A New Approach to Finite Element Modeling and Simulation of Flexible Robot Manipulators

M. Tajdari
Mech. Eng. Group
Malek-Ashtar Univ. of Tech.

M.A. Hajabasi
Mech. Eng. Dep't.
Shahid Bahonar Univ. of Kerman

A.R. Khoogar
Mech. Eng. Group
Malek-Ashtar Univ. of Tech.

ABSTRACT
Traditional robot manipulators that have large links need powerful actuators and their massive structures strongly limit their operating speed. Flexible manipulators having lightweight links are designed to overcome these disadvantages and in this case their flexibility is an important and unavoidable characteristic. In this paper, a new approach to finite element modeling of flexible manipulators, using Hamiltonian mechanics and simulation of their dynamic behavior is presented. The finite element model includes all non-linear terms, such as dynamic interactions between linkages. A computer program is developed in the MATLAB medium to simulate the effects of flexibility on the robot's motion quality. Our results indicate the importance of flexibility and existence of considerable errors in the end-effector's positions.

Key Words: Flexiblity, Manipulators, Robot, Finite Element, Modeling, Simulation

ارائه یک روش جدید مدلسازی و شبیه‌سازی اجزای محدود برای بازوهای روباتیک انعطاف‌پذیر

مهدی تاجداری[۱]
گروه مهندسی مکانیک
دانشگاه صنعتی مالک اشتر

محمد علی حاج عباسی[۲]
دانشکده مهندسی مکانیک
دانشگاه شهید باهنر کرمان

احمد رضا خوگر[۳]
گروه مهندسی مکانیک
دانشگاه صنعتی مالک اشتر

(تاریخ دریافت: ۱۳۸۶/۰۹/۲۱؛ تاریخ پذیرش: ۱۳۸۷/۰۹/۱۲)

چکیده
روبات‌های بزرگ برای حرکت بازوهای سنگین خود نیاز به عملگرهای قدرتمند دارند. از طرفی، بزرگی سازهٔ بازوها سرعت حرکت آنها را به شدت محدود می‌سازد. برای رفع این محدودیت‌ها، روبات‌های انعطاف‌پذیری با بازوهای سبک طراحی می‌شوند. در این مقاله، یک روش جدید اجزاء محدود برای مدلسازی و شبیه‌سازی رفتار دینامیکی بازوهای انعطاف‌پذیر با استفاده از مکانیک هامیلتونی ارائه شده است. در این مدل، تمامی عبارات غیرخطی، مانند آثار دینامیکی بین بازوها در نظر گرفته شده است. یک برنامه MATLAB برای مدل‌سازی و شبیه‌سازی انعطاف‌پذیری روبات و اثرات آن در رفتار دینامیکی حرکت بازوها توسعه داده شده است. نتایج شبیه‌سازی اهمیت اثر انعطاف‌پذیری و تأثیر آن بر خطای مجری نهایی را نشان می‌دهد.

واژه‌های کلیدی: انعطاف پذیری، روبات، بازوهای مکانیکی، اجزاء محدود، مدلسازی، شبیه‌سازی

1 - Associate Professor (Corresponding Author): tajdari@yahoo.com
2 - Assistant Professor: hajabasi@mail.uk.ac.ir
3 - Assistant Professor: Khoogar@yahoo.com

1. Introduction

With the growing need for advanced robot manipulators, the requirement for higher speed, higher efficiency, smaller actuators or motors, and greater maneuverability robots are more in demand. Therefore, more powerful analytical and numerical models are required. Mass and inertia Reduction of different components and linkages of a robot usually result in a higher speed, lower energy consumption and easier system fabrication. Reduction of inertia of linkages is also especially important in the design of robot control policies. This many criteria must be considered. For instant, using thin member linkages is especially helpful. In this case, flexibility of the members becomes important and a new branch of robotics under the title of "flexible manipulators" is evolved. However, the disadvantages of flexible manipulators are undesirable dynamic behavior and the difficulty for their identification and prediction. In fact, when a mechanical manipulator with thin linkages moves at a high speed, including the flexibility and motion due to elasticity of members in the dynamic model is necessary. This makes the dynamic model more complex than the case of rigid assumed linics. Although including flexibility makes the control system design more complex, its usage is growing because of the earlier mentioned this, advantages.

Research into the area of flexible manipulators has been considered by many researchers in the past two decades. Regarding this, Book [1] has introduced homogenous transformation matrices for describing the motion of revolute and prismatic joints and deformation due to flexibility of the links. In this work, links deformation is assumed as the sum of the modal vectors obtained from the governing equation using the Lagrangian method. Usoro etal [2] divided each mechanical linkage with flexible and light members into discrete finite elements. The governing equations were then deduced using the potential and the kinetic energy. They extended their work to manipulators with two members. Low and Vidyasygar [3] introduced a procedure for obtaining the dynamic equations for manipulators with rigid and flexible members using the Hamiltonian principle. Wen [4] simulated the deformation by considering hypothetical modes for the members. He assumed that a flexible member behaves like the Euler-Bernoulli beam. In this manner, the equations of the system were determined using the Langrargian method. Zhang and Yu [5] developed the dynamic equations by considering the effects of flexibility in the members and joints. Their technique was applied to a four-member manipulator, and the results clarified the importance of the joints and members flexibility. Plosa and Wojciech [6] developed a mathematical model for a three-dimensional beam with variable stiffness under torsion and bending. The equations of the system were obtained using the finite element method. Biswas and Klafer [7] considered a one-dimensional manipulator and used a servomotor for actuation of the member. And assuming this the governing equations were derived. Pfeiffer and Gebler [8] described the dynamics and control of flexible manipulators using their end-effector flexibility in a given path and derived the governing equation using this technique. Everett [9] derived the equations of motion for a flexible manipulator using the energy method and state space vectors. Readman and Belanger [10] modeled their flexible links as a helical linear spring. They obtained the dynamic equations for flexible by this assumption.

