Numerical Simulation of Waves Generated by ships in Shallow Water

Mahmood Rostami Varnousfaaderani[1], Mohammad Javad Ketabdari[2]
AmirKabir University of Technology (AUT), Faculty of Marine Technology
Rostamivf@yahoo.com

ABSTRACT
Shallow-water ship-waves, known as wash waves, are important in marine engineering. These waves can affect coastal structures and change near-shore morphology. They can also cause damage to ship itself in finite depth channels. There are different theoretical methods to consider these waves. In this paper shallow-water ship-waves are simulated numerically. Applying Michell's thin-ship theory, flow field far from the ship is investigated. The relevant ship is considered thin and chosen from Series 60. The numerical simulation is performed in subcritical, critical and supercritical regimes for different depth Froude numbers, constant ship speeds and water depths. In this study the flow is considered incompressible and irrotational. However for the accuracy of simulation the effect of eddy viscosity is then considered. Furthermore the effects of the boundary layer are considered. The numerical results were compared with other models and experimental results. It showed that Michell's thin-ship theory could simulate this kind of waves with grate accuracy and reliability.

Keywords: Ship waves, Shallow water, Far field, Michell's thin-ship theory

1. Introduction

Shallow-water ship-waves, or wash waves can be generated by a fast ship at high speeds or by a large ship at moderate speeds, operating on a near-shore fairway or on an inland waterway. Generally these waves called vessel-induced effects due to having the following most important effects and problems [1, 2, 3]:

- Cause extreme water level drawdown and return currents in confined channels
- Generate surge waves on shallow banks
- Present higher risk of ship transit in confined waterways and shallow estuaries
- Disturbance of tranquility and accessibility of marinas, ports and harbors

And also ship-generated waves have the most important following consequences:

- Bank stability and shoreline erosion
- Potential impact on marine life in coastal wetlands
- Endanger the smaller boats moving in waterways
- Swimmer safety
- Berthing/mooring and on-offloading difficulties for cargo vessels in inland ports
- Hazard to recreational and commercial boat marinas

Due to the great importance of wake-wash effects, considerable research efforts have been devoted to the wash problem during last few years, both experimentally and theoretically. It seems that Michell [4] was the first who solved the Linearized problem of ship-waves for a ship moving steadily forward in a calm shallow sea. In general Michell formulated his thin-ship theory in water of finite depth. However in his theory the potential and wave resistance was

[1].M.Sc. Student.
[2]Professor Assistant

derived for infinite depth. For convenience Tuck et al., (2000) [6] re-derived full finite-depth ship theory, and in particular the free-wave pattern far behind the ship.
A more general shallow-water approximation is the equation of Boussinesq type, which is valid for almost arbitrarily unsteady cases. A set of modified Boussinesq Equations, was applied to compute ship waves in shallow water, using slender-body theory to approximate the near ship flow By Jiang, (2000) [1]. But in this kind of simulation, there are some limitations such as [3]:

- Dispersive Limit for short-period waves, i.e., water depth/wavelength must be less than 0.5 ($h_0/L_W < 0.5$).
- Depth-based Froude Number must be grater than 0.65 ($F_h = V/\sqrt{gh_0} > 0.65$).
- The effects of viscosity are neglected.

In the present study to simulate shallow-water ship-waves Michlet code is used. This code is based on the Michell's thin-ship theory. Also to validate the study, the results were compared with previous work in literature. Moreover the effects of eddy viscosities are considered. The incidental benefit of introduction viscosity to the model is an improvement and increasing accuracy of final results of simulation.

2. Problem formulation

Describing the flow generated by a ship at a uniform speed of V in shallow water of constant h_0, a Cartesian Coordinate System of Oxyz is used (see Fig.1). The plane Oxy is on the calm free surface with the axes x in the direction of ship motion. On the assumption that the fluid is incompressible and inviscid, the irrotational flow generated by moving ship can be described by a potential $\Phi(x, y, z, t)$ governed by the Laplace equation in the whole fluid domain as:

$$\Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0 \qquad (1)$$

Kinematic and dynamic boundary conditions for this problem on the free surface at $z = \varsigma(x, y, t)$ are:

$$\varsigma_t - V\varsigma_x + \Phi_x\varsigma_x + \Phi_y\varsigma_y = \Phi_z \qquad (2)$$

$$\Phi_t - V\Phi_x + \frac{|\nabla\Phi|^2}{2} + gz = 0 \qquad (3)$$

where g is the acceleration due to gravity. The no-flux condition on the hull-surface $F(x, y, z, t) = 0$ can be expressed as:

$$F_t - VF_x + \nabla\Phi \cdot \nabla F = 0 \qquad (4)$$

And no-flux condition on the water bottom at $z = -h_0$ is:

$$\Phi_z = 0 \qquad (5)$$

Ship waves in shallow water can be obtained solving the governing equation using the above boundary conditions.

