

# Identification of Railway Car Body Model Using Operational Modal Analysis

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## Abstract

In this paper operational modal analysis technique is employed in model identification of a railway car body structure which uses recorded responses of the structure for identification of modal parameters of the body. Since excitation of the railway car body with conventional shakers is difficult due to the large mass of the structure, the classical modal analysis techniques are not commonly used for mode identification of these structures. To overcome this problem a different technique known as Operational Modal Analysis (OMA) which uses responses of the structure during its working conditions is used. Two sets of operational measurements were performed on the structure and results are used to extract mode shapes and natural frequencies of the structure. Validation of obtained modes is performed by comparing the modes obtained from the two sets of measurements.

**Keywords:** Railway, Car Body, Identification, Operational Modal Analysis.

## 1. Introduction

Studies on railway dynamics have been performed for almost a century. Identification of modes of the car body to obtain more reliable modeling is one of the problems in railway dynamic [1-3]. Accurate models are required to give the possibility to capture effects such as determination of contact force between wheel and rail. In fact increasing accuracy of model is something that has always been favorable among researchers and as a result different methods have been proposed to increase modeling accuracy.

One way of obtaining accurate models is to validate and update the model using experimental observations. Updating process needs an objective function which is routinely characterized by the error between results of simulation and measurement [4] and model is then updated using conventional identification methods. For large structures such as a railway wagon, non-parametric system identification methods like those used in classical modal analysis are preferred upon parametric methods. Parametric methods are mainly used to identify transfer function between two points of the structure; input and output points. For large structures more than few transfer functions are likely to be identified to guarantee the accuracy and other purposes of the modeling. Unfortunately all of the classical modal analysis methods are limited to the fact that one should be able to excite the structure with some standard devices such as hammer or shaker [5]. As a result if it is not possible to excite the structure properly with these devices, classical modal analysis methods fail to be in hand for the case.

In such conditions different techniques have been proposed currently which do not need to excite the structure with known inputs [5]. These methods make use of the responses of the structure to natural inputs which excite the system during its working condition so there is no need to shut down the system and restart the identification procedure with desired forces [6-9]. Although there is a special condition in which these methods have their maximum efficiency mainly when the input to the system guarantees excitation of all of the natural frequencies or those in the range of interest such as a white noise.



Figure 1. Picture of the tested wagon

Keeping in mind above descriptions it seems that conditions of a railway wagon satisfy all of the above requirements. The interaction force between wheel and rail is abound with small impacts therefore is possible to excite all of the modes under 45Hz of the structure which is the desired frequency range for identification of operational modes of the structure. Based on the above discussion two sets of measurements are performed on the structure. First set is recorded while wagon has been running with constant speed of 80Km/h in a newly built straight railway. This test case is referred as "dynamic test" in the paper. Although results of the dynamic test are sufficient for purpose of identification of mode shapes and natural frequencies, a static test is also performed for verification of the results of the dynamic test. Details of static test conditions and measurements are explained in the following sections.

## **2. Identification of Car Body Structure**

As described before for identification of wagon structure 2 sets of measurements are performed. In both of the tests accelerations of 20 points on the wagon have been recorded using 20 uni-axial accelerometers. Locations and directions of accelerometers are shown in Figure 1.