In this paper a new approach to finite element modeling for two degrees of freedom of flexible manipulators is presented. In this modeling, all the nonlinear terms such as dynamic interactions between linkages are considered in simulating their dynamic behavior. In this manner, the deduced dynamic equations and the dynamic behavior of flexible manipulators are more realistic than the previous studies.

2. Formulation of the Problem

In this section, the dynamic model of the two degrees of a freedom of flexible manipulator is derived. Each member is divided to several elements and for each one, kinetic and potential energies are considered using the Euler - Bernoulli beam theory. The manipulator is shown in figure 1. In this research, the joints are assumed rigid and members are considered flexible. Each member length is bounded and the Euler- Bernoulli beam theory is used. The mechanical linkage moves in a horizontal plane. The system parameters are defined as follows:

v_1^e : Displacement in the first node,

v_1^e : Displacement in the second node,

φ_1^e : Rotation in the first node,

φ_2^e : Rotation in the second nodem,

$\mu(x)$: Mass per unit length,

L : Length of link,

l^e : Length of element,

EI : Flexural rigidity,

τ_1^e : Actuating torque at first joint,

τ_2^e : Actuating torque at second joint,
θ : Joint variables,
$\mathbf{r}$: Position vector in local coordinate system, and
$\mathbf{R}$: Position vector in global coordinate system.

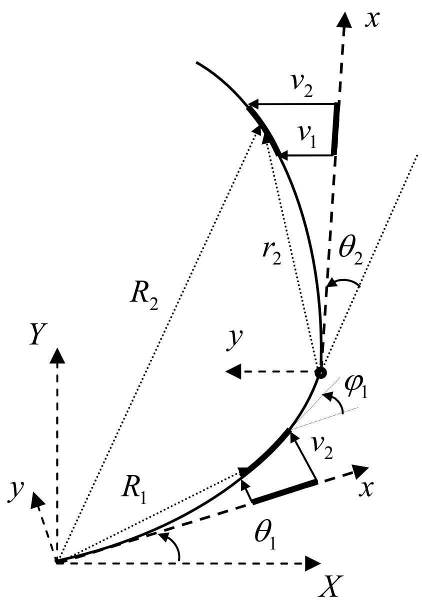

Fig. (1): Two degrees of freedom flexible manipulator and parameters.

Velocity vectors can be expressed in the link coordinate system as:

$$\mathbf{r}_1 = x\mathbf{i} + v\mathbf{j}, \tag{1}$$

$$\boldsymbol{\omega}_1 = (\dot{\theta}_1 + v')\mathbf{k}. \tag{2}$$

The absolute position and velocity of the particle in the first link coordinate system is represented by the following equations:

$$\mathbf{R}_1 = x\mathbf{i} + v\mathbf{j}, \tag{3}$$

$$\dot{\mathbf{R}}_1 = \dot{v}\mathbf{j} + \dot{\theta}_1 \mathbf{k} \times (x\mathbf{i} + v\mathbf{j}). \tag{4}$$

To calculate the first link's total kinetic energy, the definition of kinetic energy can be used by integrating along the element length as:

$$T_1^e = \frac{1}{2}\int_{l^e} \mu \, \dot{\mathbf{R}}_1 \cdot \dot{\mathbf{R}}_1 \, dx \quad . \tag{5}$$

Substituting equation (4) into equation (5), we obtain:

$$T_1^e = \frac{1}{2}\int_{l^e} \mu (v^2\dot{\theta}_1^{\,2} + \dot{v}^2 + x^2\dot{\theta}_1^{\,2} + 2\dot{v}x\dot{\theta}_1)\, dx \; . \tag{6}$$

Equation (6) gives the total kinetic energy of the first link. According to Fig. **1**, the displacement and angular velocity vectors for an element of the second link can be expressed by:

$$\mathbf{r}_2 = x\mathbf{i} + v\mathbf{j}, \tag{7}$$

$$\boldsymbol{\omega}_2 = (\dot{\theta}_2 + v')\mathbf{k}. \tag{8}$$

The absolute position and velocity of a particle of the second link in its coordinate system can be written by the following equations:

$$\mathbf{R}_2 = \mathbf{R}_{o2} + \mathbf{r}_2, \tag{9}$$

$$\dot{\mathbf{R}}_2 = \dot{\mathbf{R}}_{o2} + \dot{\mathbf{r}}_2, \tag{10}$$

$$\dot{\mathbf{R}}_2 = V_{ox}\mathbf{i} + V_{oy}\mathbf{j} + \dot{\mathbf{r}}_2, \tag{11}$$

$$\dot{\mathbf{R}}_2 = (V_{ox} - v\dot{\theta}_2)\mathbf{i} + (V_{oy} + x\dot{\theta}_2 + \dot{v})\mathbf{j}. \tag{12}$$

$\mathbf{R}_{o2}$ is the position vector of the second joint, which is located at the beginning of the second link. V_{ox} and V_{oy} are the components of the absolute linear velocity at the beginning point of the second link in the local coordinate system. The kinetic energy for an element of this link is defined as:

$$T_2^e = \frac{1}{2}\int_{l_e} \mu \, \dot{\mathbf{R}}_2 \cdot \dot{\mathbf{R}}_2 \, dx \; . \tag{13}$$

Now, by substituting equation (12) into equation (13),

$$T_2^e = \frac{1}{2}\int_{l^e} \mu (V_{ox}^2 + v^2\dot{\theta}_2^{\,2} - 2V_{ox}v\dot{\theta}_2 + V_{oy}^2 + x^2\dot{\theta}_2^{\,2} + \dot{v}^2 + 2V_{oy}x\dot{\theta}_2 + 2V_{oy}\dot{v} + 2x\dot{\theta}_2\,\dot{v})\, dx. \tag{14}$$

Equation (14) gives the total kinetic energy of the second link.

