Tarbiat Moallem University, 20^{th} Seminar on Algebra,
2-3 Ordibehesht, 1388 (Apr. 22-23, 2009) pp 190-193

PRIME FUZZY SUBNEXUSES

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Abstract. The notion of prime fuzzy subnexus of a nexus is defined, and some related result are obtained. In particular by considering the concept of homomorphism some theorems about the coimage and preimage are proved.

1. Introduction and preliminaries

The space Structure Research Center of the university of Surrey was founded by Z.S. Makoswski as part of Department of Civil Engineering in 1963. The aim of the Center is to carry out research into the design and analysis of space structures. Space structures include structural forms such as single and double layer girds,barrel vaults,shells and various forms of tension structures.
The basic idea of a nexus has been further developed as a mathematical object for general use. The aim of recent study has been to evolve a mathematical object that allows complex processes on groups of mathematical objects to be formulated with ease of elegance. This notion is very useful for study of space structure. In this paper the notion of a prime fuzzy subnexus of a nexus is defined and some related results are obtained.

Definition 1.1. [2](i) An address, is a sequence of $\aleph^* = \aleph \cup \{0\}$ such that $a_k = 0$ implies that $a_i = o$ for all $i \geq k$. the sequence of zero is called the empty address and denoted by (). In other word, every nonempty address is of the form

$$(a_1, a_2, \ldots, a_n, 0, 0, \ldots)$$

Where n are belong to $\aleph$.
Hereafter, this address will be denoted by

$$(a_1, a_2, \ldots, a_n)$$

(ii) A nexus N is set of address with the following properties:

$$(a_1, a_2, \ldots, a_n) \in N \Rightarrow (a_1, a_2, \ldots, a_{n-1}, t) \in N, \quad \forall\, 0 \leq t \leq a_n$$

$$\{a_i\}_{i=1}^{\infty} \in N, a_i \in \aleph \Rightarrow \forall n \in \aleph, (a_1, a_2, \ldots, a_n) \in N$$

Definition 1.2. [2] Let $w \in N$.The level of w is said to be :
(i) n, if $w = (a_1, a_2, \ldots, a_n)$, for some $a_n \in \aleph$,
(ii) ∞, if w be an infinite sequence of $\aleph$,
(iii) 0 if $w = ()$.
The level of w is denoted by $l(w)$.

Definition 1.3. [2] let $w = \{a_i\}\ \ i \in \aleph$ and $v = \{b_i\}\ \ i \in \aleph$ be addresses. then $w \leq v$ if $l(w) = 0$ or one of the following cases satisfies:
Case 1. If $l(w) = 1$, That is, $w = (a_1)$ for all $a_1 \in \aleph$ and $a_1 \leq b_1$.
Case 2. If $1 < l(w) < \infty$, then $l(w) \leq l(v)$ and $a_{l(w)} \leq b_{l(v)}$ and for any $1 \leq i \leq l(w), a_i = b_i$.
Case 3. If $l(w) = \infty$, then $w = v$.
For example, consider the nexus

$$N = \{(), (1), (2), (1,1), (1,2), (1,3), (1,3,1), (1,3,2)\}$$

Then

$$(1) \leq (2), (1,2) \leq (1,3,1), (1,3,1) \leq (1,3,2).$$

Proposition 1.4. [2] *$(N, \leq)$ is a lower semi lattice.*

Proposition 1.5. [2] *Suppose that N is a set of addresses. Then N is a nexus if and only if, $v \in N$ and $w \leq v$ implies that,$w \in N$.*

Definition 1.6. [2] *A nonempty subset S of N is called a subnexus of N provided that S itself is a nexus. The set of all subnexus of N is denoted by $SUB(N)$.*

Definition 1.7. [2] Let $\emptyset \neq X \subseteq N$. Then the smallest subnexus of N containing X is called the sub nexus of N generated by X and denoted $\langle X \rangle$.

Definition 1.8. [1] A proper subnexus P of a nexus N is said to be a prime subnexus of N if $a \wedge b \in P$ implies that $a \in P$ or $b \in P$ for any $a, b \in N$.

Definition 1.9. [3] Let $\widetilde{P}$ be fuzzy subset of nexus N. Then $\widetilde{P}$ is called a fuzzy subnexus of N,if $w \leq v$ implies $\widetilde{P}(v) \leq \widetilde{P}(w)$ for all $v, w \in N$.

The set of all fuzzy subnexus of N is denoted by $FSUB(N)$.

2. Main results

Theorem 2.1. *Let A be a nonempty subset of nexus N.*
$A \in N$ if and only if $\chi_A \in FSUB(N)$. Where that χ_A is the characteristic function of A.

Theorem 2.2. *$\widetilde{P} \in FSUB(N)$ if and only if $\widetilde{P}_t \in SUB(N)$ for all $t \in [0,1]$, Where $\widetilde{P}_t \neq \emptyset$.*

Theorem 2.3. *Let N be a nexus and $\mathcal{A} = \{\widetilde{P}_\alpha \mid \widetilde{P}_\alpha \in FSUB(N)\}$. Then*
(i) $\bigcap_{\alpha \in I} \widetilde{P}_\alpha \in FSUB(N)$.
(ii) $\bigcup_{\alpha \in I} \widetilde{P}_\alpha \in FSUB(N)$.