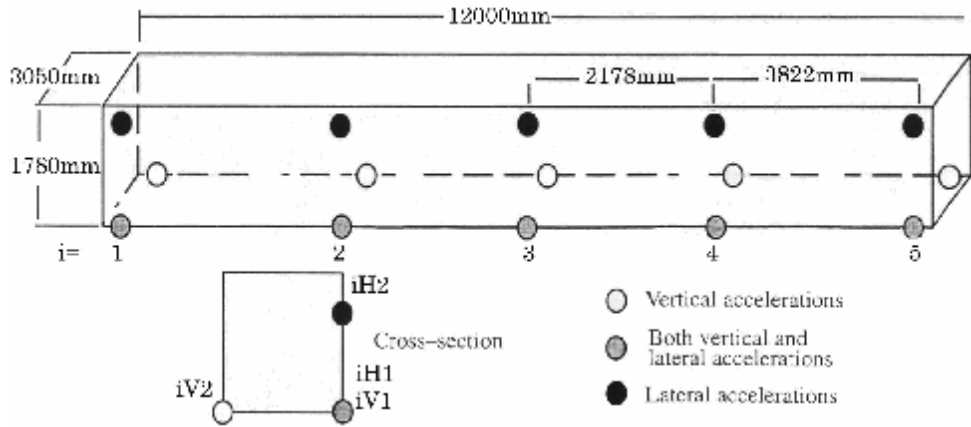


Figure 1. Location and direction of the uni-axial accelerometers

Both dynamic and static tests are performed with a data logger which records signals with sampling frequency of 2KHz. Using the data logger accelerations of the specified points are recorded for about 30 seconds. The static test was performed while the wagon has been at rest and excited by a big hammer. Since it was not suitable to target the wagon directly with hammer due to its destructive effects on the body and even poor excitation of suspension modes, the hammer targeted a lever located between bogie and wagon. This configuration helps with better excitation of suspension modes. For both tests the Frequency Domain Decomposition (FDD) method [6-10] is used for identification of natural frequencies and mode shapes of the structure and results of both tests are compared together. Dynamic test was performed while the train was running with speed of 80km/h. Since there is an excitation from rail track to the structure responses of the structure are measured. Two samples of the recorded data are shown in figure 2. Left hand side picture shows recorded accelerations of accelerometer 1 during static test while right hand side picture shows response of the same accelerometer whilst the train has been running through its track for dynamic test. For the purpose of identification recorded data of the first 3 seconds are taken into account.

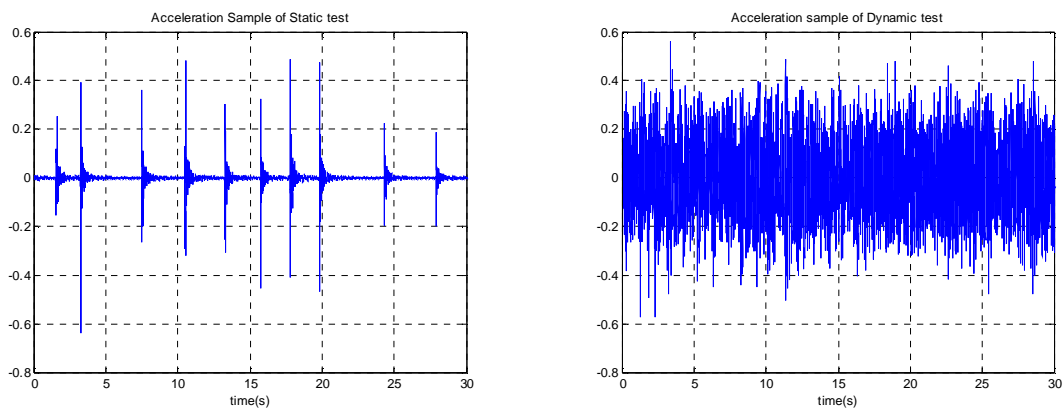


Figure 2. Sample of recorded accelerations

Left: response of the 1<sup>st</sup> accelerometer during static test

Right: response of the 1<sup>st</sup> accelerometer during dynamic test

One of the widely used methods for Operational Modal Analysis is called Frequency Domain Decomposition (FDD) which uses the classical Complex Mode Indicator Functions (CMIF) together with Cross Power Spectral Density matrix of measured

responses [9-10]. In this method after calculation of correlation matrix in each spectral line, curves of singular values and their dependent singular vectors are used to calculate natural frequencies and mode shapes of the structure. The cross spectral density of two signals  $\{s(t)\}$  and  $\{w(t)\}$  is defined as:

$$R_{sw}(t) = E\{s(t).w(t-t)\} \quad (1)$$

$$j_{sw}(w) = \sum_{t=-\infty}^{\infty} R_{sw}(t)e^{-iwt} \quad (2)$$

Curve of the singular values demonstrate a peak near natural frequencies while right singular vectors in each spectral line estimate the related mode shape of that frequency. Using  $y(t)$  for outputs or responses of the system and  $f(t)$  for inputs or forces of the system and by referring to *Modal Expansion* theory the input-output relation can be written in the following form form:

$$\{y(t)\} = [\Phi]\{q(t)\} \quad (3)$$

$$[C_{yy}(t)] = E\{y(t)*y(t+t)^T\} \quad (4)$$

Where,  $E\{s\}$  is expected value of vector  $\{s\}$ ,  $\{q(t)\}$  is vector of time dependent modal coefficients,  $[C_{yy}(t)]$  is the correlation matrix, and  $\{*\}$  sign denotes convolution integral. Substituting  $y(t)$  from modal expansion theory into the correlation formulation one obtains:

$$\begin{aligned} [C_{yy}(t)] &= E\{[\Phi]q(t)*q(t+t)^T[\Phi]^T\} \\ &= [\Phi]E\{q(t)*q(t+t)^T\}[\Phi]^T = [\Phi][C_{qq}(t)][\Phi]^T \end{aligned} \quad (5)$$

By taking Fourier transform of the correlation matrix of equation (5) following expression is obtained in frequency domain. The same relation holds for singular values of the correlation matrix shown in equation (7):

$$[H_{yy}(w)] = [\Phi][H_{qq}(w)][\Phi]^T \quad (6)$$

$$[H(w)] = [U][\Sigma(w)][U]^T \quad (7)$$

It should be noted that vectors of  $[U]$  are orthogonal and unitary. In this situation, vectors of  $[U]$  estimate mode shapes of the structure and the number of non-zero diagonal singular values is a useful indicator for determination of rank of the system.