2.2 Potential Energy

It is assumed that bending is the only source of potential energy in the first and second linkages. The euler-Bernoulli beam theory is applied to calculate the potential energy resulting:

$$U_1 = \frac{1}{2}\int_{l^e} EI\,(\frac{\partial^2 v}{\partial x^2})dx\ , \tag{15}$$

$$U_2 = \frac{1}{2}\int_{l^e} EI(\ \frac{\partial^2 v}{\partial x^2})dx\ . \tag{16}$$

2.3 Finite Element Modeling

There are several methods for modeling the system. These methods include the assumed mode method, lumped method, and the finite element method. Among these the finite element method is the most accurate and popular one, thus, this method has been selected for describing the movement of the flexible members. In this method, the deformations are usually written as following:

$$v(x,t) = [\mathbf{N}(x)]\{\mathbf{q}(t)\}\ , \tag{17}$$

where, $[\mathbf{N}(x)]$ is the row matrix of the shape functions and $\{\mathbf{q}(t)\}$ is the nodal variable vector. Substituting $v(x,t)$ into the kinetic energy formula, and evaluating the integral, the kinetic energy for the elements of the first and the second linkages can be obtained as:

$$T_1 = \frac{1}{2}\mathbf{q}^T\mathbf{M}_1\mathbf{q}\dot{\theta}_1^{\,2} + \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}_1\dot{\mathbf{q}} + \frac{1}{2}I\dot{\theta}_1^{\,2} + \dot{\mathbf{q}}^T\mathbf{a}^T\dot{\theta}_1\ , \tag{18}$$

$$\begin{aligned} T_2 = \frac{1}{2}(&M_4V_{ox}^2 + \mathbf{q}^T\mathbf{M}_1\mathbf{q}\dot{\theta}_2^{\,2} - 2V_{ox}\mathbf{q}^T\mathbf{M}_2^T\dot{\theta}_2 \\ &+ M_4V_{oy}^2 + \dot{\mathbf{q}}^T\mathbf{M}_1\dot{\mathbf{q}} + I\dot{\theta}_2^{\,2} + 2V_{oy}\dot{\mathbf{q}}^T\mathbf{M}_2^T + \\ &2V_{oy}M_3\dot{\theta}_2 + 2\dot{\mathbf{q}}^T\mathbf{a}^T\dot{\theta}_2), \end{aligned} \tag{19}$$

in which, $\mathbf{M}_1$, $\mathbf{M}_2$, M_3, M_4, I and $\mathbf{a}$ are defined as:

$$\begin{aligned} &\mathbf{M}_1 = \int_{-1}^{1} \mu(\eta)(\mathbf{N}^T(\eta).\mathbf{N}(\eta))\,J\,d\eta\ , \\ &\mathbf{M}_2 = \int_{-1}^{1} \mu(\eta)\mathbf{N}^T(\eta)J\,d\eta\ , \\ &M_3 = \int_{-1}^{1} \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)\,J\,d\eta\ , \\ &M_4 = \int_{-1}^{1} \mu(\eta)\,J\,d\eta\ , \\ &I = \int_{-1}^{1} \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)^2\,Jd\eta\ . \\ &\mathbf{a} = \int_{-1}^{1} \mu(\eta)(\frac{1}{2}(2i-1+\eta)l^e)\mathbf{N}(\eta)\,J\,d\eta\ . \end{aligned} \tag{20}$$

The potential energy of the first and second links can be calculated using the same procedure as:

$$U_1 = \frac{1}{2}\mathbf{q}^T\mathbf{K}_1\mathbf{q}\ , \tag{21}$$

$$U_2 = \frac{1}{2}\mathbf{q}^T\mathbf{K}_2\mathbf{q}\ , \tag{22}$$

where, $\mathbf{K}_1$ and $\mathbf{K}_2$ are defined as:

$$\mathbf{K} = \int_{-1}^{1} EI(\eta)\mathbf{N}''(\eta)\mathbf{N}''(\eta)J^{-3}d\eta\ . \tag{23}$$

To derive the governing equations, Hamiltonian principle, which is defined by the following formula, is used:

$$\delta\int_{t_1}^{t_2}(T-U)dt = 0, \tag{24}$$

where, T and U are the kinetic and potential energies, respectively. Now, substituting equations (20) and (22) into equation (24), the governing equations are obtained in the form:

$$\begin{aligned} &\mathbf{M}_{11}\ddot{\mathbf{q}}_1 + \mathbf{a}_1{}^T\ddot{\theta}_1 + [\mathbf{K}_1 - \mathbf{M}_{11}\dot{\theta}_1{}^2]\mathbf{q}_1 = \mathbf{F}_{11}^e, \\ &\mathbf{a}_1\ddot{\mathbf{q}}_1 + I_1\ddot{\theta}_1 + 2\dot{\mathbf{q}}_1{}^T\mathbf{M}_{11}\mathbf{q}_1\dot{\theta}_1 + \mathbf{q}_1{}^T\mathbf{M}_{11}\mathbf{q}_1\ddot{\theta}_1 = F_{21}^e, \end{aligned} \tag{25}$$

$$\begin{aligned} &I_2\ddot{\theta}_2 + \mathbf{q}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\ddot{\theta}_2 + \mathbf{a}_2\ddot{\mathbf{q}}_2 - \mathbf{q}_2{}^T\dot{V}_{ox}\mathbf{M}_{22} + \\ &\dot{V}_{oy}M_{32} + 2\dot{\mathbf{q}}_2{}^T\mathbf{M}_{12}\mathbf{q}_2\dot{\theta}_2 - \dot{\mathbf{q}}_2{}^TV_{ox}\mathbf{M}_{22} = F_{22}^e \\ &\mathbf{a}_2{}^T\ddot{\theta}_2 + \mathbf{M}_{12}\ddot{\mathbf{q}}_2 + \mathbf{K}_2\mathbf{q}_2 - \mathbf{M}_{12}\dot{\theta}_2{}^2\mathbf{q}_2 + \mathbf{M}_{22}\dot{V}_{oy} \\ &+\mathbf{M}_{22}V_{ox}\dot{\theta}_2 = \mathbf{F}_{12}^e. \end{aligned} \tag{26}$$