3. Michell's thin-ship theory

Taking the vertical median plane of the ship as $y = 0$, it may suppose that the ship remains at rest and the water moving backwards with uniform velocity V apart from the wave-disturbance. The motion is assumed steady. Moreover it is assumed that a disturbance source having potential function of $\phi(x, y, z)$ is located on Centerplane of the ship. Therefore the total velocity potential can be written as $\phi^T = \phi - Vx$. Since the inclination of the ship's surface to the plane $y = 0$ is small everywhere, ϕ will be small as well, and therefore the squares of the velocities which are differentiation of ϕ, in comparison with their first powers can be neglected. The consequent linearized kinematic boundary condition can be changed to:

$$\phi_z = -(-V + \phi_x)\zeta_x - \phi_y \zeta_y = V\zeta_x \qquad (6)$$

The dynamic boundary condition can be written as :

$$\frac{p}{\rho} + \frac{1}{2}q^2 + g\zeta = const. \qquad (7)$$

where:

$$q^2 = |\nabla \phi_t|^2 = (-V + \phi_x)^2 + (\phi_y)^2 + \phi_z^2 = V^2 - 2V\phi_x \qquad (8)$$

and finally Eq.7 can be written as:

$$\zeta(x, y) = \frac{V}{g}\phi_x(x, y, 0) \qquad (9)$$

Eqs. (9) and (6) lead to the Kelvin free-surface condition on $z = 0$ as:

$$\frac{g}{V^2}\phi_z + \phi_{xx} = 0 \qquad (10)$$

No-flux condition on hull surface (Eq. 3) can be also be linearized to Michell's hull boundary condition. Considering the ship, with offsets $y = \pm Y(x, z)$, which is supposed to be laterally (y-wise) symmetric on $y = 0$ by putting Y instead of F in Eq. (3) and some linearizations the following relation can be obtained:

$$\phi_y = \pm V Y_x(x, z) \qquad (11)$$

No flux-condition on the bottom (Eq. 5) can be modified on $z = -h_0$ as:

$$\phi_z = 0 \qquad (12)$$

Now Laplace Equation with new velocity potential $\phi(x, y, z)$ with three boundary conditions (Eqs.10 to12) give Michell boundary-value problem. Solving this boundary-value problem using Fourier transformation (Tuck et al., 2000 [6]) leads to:

$$\bar{\phi} = \frac{2V}{\pi k}\int_{-h_0}^{z} \bar{Y}_x(\zeta;\lambda)\sinh k(z-\zeta)\,d\zeta + C\cosh k(z + h_0) \qquad (13)$$

where $\bar{\phi}$ and $\bar{Y}_x$ are the Fourier transform of ϕ and Y respectively. C is a constant obtained using Eq. 10:

$$C = \frac{2V}{\pi k}\int_{-h_0}^{0} \bar{Y}_x(\zeta;\lambda)\frac{k_0 k\cosh k\zeta + \lambda^2 \sinh k\zeta}{k_0 k \sinh kh_0 - \lambda^2 \cosh kh_0}\,d\zeta \qquad (14)$$

where $k_0 = g/V^2$ and the wave number of the wave component traveling at angle θ is:

$$k = k_0 \sec^2\theta \tanh kh_{.0} \qquad (15)$$

And also:

$$\bar{Y}_x(z;\lambda) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx e^{i\lambda x} Y_x(x, z) \qquad (16)$$

$\phi(x, y, z)$ can be obtained using inverse Fourier transform as:

$$\phi(x, y, z) = \int_{-\infty}^{\infty} d\lambda e^{-i\lambda x}\int_{0}^{\infty} d\mu \cos\mu y\,\bar{\phi}(z;\lambda,\mu) \qquad (17)$$

where $\lambda = k\cos\theta$ and $\mu = k\sin\theta$ or $k^2 = \lambda^2 + \mu^2$.