Figure 3 shows first CMIF curve of the static and dynamic tests plotted linearly to show its peaks in an amplified manner. It is obvious that both plots have some peaks in common. These repetitive peaks represent primary candidates for natural frequencies keeping in mind that forces are different in two tests. First all of the strong peaks and also those which are appeared commonly in both plots are selected. Figure 4 shows CMIF curve of dynamic test; as expected this curve has more fluctuations than curve of the static test remembering that contact force between rail and wheel is always covered with a significant amount of impacts.

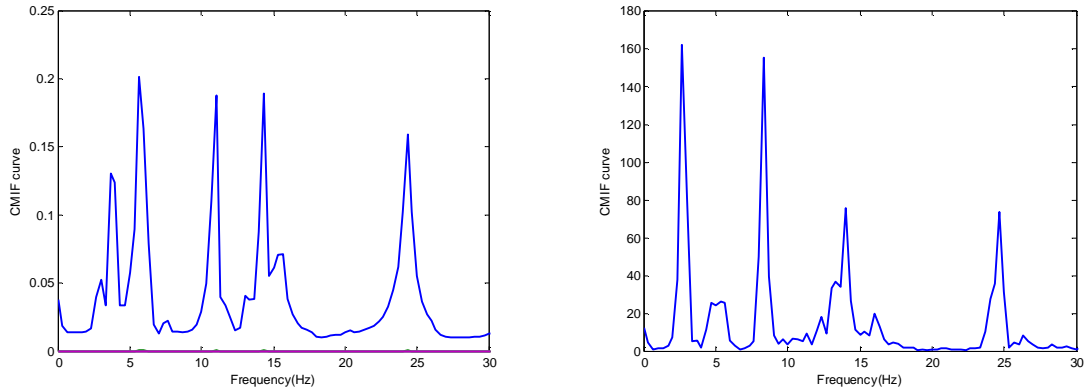


Figure 3. Curves of CMIF for static and dynamic tests  
 Left: CMIF of static test , Right: CMIF of dynamic test

Table 1 shows identified natural frequencies of the system from both static and dynamic test. In the static test some modes are not excited significantly like 4<sup>th</sup> mode; this is due to the excitation type which was not able to excite system around this mode. Another criterion which is used for distinguishing natural modes is the AutoMAC of derived mode shapes described in section 3.

### 3. Mode Extraction Procedure

Orthogonality of modes of a structure with uniform mass distribution requires that the extracted mode shapes should have MAC values near zero with each other. MAC values between preliminary mode shapes are calculated and plotted in Figure 5. Some of the modes seem to have high MACs; these might be operational deflection shapes of the structure and not the mode shapes. To make sure of identified mode shapes these deformed shapes were rejected from preliminary candidates of modes while others with low MAC number are retained. Using this method, 3 candidates with high MAC value are rejected and 5 modes of the structure are identified. Natural frequencies of identified modes are shown in table 1 and their corresponding mode shapes are shown in figure 5.

mode number	Static test	Dynamic test	Description
1	3	2.66	1st mode
2	3.66	4.66	2nd mode
3	5.66	5.66	ODS <sup>1</sup>
4	-	8.33	3rd mode
5	11	12.33	ODS
6	14	14	4th mode
7	15.66	16	ODS
8	24.33	24.66	5th mode

Table 1. Identified modes from static and dynamic tests.