Assembling the mass and the stiffness matrices, the dynamic equations of the manipulator will have the following forms:

$$\mathbf{M}_1^*\begin{Bmatrix}\ddot{\theta}_1 \\ \ddot{\mathbf{q}}_1\end{Bmatrix} + \mathbf{K}_1^*\begin{Bmatrix}\theta_1 \\ \mathbf{q}_1\end{Bmatrix} = \mathbf{F}_1^*\ . \tag{27}$$

$$\mathbf{M}_2^*\begin{Bmatrix}\ddot{\theta}_2 \\ \ddot{\mathbf{q}}_2\end{Bmatrix} + \mathbf{K}_2^*\begin{Bmatrix}\theta_2 \\ \mathbf{q}_2\end{Bmatrix} + \mathbf{C}_2^* = \mathbf{F}_2^*\ .$$

It should be noted that $\mathbf{q}$ is the nodal displacement and the other parameters are defined as:

$$\mathbf{M}_1^* = \begin{bmatrix} I_1 + \mathbf{q}_1{}^T \mathbf{M}_{11} \mathbf{q}_1 & \mathbf{a}_1 \\ \mathbf{a}_1{}^T & \mathbf{M}_{11} \end{bmatrix},$$

$$\mathbf{K}_1^* = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{K}_1 - \mathbf{M}_{11} \dot{\theta}_1{}^2 \end{bmatrix}$$

$$\mathbf{F}_1^* = \begin{Bmatrix} F_{21} - 2\dot{\mathbf{q}}_1{}^T \mathbf{M}_{11} \mathbf{q}_1 \dot{\theta}_1 \\ \mathbf{F}_{11} \end{Bmatrix},$$

$$\mathbf{M}_2^* = \begin{bmatrix} I_2 + \mathbf{q}_2{}^T \mathbf{M}_{12} \mathbf{q}_2 & \mathbf{a}_2 \\ \mathbf{a}_2{}^T & \mathbf{M}_{12} \end{bmatrix},$$

$$\mathbf{K}_2^* = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{K}_2 - \mathbf{M}_{12} \dot{\theta}_2{}^2 \end{bmatrix}, \qquad (28)$$

$$\mathbf{C}_2^* = \begin{bmatrix} -\mathbf{q}_2{}^T \dot{V}_{ox} \mathbf{M}_{22} + \dot{V}_{oy} M_{32} \\ \mathbf{M}_{22} \dot{V}_{oy} \end{bmatrix},$$

$$\mathbf{F}_2^* = \begin{Bmatrix} F_{22} - 2\dot{\mathbf{q}}_2{}^T \mathbf{M}_{12} \mathbf{q}_2 \dot{\theta}_2 + \dot{\mathbf{q}}_2{}^T V_{ox} \mathbf{M}_{22} \\ \mathbf{F}_{12} - \mathbf{M}_{22} V_{ox} \dot{\theta}_2 \end{Bmatrix},$$

$$q = \{v_2^1 \ \varphi_2^1 \ v_2^2 \ \varphi_2^2 \ \ldots\}.$$

3. Numerical Solution and Simulation Results

In this section, the numerical solutions and the simulation results of a flexible manipulator is presented. A computer program was developed for solving the system of equations using the finite difference method. Table **1** indicates the parameters used in the numerical solution. The computer code was executed with quadratic torques applied at joints 1 and 2. The profiles of the quadratic torque used in this simulation are shown in Fig's **2-3**. Under this condition, variation of the joint angles and the angular velocity of the two links are considered. Figures 4 and 5 show the first and second joint angle variations with respect to time. Fig's **6-7** show the displacement of two points located at the end of the two links. The rotation of the tangent line at end points is shown in Fig's **8-9**. Fig's **10- 15** show the rate of the mentioned quantities. These are the angular velocities of the first and second links, the linear velocity of the ends of two links, and their angular velocity of the tangent line at the end, respectively.

Table (1): Problem parameters.

EI ($N.m^2$)	μ (kg/*m*)	*L* (*m*)
200	1	1

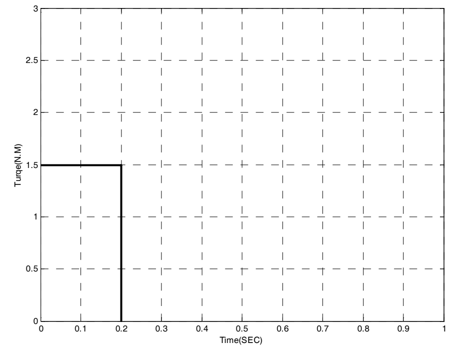

Fig. (2): Actuating torque applied to joint one.

