# Mathematics application in sound propagation modelling in sea water

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**Abstract**
Mathematics is an important science in Calculations related to professional and researching subjects[۲]. Sound propagation in sea water and its applications is being important and more important in science researching and fisheries tomography and solitary fronts[٤]. In this article, we are following the importance of mathematical solutions to problems related to sound propagation in sea water. The wave equation is solved and noise directionality is modelled from distributed surface noise sources over a uniformly sloping sea bed taking account of multipaths. With paying attention to this article and those similar, we will understand the importance of mathematics in calculations related to sound propagation systems and their applications.

**Keywords:** math, sound, propagation, modeling.

## Introduction

For being successful in facing with scientific, searching and professional section dependent on sound propagation in sea water, we must apply mathematical methods and theories. Numerical methods such as finite differences or finite elements are becoming increasingly popular in obtaining solutions to the elastic wave equation without making these assumptions. However, different methods frequently produce different results for the same problem, and it is essential to confirm the validity of a particular method before placing confidence in the results. Noise directionality (noise power per unit solid angles as a function of angle) is an important quantity for determining array performance. Directly multiplying it by and array's steered beam pattern and integrating over all angles gives the array's noise response for that steer direction[۳]. Sound as a mechanical wave propagates more rapidly in sea water rather than in air. We use sound signals in sea water for different purposes.We use mathematical calculations and formulas to problems related to it.Using this method, we coat the searches on application of sound propagation more better[٥].

Wave equation

Because of the simplicity and speed, a particular finite difference code can be greatly improved by judiciously simplifying the elastic wave equation, a number of different wave equations are treated by ismologists[٦]. Some of the wave equations which have been used for finite difference work are listed below:

$$\rho\ddot{u}=(\lambda+\mu)\nabla(\nabla.u)+\mu\nabla^{2}u+[\nabla\lambda(\nabla.u)+\nabla\mu\times(\nabla\times u)+2(\nabla\mu.\nabla)u] \quad (۱)$$

$$\rho\ddot{u}=\lambda\nabla(\nabla.u)+[\nabla\lambda(\nabla.u)]=\nabla(\lambda\nabla.u) \quad (۲)$$

$$\rho P=\lambda\nabla^{2}p+[(\lambda\rho\nabla(1/\rho)).\nabla p]=\lambda\rho\nabla.((1/\rho)\nabla p) \quad (۳)$$

$$\rho V=\mu\nabla^{2}V+[\nabla\mu.\nabla V]=\nabla.(\mu\nabla V) \quad (٤)$$

$$-\rho\omega^{2}\hat{u}=(\lambda+\mu)\nabla(\nabla.\hat{u})+\mu\nabla^{2}\hat{u}+[\nabla\lambda(\nabla.\hat{u})+\nabla\mu\times(\nabla\times\hat{u})+2(\nabla\mu.\nabla)\hat{u}] \quad (٥)$$

$-\rho\omega^{۲} \hat{p} = \lambda\nabla^{۲}p+[\lambda\rho\nabla(۱/\rho).\nabla\hat{p}]$ (٦)

The terms in brackets apply to heterogeneous media only. In the above equation, we find speed of sound with paying attention to viscosities of sea water. Density of sea water is important in variations of the speed. Pressure is an important parameter in locations of sound signals too which in different depths of water affects on values of sound speed. The above appointed parameters are u,λ,μ,ρ and arrangely. û is unit vector of sound propagating speed in water. ω, is angular frequency of sound signal[٦].

Formulation of noise directionality

Following Harrison's extention of the below relation to range and azimuth dependence, we define the noise directionality $D(\phi,\theta_r)$, at azimuth $\phi$ and elevation angle $\theta_r$ and for unit source strength per unit area, as the product of two terms:

$D(\phi,\theta_r)=Q(\theta_r)S(\phi,\theta_r)$ (۷)

The function Q is the residual attenuation due to bottom reflection (power reflection coefficient $R_b$ ) and volume absorption (a) between the last ray upper turning point and the receiver. For the upward ray the partial path length is $S_p$ and for the downward path it is $(S_c - S_p)$, where $S_c$ is the complete cycle path length. So for a path steep enough to hit the surface,Q is given by:

$Q(\theta_r)= e^{-aS_p}$ ;$\theta_r \geq ۰$

$=R_b(\theta_b)e^{-a(S_c-S_p)}$ ;$\theta_r < ۰$ (۸)

Central to this paper is the function S which represents the contribution along one ray from dipole sources.

$S(\phi,\theta_r) =\Sigma \sin(\theta_s)n \exp(-\Sigma Lj)$ (۹)

N n

With the attenuation factor $R_j$ per cycle given by:

$R_j = e^{-Lj} \equiv R_s((\theta s)j)R_b((\theta_b)j).\exp(-a(S_c)j)$ (۱۰)

Here the surface and bottom reflection coefficients $R_s$ and $R_b$ (evaluated at their respective grazing angels $(\theta_s)j$ and $(\theta_b)j$ are treated as symmetrical functions of angel . If we then take the joint boundary loss L to be either dominated by surface losses (or simply dependent on $\theta_s$)through $L= \alpha_s \sin\theta_s$ (with $\alpha_s$ being a constant) and treat the angels as a continuum we find (۹).

