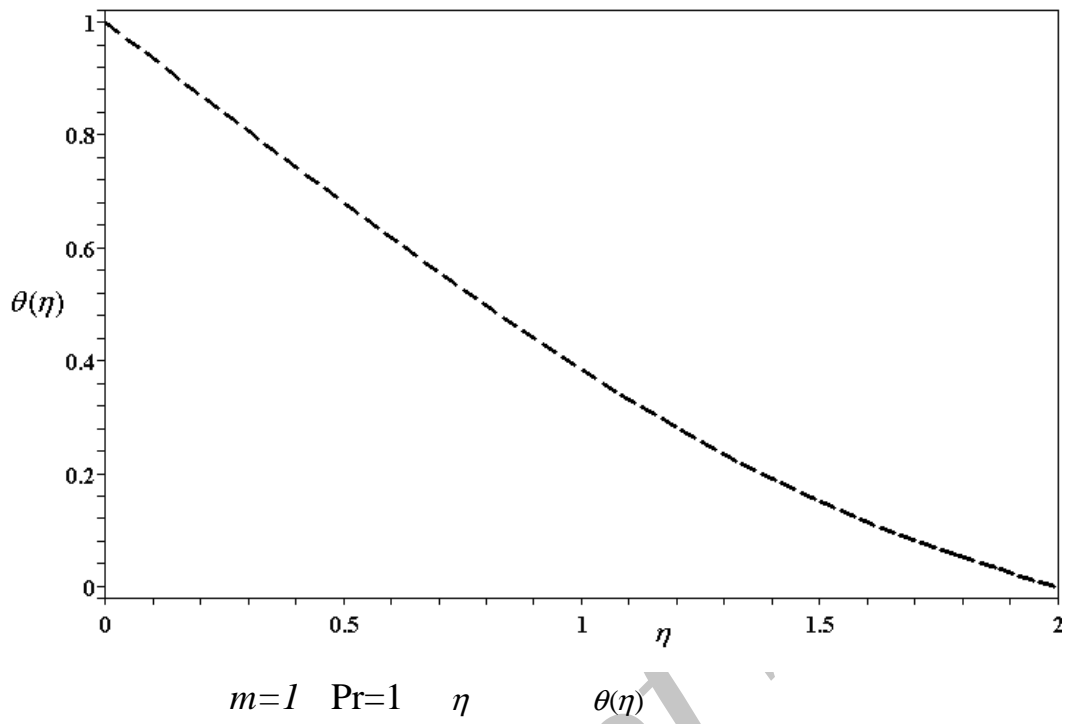
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$m=0.5$ $Pr=1$ η $\theta(\eta)$



$m=1$ η $f'(\eta)$



Abstract

One of the most advanced analytical methods to solve non-linear equations is the application of homotopy and perturbation techniques simultaneously. In the present paper the laminar flow over a wedge which is equivalent to the flow over a flat plate in the presence of non-zero pressure gradient, is solved using the homotopy perturbation method, and the results are compared with numerical results. The advantage of the HPM is the remove of restrictions of the two methods of homotopy and perturbation. In the perturbation method a small parameter should be obtained, which is a difficult task. The combination of the two methods eliminates the difficulty. The obtained solutions are explicit functions. The comparison of the present results with the results reported in literature is in good agreement. The accuracy and convenience of the computations are the main advantages of the HPM, which makes it suitable to be used in engineering problems.

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