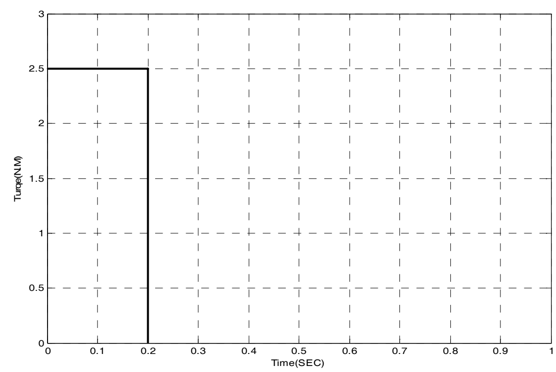

Fig. (3): Actuating torque applied to joint two.

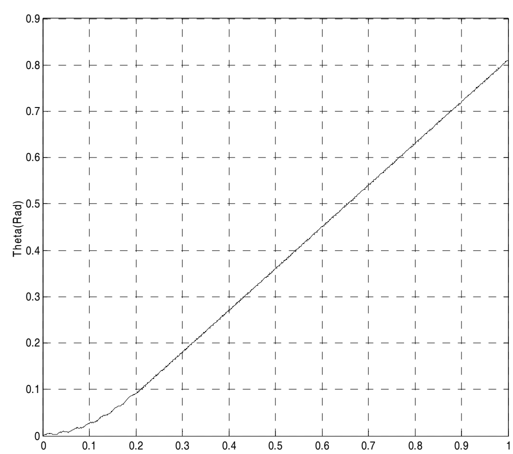

Fig. (4): Joint angle versus time at first joint.

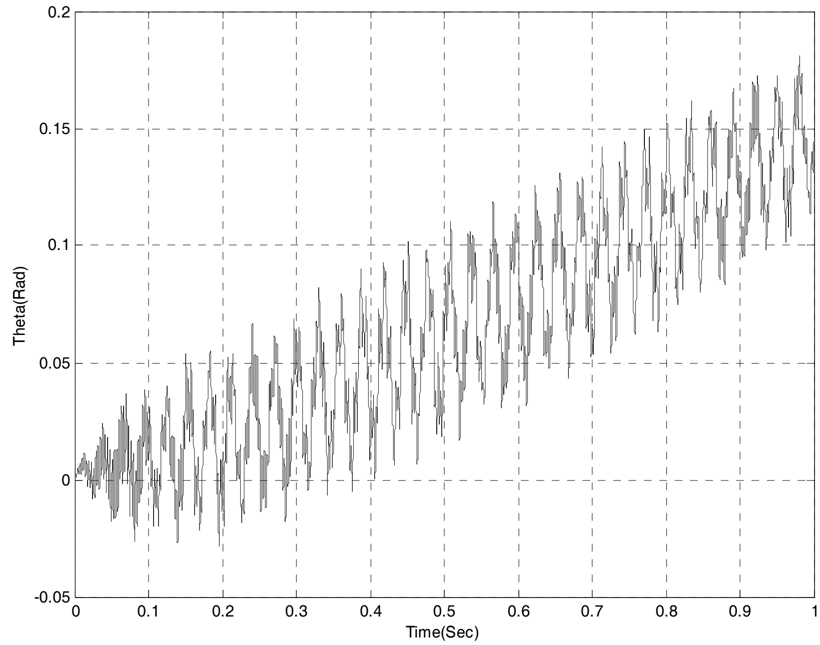

Fig. (5): Joint angle variations versus time at second joint.

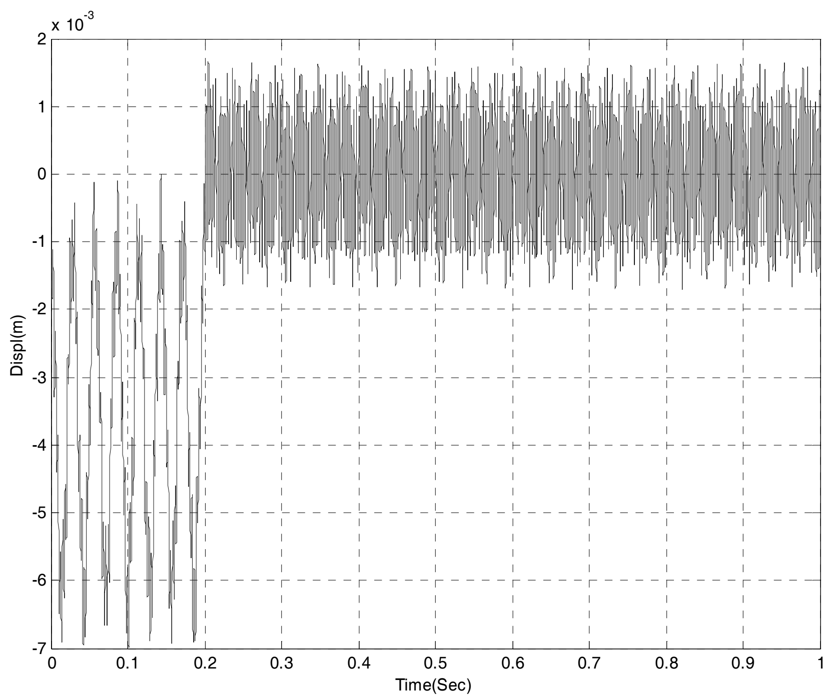

Fig. (6): End point displacement of the first link.

