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# A PRESENTATION FOR EXTENDED AFFINE LIE ALGEBRAS

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## 1. INTRODUCTION

For a complex finite dimensional simple Lie algebra  $\mathscr{G}$  and a field  $\mathbb{K}$ , one can define a Lie algebra  $\mathscr{G}(\mathbb{K}) := \mathscr{G}_{\mathbb{Z}} \otimes_{\mathbb{Z}} \mathbb{K}$  over  $\mathbb{K}$  where  $\mathscr{G}_{\mathbb{Z}}$  is the *Chevalley*  $\mathbb{Z}$ -form of  $\mathscr{G}$  with respect to a given *Chevalley basis* of  $\mathscr{G}$ . In the case that the rank of  $\mathscr{G}$  is greater than 1 and  $ch(\mathbb{K}) \neq 2,3$ , Stienberg [St] proves that  $\mathscr{G}(\mathbb{K})$  is centrally closed and gives a presentation of  $\mathscr{G}(\mathbb{K})$  by generators and relations. Kassel [K] generalizes this concept by considering a unital commutative algebra A over a commutative ring Rin place of the field  $\mathbb{K}$  and defines the Lie algebra  $\mathscr{G}(A) := \mathscr{G}_{\mathbb{Z}} \otimes_{\mathbb{Z}} A$  over R. He proves that the universal covering algebra of  $\mathscr{G}(A)$  is  $\mathscr{G}(A) := \mathscr{G}(A) \oplus C$  where C is linearly isomorphic to  $\Omega_A^1/dA$ , the module of Kähler differentials of A modulo exact forms. He also gives a presentation of  $\mathscr{G}(A)$  by generators and relations. When  $R = \mathbb{C}$  and A is the algebra of Laurent polynomials in *n*-variables, the algebra  $\mathscr{G}(A)$  is called, by Moody, Rao and Yokonuma [MRY], an *n-toroidal* Lie algebra. They give an abstract infinite presentation of a 2-toroidal Lie algebra in terms of generators and relations involving the extended Cartan matrix of  $\mathscr{G}$ . They use their presentation to construct a great number of representations of  $\mathscr{G}(\mathbb{C}[t_1^{\pm 1}, t_2^{\pm 1}])$  for a simply laced algebra  $\mathscr{G}$ . Saito and Yoshii [SaY] introduce a class of Lie algebras whose cores are 2-toroidal Lie algebras. They call their class elliptic Lie algebras as they are used in the study of *elliptic singularities.* They give a Serre-type presentation of a simply laced elliptic Lie algebra in term of the elliptic Dynkin diagram (R,G) attached to its elliptic root system R (an extended affine root system of nullity 2) with marking G which is a rank 1 subspace of the radical of the semi-positive symmetric bilinear form defining R. Yamane [Ya] extends the presentation given by Saito and Yoshii to elliptic Lie algebras in general. More precisely, he gives a Serre-type theorem for the elliptic Lie algebras associated to the (reduced marked) elliptic root systems with rank greater than 2. A toroidal Lie algebra is centrally isogenous to the centerless core of an *extended affine* Lie algebra which is in turn a Lie torus. Now the question is whether one could find a (finite) presentation of the universal covering algebra of a Lie torus for a given nullity and type. In this work we give an affirmative answer to this question.

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### 2. MAIN RESULTS

**Theorem 2.1.** The universal covering algebra of a Lie torus of type  $X \neq A_1, C$ , is a finitely presented Lie algebra.

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