Finally the free-surface elevation or shallow-water ship-wave profile can be obtained using Eq. 9. It is supposed that the steady wave pattern $z = \zeta(x, y)$ is the form of a sum of plane waves traveling at various angles θ of propagation relative to the direction of motion of body. Thus:

$$\zeta(x, y) = \Re\int_{-\pi/2}^{\pi/2} \zeta_A(\theta)e^{-i\Omega(\theta)}\,d\theta \qquad (18)$$

where the phase function is:

$$\Omega(\theta) = k(\theta)[x\cos\theta + y\sin\theta] \qquad (19)$$

$\zeta_A(\theta)$ is the (complex) wave amplitude, and $k(\theta)$ is the wave number of the wave component traveling at angle θ, which can be obtained using Eq. 15. The (complex) amplitude function $\zeta_A(\theta)$ which is sometimes known as the free wave spectrum or Kochin Function, can be computed in different ways. For example it can be obtained using Michell's thin-ship theory, or empirical or experimental measurements [5]. Michell's theory indicates that $\zeta_W(\theta)$ in terms of the hull slope $Y_x(x,z)$ can be obtained as:

$$\zeta_A(\theta) = \frac{2}{\pi}\sec\theta \frac{k}{1 - k_0\sec^2\theta \sec h^2 kh_0}\iint Y_x(x,z)\frac{\cosh k(z+h_0)}{\cosh kh_0}e^{ikx\cos\theta}dxdz \qquad (20)$$

4. Eddy Viscosity

If the usual molecular kinematic viscosity for water, of the order of $\nu = 10^{-6} m^2 s^{-1}$ is used, there shall be no discernible effect. Even with large viscosities of order of $\nu = 10^{-3} m^2 s^{-1}$, the viscous effect seems to be quite small. However physical nature of the phenomena dictates the existence of very large viscosities (though we believe unlikely) [7] as ambient eddy viscosities in the ocean. However, we are more interested in eddy viscosity effects produced by the vessel itself, especially in its wake, where the highly vortical flow might indeed correspond to effective eddy viscosities of up to that order of magnitude. It should be noted that oceanographically relevant eddy viscosities of the order of $\nu = 5\times10^{-3}$ and also high frequency waves do damp out the shortest diverging waves as $|\theta| \to \pi/2$. The empirical viscous correction factor was derived from a formula for damping of plane water waves given by Lamb [8]. Lamb suggested that a plane wave of wavenumber k (k can be obtained from Eq. 15) would be damped with respect to time t (that equals x/V , since the problem is steady) as it travels on sea surface. In this regard the damping factor can be given as:

$$D = \exp(-2\nu \frac{x}{V} k^2) \qquad (21)$$

Or $D = \exp(-2\nu \frac{x}{V} k_0{}^2 \sec^4\theta)$ (22)

Therefore the viscosity can be included to the problem applying Eq. 21 in the $\theta-$integrand for the far-field waves in Eq. 18.

5. Numerical model and results

A Series 60 hull with a block coefficient $C_B = 0.594$ was chosen for numerical simulation. The model has a length of $L = 4.689$ m , a beam $B = 0.625$ m and a draft $T = 0.25$ m (see Fig. 2). The water depth was $0.5m$, which leads to a ratio of water depth to ship draft $h_0/T = 2.0$,being representative of shallow water dynamics. The length of calculation domain is 6 times the ship length behind the ship and the width of calculation domain is 26 times the ship beam. The calculation domain is divided by 101×101 panels to give generally good results. Water depth can have a significant effect on the far-field waves created by ships. For infinitely deep water, i.e. $F_h = 0$ and at the Length-based Froude number $F_L = 0.35$ (see Fig. 3.a), the wave system is the Kelvin wave pattern. At a subcritical speed, $F_h = 0.5$ (see Fig. 3.b), the wave system is close to a Kelvin-Havelock wave pattern and the enveloping wedge is a little wider, but the half-angle (the Kelvin angle) is about 19 deg 28 min. Moreover the transverse wavelength is becoming longer.

At critical speed, $F_h = 1$ (see Fig. 3.c), the wave system is characterized by significant diverging waves. The angle of the enveloping wedge widens so that the Kelvin angle increases up to about 90 deg. The transverse wavelength is almost twice the length of the infinitely-deep water. At a supercritical speed, $F_h = 1.5$ (see Fig. 3.d), the wave system comprises only divergent waves, i.e., transverse waves or cusps have disappeared. The Kelvin angle decreases and is narrower than those of for the cases close to the critical speed. In this case the Kelvin angle can be given by $\pm \arcsin F_h^{-1}$.

Fig.4 compares the calculated wave records in the present study with Boussinesq simulation of shallow-water ship-waves and also with experimental wave records. To validate the present study، two longitudinal wave cuts at y=1.0 and 2.45 in critical speed are presented. The good agreement between the present study and the literature and also experimental data is evident. It can be seen that in the present study there are not large differences beyond two ship lengths behind the ship between calculation and experimental data. It is due to this fact that the effect of boundary layer and viscosity have been considered in the model.