<sup>1</sup> Operational Deflection Shape

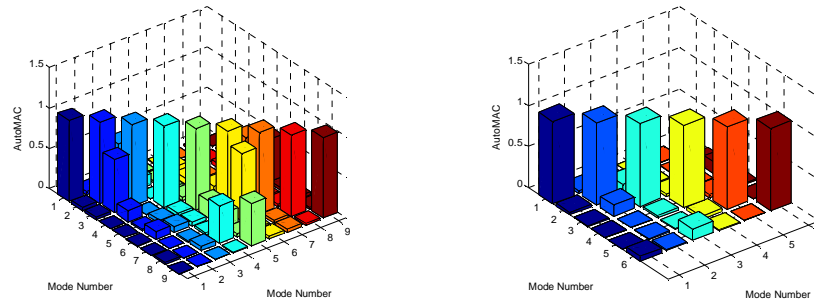
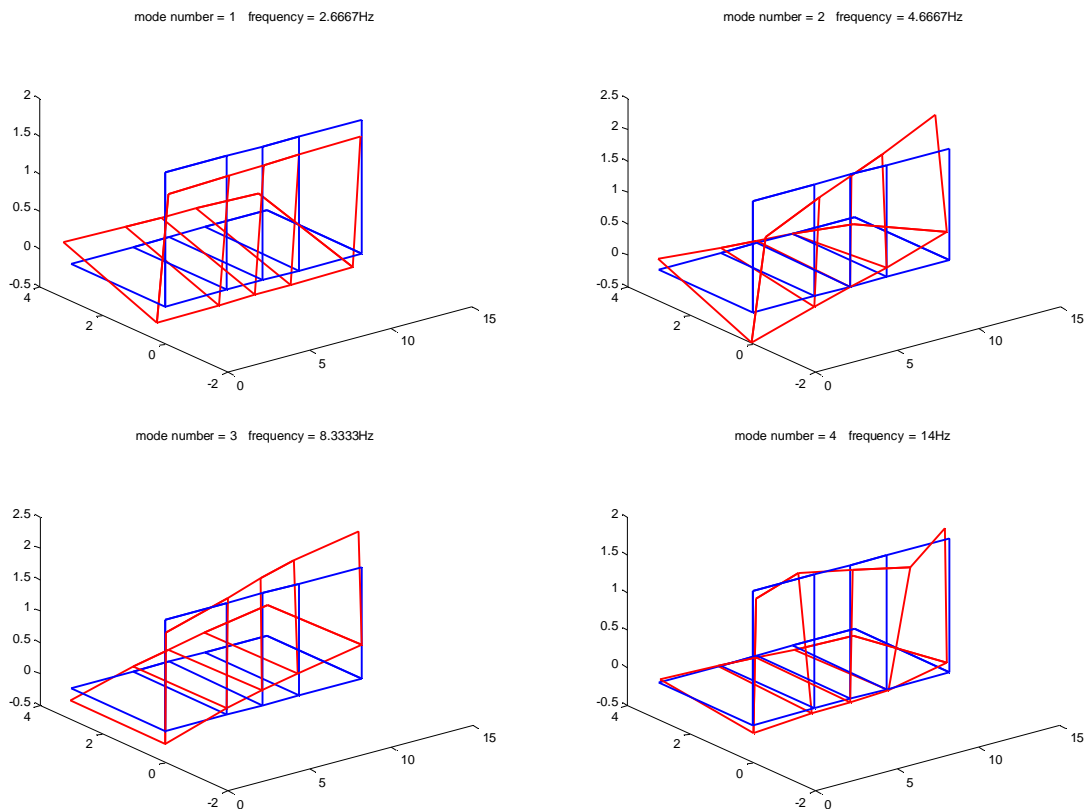


Figure 4. Plot of MAC before (left plot) and after (right plot) selecting the modes

Figure 4 shows plots of MAC between identified mode shapes before and after rejection of deformed shapes. Improvement of MAC plot is remarkable in this figure. Figure 5 shows the 5 identified modes which 3 of them are elastic modes and remaining 2 are suspension modes. The first mode occurs at 2.66Hz which represents a strong roll mode of wagon on its suspension system. Second mode is the first elastic mode of the body and is a twisting mode of body. Third mode like the first mode is a suspension pitch mode while higher modes are elastic modes due to vibration of side panels of the wagon. According to the results, suspension and elastic modes of the structure are highly coupled together and are mixed with each other. Identified modes can be used for more accurate modeling of the structure which is the primary requirement for determination of the induced force to the wagon.



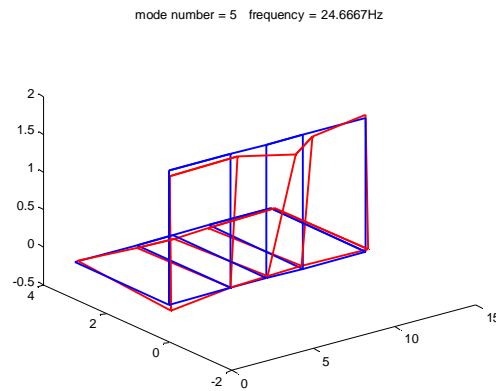


Figure 5. Selected mode shapes of static and dynamic tests

#### 4. Conclusion

Two sets of vibration measurements are performed on a wagon structure comprising a dynamic test during running of a car body on its track and another static test while the wagon has been at rest. Modal properties of the structure are identified using the FDD method on time histories of recorded accelerations of the body. Time histories with duration of 3 seconds for both static and dynamic tests are used for identification task using FDD method. Each test introduces some candidates for natural frequencies but some of them are common in both tests. Common frequencies are selected as actual modes of the structure since in two different excitations they have been present so can not be operational deformed shapes. Another criterion is also used for modification of candidates which is the AutoMAC criteria between identified modes. Eventually five modes are identified comprising two suspension modes and three elastic modes of the car body.

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