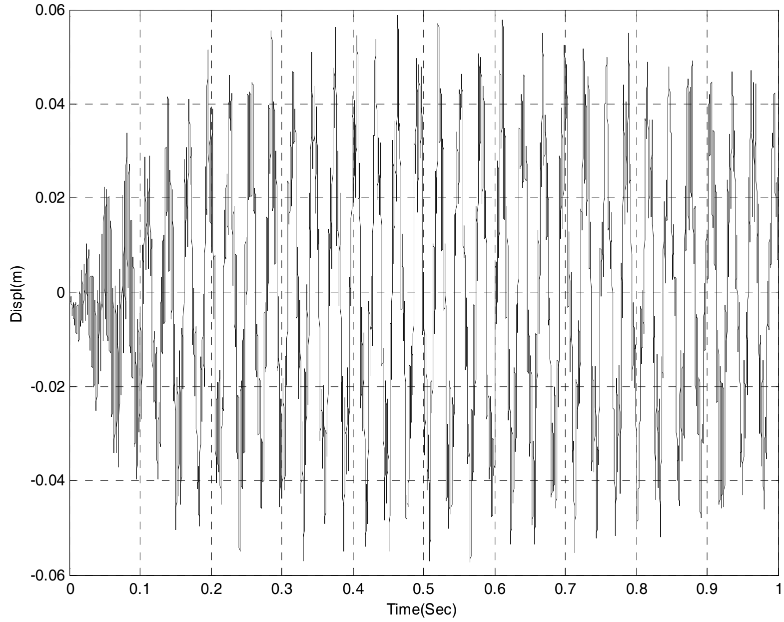

Fig. (7): End point displacement of the second link.

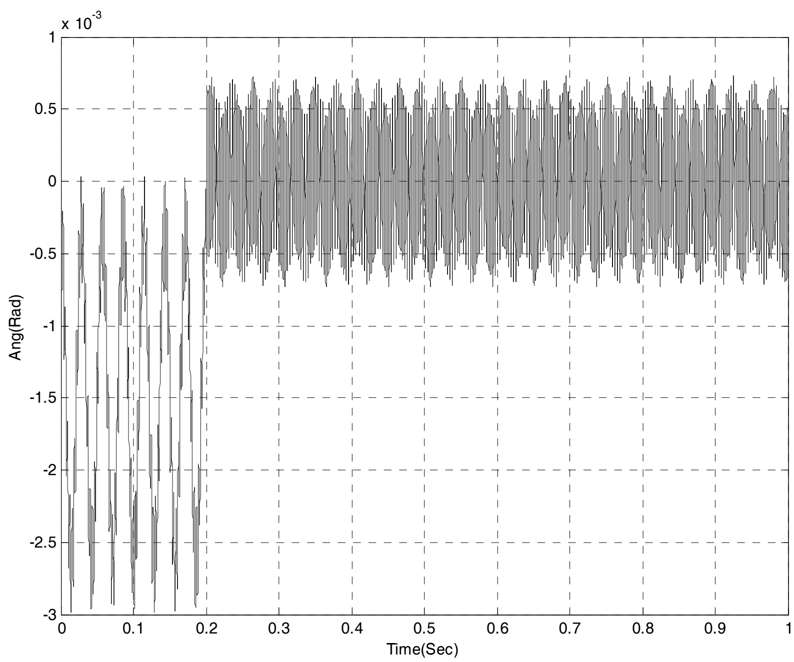

Fig. (8): Rotation of tangent line at the end point of first link.

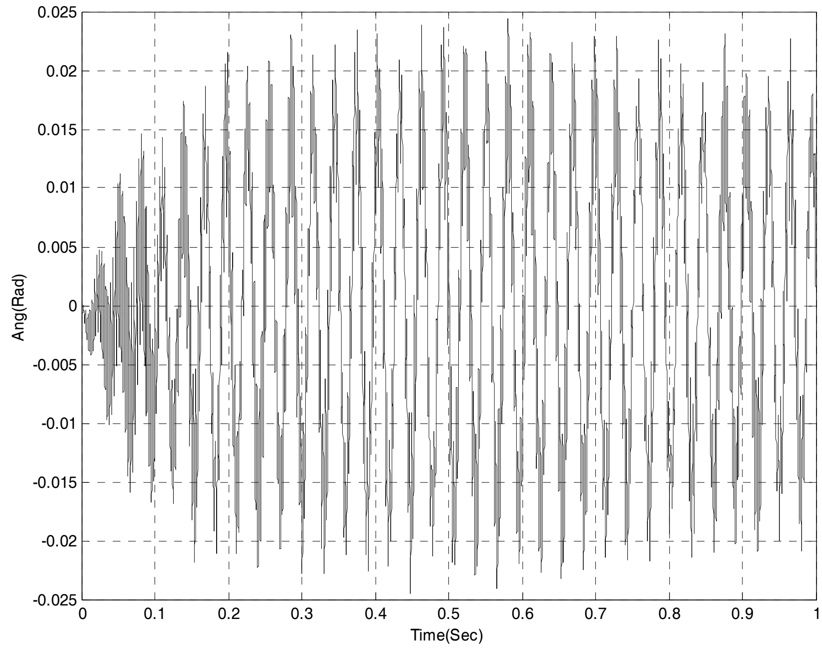

Fig. (9): Rotation of tangent line at the end point of second link.

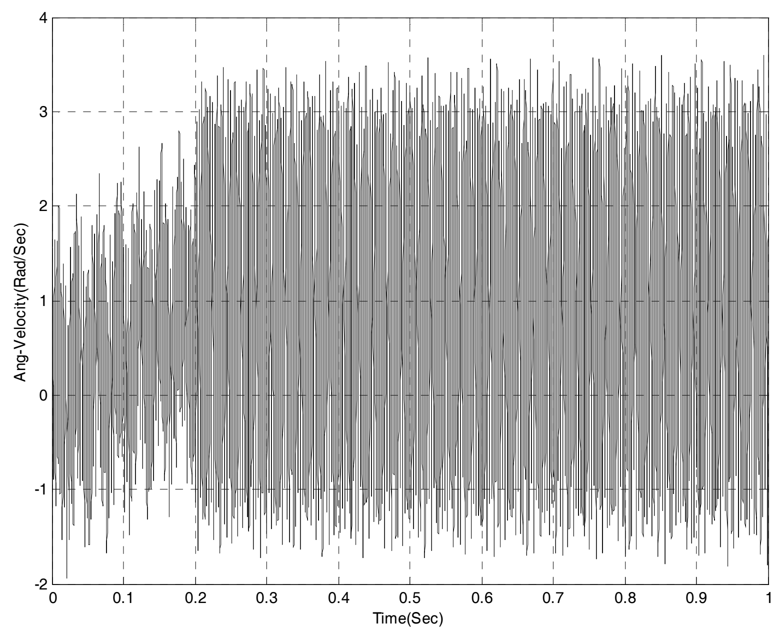

Fig. (10): Angular velocity of the first link.