Fig.5 compares the contour plots of wave patterns in each speed with and without considering eddy viscosity. The exaggerated value of $\nu = o(10^{-2}) m^2 s^{-1}$ in Fig.5. is used to magnify the values for comparison. It can be seen that comparison of this results with that of the zero-viscosity (left figures) reveals that the finest diverging ripples in the pattern have disappeared. These ripples propagate almost perpendicular to the ship's track. In this case the eddy viscosity affects is mostly due to the $\sec^4\theta$ term in Eq. 22. Larger viscosities can eliminate even more detail, eventually even inappropriately damping transverse waves. It seems that a viscosity magnitude of $\nu = o(10^{-3}) m^2 s^{-1}$ shows a good compromise.

Fig.6 shows the computed free wave spectrum at port and starboard of a Series 60 hull by Michell's thin-ship theory in different speeds. It can be seen that the wave amplitudes at critical and supercritical speeds are very larger than that of the subcritical speed.

6. Conclusions

Michell's thin-ship theory is a reliable and relatively accurate method of simulating ship-waves, which can be applied not only for deep water, but also for shallow water ship motion. This method can be applied for ships, which have the length to beam ratio more than 6. Therefore Michell's thin-ship theory is suitable for the most modern ships. Using this method has some advantages to other models in relations to fewer limitations in application of this model. For example unlike the other models in this model there are no limitations such as a minimum value for depth-based Froude number or dispersive limit of short-period waves. Moreover as mentioned and applied in the model the effect of viscosity can be considered by using an empirical method for damping of plane water waves.

7. References

[1] Jiang, T., "Investigation of Waves Generated by Ships in Shallow Water", 22nd Symposium on Naval Hydrodynamics, 2000.

[2] Jiang, T., Henn, R. and Sharma, S.D., "Wash Waves Generated by Ships Moving on Fairways of Varying Topography", 24th Symposium on Naval Hydrodynamics, Fukuoka, Japan, 8-13 July 2002

[3] Nwogu, O.G. and Demirbilek, Z., "Numerical Modeling of Ship-Induced currents in Confined Waterways", J. of Coastal Engineering, 2004.

[4] Michell, J.H., "The wave resistance of a ship", Phil.Mag. (5) 45,106-123, 1898.
[5] Tuck, E.O., Lazauskas, L. and Scullen, D.C., Sea Wave Pattern Evaluation, Part 1 report, Scullen & Tuck Pty Ltd, 1999.
[6] Tuck, E.O., Lazauskas, L. and Scullen, D.C., Sea Wave Pattern Evaluation, Part 4 report, Scullen & Tuck Pty Ltd, 2000.
[7] Tuck, E.O., Lazauskas, L. and Scullen, D.C, "Ship-Wave Patterns in the Spirit of Michell", Scullen & Tuck Pty Ltd, 2002.
[8] Lamb, H., "Hydrodynamics", 5th Edition, Cambridge , 1932.

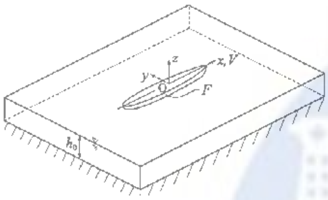


Fig. 1 Schematic of the problem and coordinate system

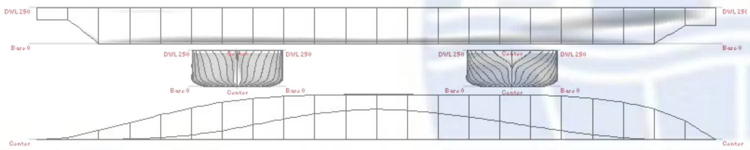
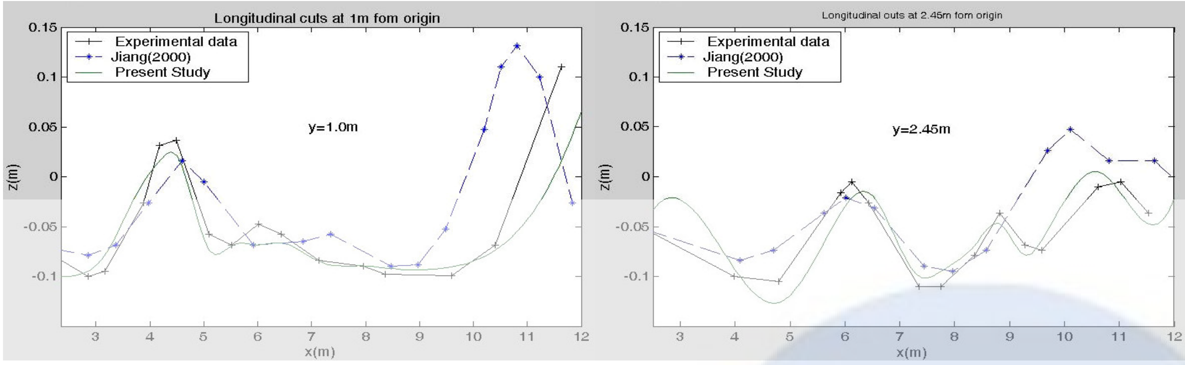


