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CENTRAL AUTOMORPHISMS OF SEMIDIRECT PRODUCT OF FINITE GROUPS

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Abstract. In this paper we find the order and structure of $\mathrm{Aut}_c(G)$ for $G = K \rtimes H$ by imposing some conditions on H and K. As a result, we show that if M is a non-normal maximal subgroup of a solvable group G, such that all central automorphisms of G fix $Core_G(M)$ pointwise, then $\mathrm{Aut}_c(G)$, is meta-abelian.

1. Introduction

An automorphism σ of a group G is central if σ commutes with every automorphism in $\mathrm{Inn}(G)$, the group of inner automorphisms of G, or equivalently, if $g^{-1}\sigma(g)$ lies in the center $Z(G)$ of G, for all g in G. The central automorphisms fix the commutator subgroup G' of G pointwise, and form a normal subgroup, denoted by $\mathrm{Aut}_c(G)$, of the full automorphism group $\mathrm{Aut}(G)$. The group of central automorphisms of a finite group G is of great importance in the investigation of $\mathrm{Aut}(G)$, and has been studied by several authors (see, for example, [1-6]).

In [1] Adney and Yen has shown that if G is a finite purely non-abelian group then $|\mathrm{Aut}_c(G)| = |\mathrm{Hom}(G/G', Z(G))|$. Suppose $G = K \times H$ where K is purely non-abelian and H an abelian subgroup of G. In [5] Jamali and Jafari introduced some subgroups for $\mathrm{Aut}_c(G)$ to find order and structure of $\mathrm{Aut}_c(G)$.

In this paper we find the order and structure of $\mathrm{Aut}_c(G)$ for $G = K \rtimes H$ by imposing some conditions on H and K. As a result, we show that if M is a non-normal maximal subgroup of a solvable group G, such that all central automorphisms of G fix $Core_G(M)$ pointwise, then $\mathrm{Aut}_c(G)$, is meta-abelian.

In this talk, G is a finite group, π_H and π_K are the projection maps from G into H and K respectively and σ_H, σ_K are restriction of σ on H and K

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respectively. Also we set:

$R = \{\pi_H \sigma_H | \sigma \in \mathrm{Aut}_c(G)\}$ $S = \{\pi_K \sigma_K | \sigma \in \mathrm{Aut}_c(G)\}$

$T = \{\pi_K \sigma_H | \sigma \in \mathrm{Aut}_c(G)\}$ $U = \{\pi_H \sigma_K | \sigma \in \mathrm{Aut}_c(G)\}$

2. Main results

Theorem 2.1. *Let $G = K \rtimes H$ be a semidirect product, such that $(|H|, |K|) = 1$. then $\mathrm{Aut}_c(G) \cong R \times S$. Furthermore, if $Z(H)$ acts trivially on K then $\mathrm{Aut}_c(G) \cong \mathrm{Aut}_c(H) \times S$. In particular if $G = K \times H$, then $\mathrm{Aut}_c(G) \cong \mathrm{Aut}_c(H) \times Aut_c(K)$.*

Corollary 2.2. *Let G be a solvable group. If M is a non-normal maximal subgroup of index p^t such that $p \nmid |\frac{M}{C}|$ and $(|M/C|, |C|) = 1$, where $C = Core_G(M)$ then we can write $G = K \rtimes H$, for which $(|K|, |H|) = 1$ and $H \leqslant M$. Hence $\mathrm{Aut}_c(G) \cong R \times S$.*

Theorem 2.3. *Let $\frac{G}{N} = \frac{K}{N} \rtimes \frac{H}{N}$ be a semidirect product and the central automorphisms fix the subgroup N of G pointwise and $(|\frac{H}{N}|, |\frac{K}{N}|) = 1$. then $\mathrm{Aut}_c(G) \cong R \times S$.*

Theorem 2.4. *Let $G = K \rtimes H$ be a semidirect product, such that $\mathcal{C}_K(H) = 1$. Then $\mathrm{Aut}_c(G) \cong R \rtimes U$.*

Theorem 2.5. *Let $\frac{G}{N} = \frac{K}{N} \rtimes \frac{H}{N}$ be a semidirect product and the central automorphisms fix the subgroup N of G pointwise and $\mathcal{C}_{K/N}(H/N) = 1$. then $\mathrm{Aut}_c(G) \cong R \rtimes U'$ where $U' = \{f_\sigma \,|\, f_\sigma \in \mathrm{Hom}(K, Z(G) \cap H), \sigma \in \mathrm{Aut}_c(G)\}$.*

Corollary 2.6. *Let G be a solvable group with non-normal maximal subgroup M. If all central automorphisms fix $Core_G(M)$ pointwise, then $\mathrm{Aut}_c(G) \cong R$ and so is abelian.*

Theorem 2.7. *Let $G = K \rtimes H$ be a semidirect product, where K is purely non-abelian and $Z(H)$ acts trivially on K. then $\mathrm{Aut}_c(G) \cong RTUS$.*

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