Definition 2.4. Let N be a proper subnexus. $\widetilde{P}$ is called prime fuzzy subnexus if, $\widetilde{P}(a \wedge b) \leq max\{\widetilde{P}(a), \widetilde{P}(b)\}$ for all $a, b \in N$.
The set of all prime fuzzy subnexus of N is deoted by $FPSUB(N)$.

Example 2.5. Let

$N = \{(), (1), (2), (3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,1,1), (3,1,2), (3,2,1), (3,2,2)\}$.

Now define the fuzzy subnexus $\widetilde{P}$ of N as follows, $\widetilde{P}(()) = 1$ and if

$$w = (a_1, a_2, \ldots, a_n) \in N \quad , \quad \widetilde{P}(w) = \frac{1}{a_1 a_2 \ldots a_n}.$$

Then $\widetilde{P}$ is a fuzzy subnexus of N, but $\widetilde{P}$ is not prime. Becaus

$$\widetilde{P}((3,2) \wedge (3,1,1)) = \widetilde{P}(3,1) = \frac{1}{31} > max\{\widetilde{P}(3,2), \widetilde{P}(3,1,1)\} = \frac{1}{32}.$$

Example 2.6. let $N = \{(), (1), (2), (1,1), (1,2), (1,3), (1,3,1), (1,3,2)\}$.
Define.

$$\widetilde{P}(()) = \alpha_1, \widetilde{P}(1) = \widetilde{P}(2) = \alpha_2, \widetilde{P}(1,1) = \alpha_3$$
$$\widetilde{P}(1,2) = \alpha_4, \widetilde{P}(1,3) = \alpha_5, \widetilde{P}(1,3,1) = \alpha_6, \widetilde{P}(1,3,2) = \alpha_7$$

such that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_7$. Then $\widetilde{P}$ is a prime subnexus of N.

Proposition 2.7. *Let N be a cyclic subnexus and $\widetilde{P}$ is a fuzzy subnexus of N. Then $\widetilde{P} \in FPSUB(N)$.*

Definition 2.8. Let F be a mapping from M onto N, if $\widetilde{P}$ is a fuzzy subset of M. Then the fuzzy subset $\widetilde{Q}$ of N defined by

$$\widetilde{Q}(y) = \inf_{x \in F^{-1}(y)} \widetilde{P}(x)$$

For all $y \in N$. is called the coimage of M under F. Similarly,
if $\widetilde{Q}$ is a fuzzy subset of N. then the fuzzy subset $\widetilde{P} = \widetilde{Q}F$ of M. (i.e the fuzzysub set defined by $\widetilde{P}(x) = \widetilde{Q}(F(x))$ for all $x \in X$) is called the preimage of v under F.

Definition 2.9. A fuzzy subnexus $\widetilde{P}$ of N has inf property if for any subset T of N, there exists $t_0 \in T$ such that $\widetilde{P}(t_0) = \inf_{t \in T} \widetilde{P}(t)$.

Definition 2.10. Let M, N be two nexus. Function, $F : M \longrightarrow N$ is called homomorphism of nexuses if $w \leq v$ implies $F(w) \leq F(v)$ for all $w, v \in M$ and F is called a semilattice homomorphism of nexuses if
(i) F be a homomorphism of nexuses.
(ii) $F(w \wedge v) = F(w) \wedge F(v)$ for all $w, v \in M$.

Example 2.11. Let $M = \{(), (1), (2), (1,1)\}$, $N = \{(), (1), (2), (1,1), (2,1)\}$ and $F : M \longrightarrow N$ be a function such that
$F(()) = F((1)) = (1)$,
$F((1,1)) = F((2)) = (2)$.
(i) F is a homomorphism of nexuses.
(ii) F is not a semilattice homomorphism of nexuses.
(iii) $F(()) \neq ()$

Theorem 2.12. *An onto semilattice homomorphism coimage of a prime fuzzy subnexus with inf property is a prime fuzzy subnexus.*

Theorem 2.13. *Any onto homomorphism of nexuses preimage of a fuzzy subnexus is also a fuzzy subnexus.*

Theorem 2.14. *Any onto semilattice homomorphism of nexuses preimage of a prime fuzzy subnexus is also a prime fuzzy subnexus.*

Proposition 2.15. *Suppose* N *be a nexus and* $\widetilde{P}$ *a fuzzy subnexus arbitrary of* N.
(i) *If* N *be a chain then* $\widetilde{P}$ *is a prime fuzzy subnexus.*
(ii) *If* $\widetilde{P}$ *be a prime fuzzy subnexus and one to one then* N *is a chain.*

Proposition 2.16. *Let* B *be a subnexus of nexus* N. *Then* $B \in PSUB(N)$ *if and only if* $\chi_B \in FPSUB(N)$. *Where that* χ_B *is charatristic function of* B.

Theorem 2.17. *Suppose* N *be a nexus. Then* $\widetilde{P} \in FPSUB(N)$ *if and only if* $\widetilde{P}_t \in PSUB(N)$ *for all* $t \in [0,1]$. *Whenever* $\widetilde{P}_t \neq \emptyset$.

Theorem 2.18. *Suppose* N *be a nexus and* $\mathbf{B} = \{\widetilde{P} \mid \widetilde{P} \in FPSUB(N)\}$.
(i) $\bigcap_{\alpha\in I} \widetilde{P}_\alpha \in FPSUB(N)$.
(ii) $\bigcap_{\alpha\in I} \widetilde{P}_\alpha \in FPSUB(N)$.

References

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