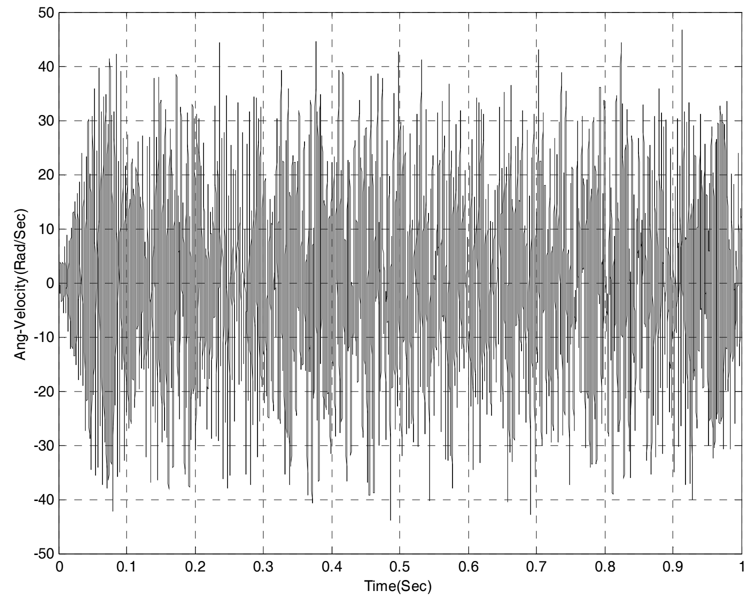

Fig. (11): Angular velocity of the second link.

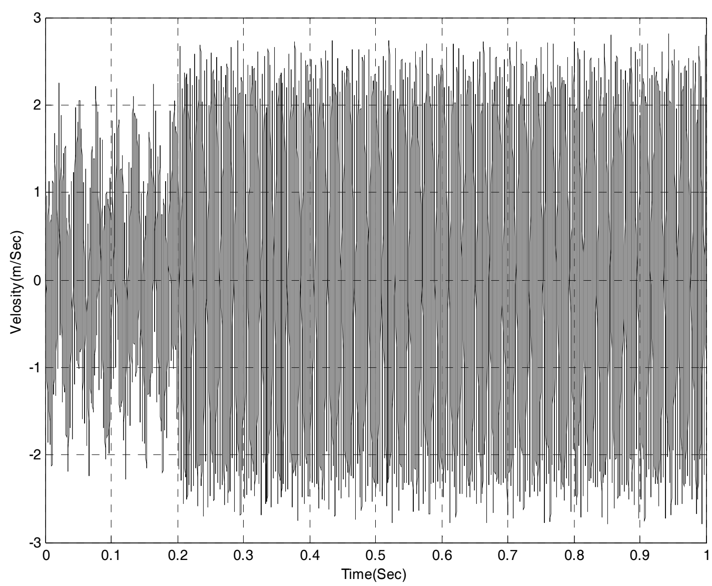

Fig.(12): Linear velocity of the end point of first link.

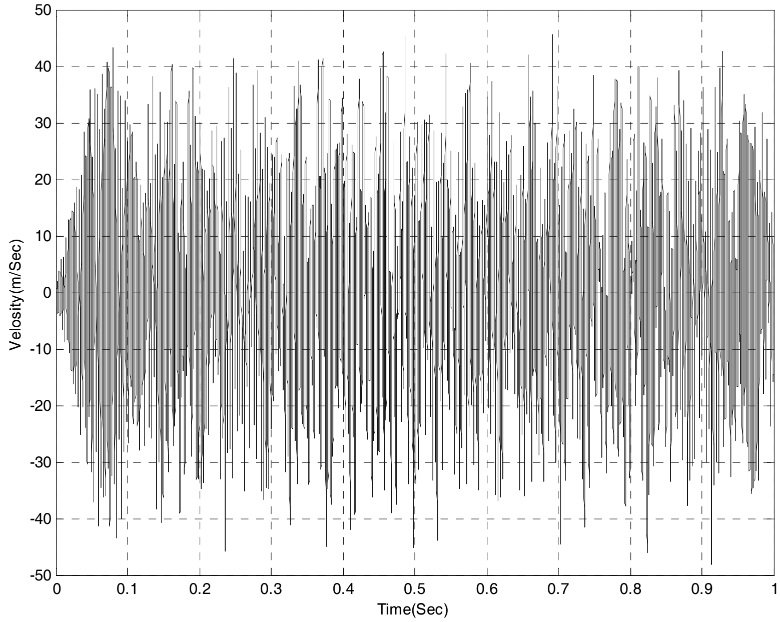

Fig. (13): Linear velocity of the end point of second link.

