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GROUPS WHOSE ELEMENTS COMMUTE WITH THEIR ENDOMORPHIC IMAGES

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Abstract. A group G is called an E-group if the Near-ring generated by the endomorphisms of G in the near-ring of maps on G is a ring. It is well known (see, e.g., Malone, 1995) that a group G is an E-group if and only if each element commutes with its endomorphic images. For any prime number p, we call an E-group which is also a p-group, a pE-group. In this paper at first we explain general properties of $E-$ groups. Also we prove that an infinite finitely generated E-group is the direct product of a central torsion-free subgroup and a finite subgroup. Next, we prove that there is no $3E$-group of nilpotency class 3 of order at most 3^{10}. Also we construct a group of class 3 which is "very close" to be an E-group.

The following questions are central ones in this paper:

(1) What is the least number of generators of a finitely generated non-abelian E-group?

(2) What is the minimum order of a finite non-abelian pE-group?

We prove that the minimal number of generators of a finitely generated non-abelian E-group is 4.

In response to the question (2), we prove that the minimum order of a finite non-abelian pE-group is p^8, for any odd prime number p and this order is 2^7 for $p = 2$.

Also we obtain a new class of E-groups.

As we have found that some of our results are valid for a very larger class of finite p-groups than pE-groups, we study a class of p-groups for every prime number p and we denote this class of p-groups by $p\mathcal{E}$. (A finite p-group G is called a $p\mathcal{E}$-group if G is a 2-Engel group and all elements of order at most p^r lie in the center of G, where p^r is exponent $\frac{G}{G'}$). We classify all 3-generator $p\mathcal{E}$-groups and $p\mathcal{E}$-groups with cyclic derived subgroup and determine endomorphisms of 3-generator $p\mathcal{E}$-groups and pE-groups. Finally we classify all pE-groups and $p\mathcal{E}$-groups of order at most p^7 for any prime number p.

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