$S(\phi,\theta_r) = ۱/\alpha_s(۱ - \exp(- \int_0 \alpha_s \sin\theta_s\, dj))$ (۱۱)

This equation is valid for arbitrary refraction and bottom bathymetry. In isovelocity water or when rays are steep, the distinction between surface and bottom angles disappears and the surface subscript(s) can be dropped.

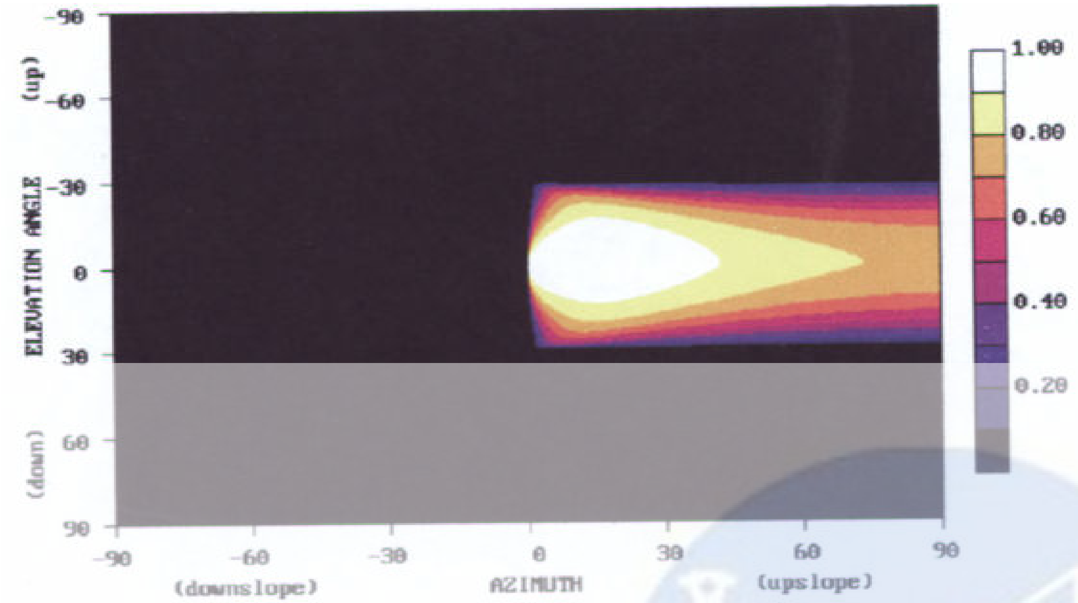
We take the bottom to be a tilted of (low) gradient εo, so that adopting a N×۲D approach the effective slope at a particular azimuth is given by $\varepsilon(\phi)= \varepsilon_o \cos\phi$. After each bottom bounce the ray angle is incremented or decremented by ۲ε $\varepsilon_o$ that θ and j are linearly related.We can then solve the integral, and looking downslope from the receiver we have:

$S_{down}(\phi,\theta_r) = ۱/\alpha(۱-e^{-(\alpha/۲|\varepsilon|)(\cos\theta_r-\cos\theta_c)})$ (۱۲)

Whereas going upslope from the receiver we have:

$S_{up}(\phi,\theta_r) = ۱/\alpha(۱-e^{-(\alpha/۲|\varepsilon|)(\cos\theta_r-\cos\theta_c)})$ (۱۳)

Where $\theta_c$ is a critical angle. It is now clear that neither $S_{up}$ nor $S_{down}$ can be greater than the range_ independent equivalent which is (۱/α)[۳].




Normalised noise directionality (αD) for a swathe of sources (distance between sources is equal to distance between sources line to receiver) bottom loss α = ۱dB/rad ; slope = ۰,۰۱

Intensity contours for normalized directionality (αD= αSQ)

Are shown in Cartesian (ϕ,θr)projection in Fig.٦. Parameters are: bottom slop $\varepsilon_o$ =۰/۰۱, bottom loss α=۰/۲۳ (i.e. l dB per radian), critical angle $\theta_c$=۳۰°. The intensity for angles steeper than $\theta_c$ at the receiver is obviously zero as indicated by the black area. The highes intensities (white) are seen slightly up_slope of across_slope. Upslope the weakest returns are in the horizontal. The up/down asymmetry is entirely due to bottom loss $R_b$ =exp(-αsin│θ│)in Q(eqn.۷) since absorption has been set to zero. At upward elevation angles greater than critical the formula reverts to that of Cron and Sherman (۱۰) leaving D=sin$\theta_s$[۳].

: Conclusions

The best tool mathematical modeling of sound propagation in sea water is solving the acoustical wave equation with numerical methods in spherical coordinate system that the most important parameters to address water domain are: molecular viscosity, density, sound wave speed and pressure in each point; μ, ρ, u and p.

With the above selections and solving the acoustic wave equation, we get form of pressure function of sound propagation under sea water as: P(x,t)=P[exp(-iωt)sin(x)] that exponential factor denotes time damping of sound propagation.

Mathematical modelling of sound propagation and numerical solving it show that sound signals path depends on sea surface slope and bottom slope for example if bottom slope ($\varepsilon_o$ =۰/۰۱) as a result bottom loss for sound following angular deviation from elementary propagation path (θ=۳۰°) will be ۰/۲۳ which it happens for more cases.

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