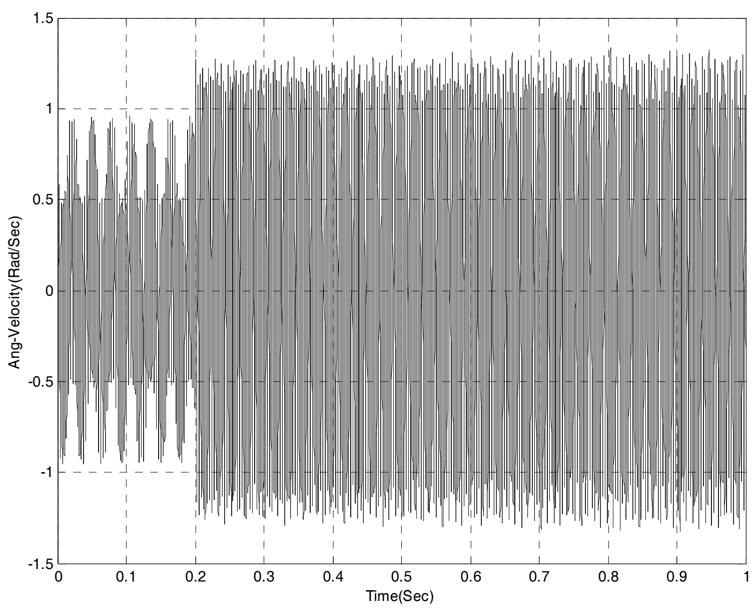

Fig. (14): Angular velocity at the end of first link.

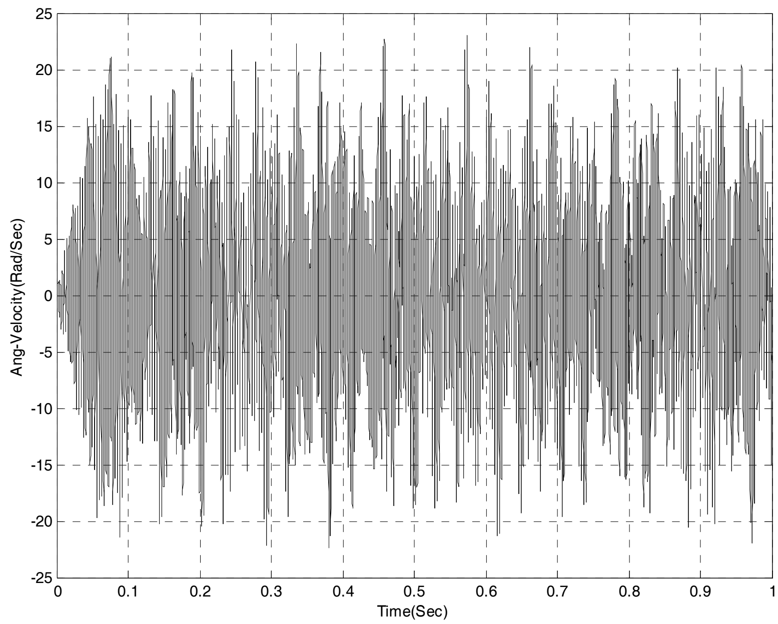

Fig. (15): Angular velocity at the end of second link.

4. Conclusions

In this paper, a new procedure is described for nonlinear finite element modeling of flexible manipulator by which the dynamic equations are deduced. The procedure for modeling a flexible manipulator uses the kinetic and potential energies and the Hamilton principle to derive the dynamic equations. The square torque is then applied to the joints of the flexible manipulator to simulate of their dynamic behavior. for a two degrees of freedom manipulator, there is no fluctuation in the angle of the first member, however, the variation of the angle for the second member is considerate. Thus, the flexible manipulator doesn't perform its task accurately. The deviation of the second member is more than the first one. Thus, the error for the exact determination of the end–effector position is important. The reason of the higher vibration in the second link is its connection to the end of the first one and also its superimposed vibration.

5. References

1. Book, W.J., "Recursive Lagrangian Dynamics of Fexible Manipulator Arms", Int. J. Robotic Research, Vol. 3, No. 3, pp. 87-101, 1984.
2. Usoro, P.B., Nadira, R., and Mahil, S.S. "A Finite Elements Lagrange Approach to Modeling Lightweight Flexible Manipulator" ASME J. Dynamics Systems, Measurements and Control, Vol. 108, No. 2, pp. 198-205, 1986.
3. Low K. and Vidyasagar, M. "A Lagrangian Formulation of the Dynamics Model for Flexible Manipulator System", ASME J. Dynamics Systems, Measurements and Control, Vol. 110, No. 1, pp. 175-181, 1988.
4. Wen, C. "Dynamic Modeling of Multi-Link Flexible Robotic Manipulators" J. Computers and Structures, Vol. 79, No. 2, pp. 183-195, 2001.
5. Zhang, X. and Yu, Y. "A New Spatial Rotor Beam Element for Modeling Spatial Manipulators with Joint and Link Flexibility", J. Mechanism and Machine Theory, Vol. 35, No. 3, pp. 403-421, 2000.
6. Plosa, J. and Wojciech, S. "Dynamics of Systems with Changing Configuration and with Flexible Beam-Like Links", J. Mechanism and Machine Theory, Vol. 35, No. 11, pp. 1515-1534, 2000.
7. Biswas S.K. and Klafer, R.D. "Dynamic Modeling and Optimal Control of Flexible Robotic Manipulators", IEEE J. Robotics and Automation, Vol. l, No. 1, pp. 15-23, 1988.
8. Pfeiffer, F. and Gebler, B. "A Multistage-Approach to the Dynamics and Control of Elastic Robots", IEEE, J. Robotics and Automation, Vol. 1, No. 1, pp. 2-12, 1988.
9. Everett, L.J. "Equation of Motion of Flexible Manipulators", J. Robotic System, Vol. 1, No. 1, pp. 553-566, 1988.
10. Readman, M.C. and Belanger, P.R. "Analysis and Control of a Flexible Joint Robot", IEEE, The 29th Conf. on Decision and Control, Vol. 4, No. 2, pp. 2551-2563, 1990.