Fig. 2 Lines plan of ship

(a) $F_h = 0$ **(b)** $F_h = 0.5$

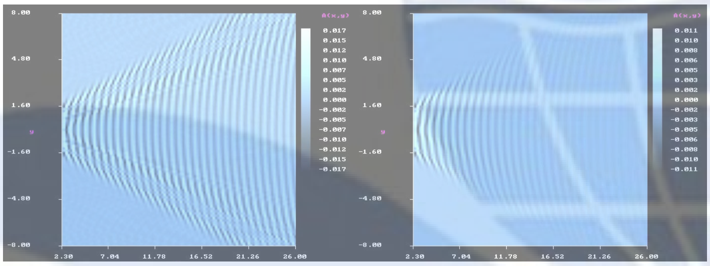
(c) $F_h = 1$ **(d)** $F_h = 1.5$

Fig. 3 Calculated wave patterns of a Series 60 hull, a) in deep water at $F_L = 0.35$, b) shallow water at subcritical speed,
c) shallow water at critical speed, d) shallow water at supercritical speed

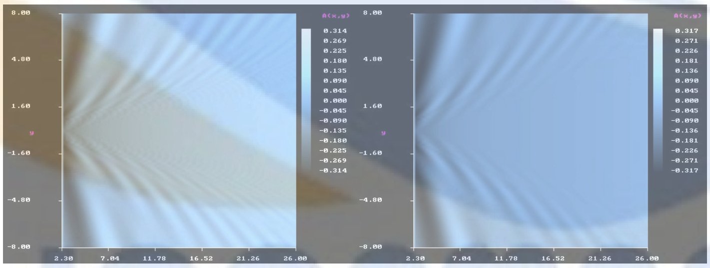


(a) (b)

Fig. 4 Wave records of a Series 60 hull passing by the critical speed a) at y=1.0m from origin, b) at y=2.45m from origin



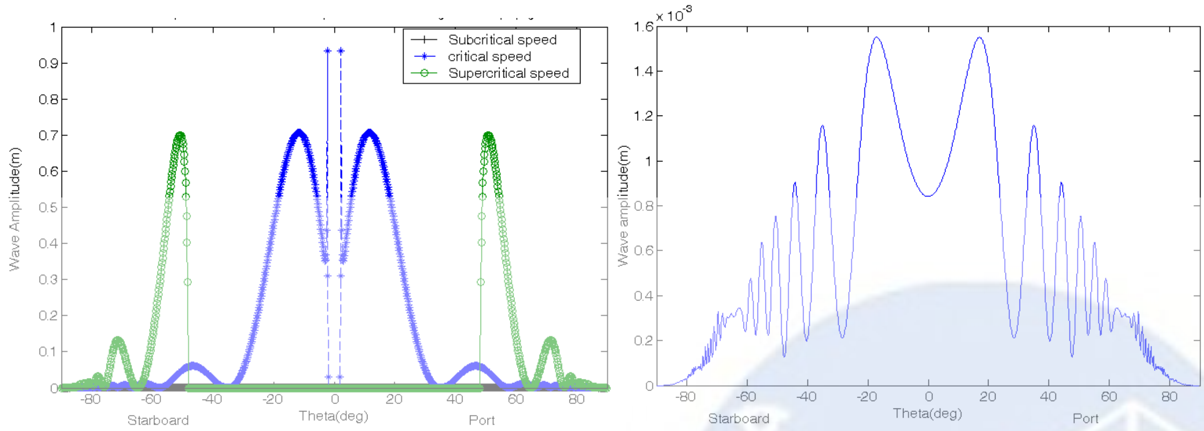
a) $F_h = 0.5$



b) $F_h = 1.0$

c) $F_h = 1.5$

Fig. 5 Comparison of wave patterns under the effect of eddy viscosity in different speeds. Left pictures at the effect of zero viscosity, right pictures are with $\nu = o(10^{-2})$



(a)

(b)

Fig. 6 Free wave spectrum of a Series 60 hull in shallow water versus the angle of wave propagation a) at different speeds b)at subcritical speed