Spectral Analysis of Wave Reflection in Model Tests

Mohammad Javad Ketabdari
Faculty of Marine Technology, Amirkabir University of Technology (AUT),
Ketabdar@cic.aut.ac.ir

Abstract
Wind waves, which can travel beyond the direct effect of the generating forces, are reflected by beaches, breakwaters, shoreline structures and submerged or floating offshore structures. The interactions between reflected and incident waves contribute to the characteristics of the wave field and flow field beneath the waves. A laboratory study carried out to consider the reflection characteristics from the beach with a constant mild slope of a wave flume and a model vertical seawall using the two dimensional method of Goda and Suzuki (1976). An extensive series of experiments set up covering different regular, irregular and groupy waves to find the percentage of reflected waves. Two methods of 1) Averaging the reflection coefficient components 2) Using the power spectra of incident and reflected waves in a suitable range of frequency were employed to obtain the reflection coefficients. The results showed that in both methods the proper handling of spurious spikes is vital to get real results for reflection coefficient. The results also showed that the rate of reflection coefficient increases with an increase in fundamental wave frequency. However the reflection coefficient get larger for groupy waves with longer duration.
Keywords: Spectral Analysis, Wave Reflection, Laboratory Model, Irregular Waves, Groupy Waves

1 Introduction

As water waves attack marine structures or sloping beaches a part of wave reflects and propagates in the opposite direction of incoming waves. Reflected waves cause increased agitation of the water in front of the structure or they may propagate some distances to become a source of disturbance in a calm area of water. In many laboratory studies it is necessary to separate the measured wave train into components of incident and reflected waves, so that the model response can be related to the parameters of the incident wave field. In a wave flume a complicated multi-reflection system of wave trains is formed as waves are reflected and re-reflected by the model structure, sloping beach and wave paddle. Some methods of analysis have been developed for determination of the reflection coefficient in laboratory wave flumes.

2 Reflection Analysis Methods

Methods of reflection analysis may substantially be divided into the two following categories:

2-1 Reflection Analysis of Regular Waves

According to linear first order wave theory, a non-breaking regular wave can be expressed as:

$$\eta(t) = a_I \cos(kx - \omega t + \varepsilon'') + a_I K_R \cos(kx + \omega t + \varepsilon'' + \theta'') \tag{1}$$

where a_I is incident wave amplitude, ε'' is the arbitrary incident wave phase angle, K_R is the reflection coefficient and θ'' is the reflection phase shift. The first term is a regular incident wave moving in the positive x-direction and the second term is the reflected wave moving in the opposite direction. Interaction of the incident and reflected waves creates a partial standing wave pattern characterised by an envelope that has uniform maxima and minima, spaced at distances one quarter of a wave length apart. Expanding the trigonometric functions and after some rearrangement the reflection coefficient is given as follows:

$$K_R = \frac{H_{\max} - H_{\min}}{H_{\max} + H_{\min}} \tag{2}$$

where $H_{\max}$ and $H_{\min}$ are the maximum and minimum wave heights.

In a simple and common method denoted as "Method of traversing gauges", by moving a wave gauge along a line, parallel to the direction of wave propagation, the maximum and minimum of the wave envelope can be measured. In another method two gauges can be used with a fixed separation distance equal to one quarter of a wave length apart ($\lambda/4$), assumed to have been located at a maximum and minimum of the standing wave envelope Isaacson (1991) states that method of traversing gauges is cumbersome when tests are carried out at a series of wave lengths. Also, the method of using two fixed probes at node and antinode, because of uncertainty of the location $\lambda/4$ and $\lambda/2$ from the beach or structure, may be inaccurate. He described three methods to determine three unknowns of a_I, K_R, θ'' as follows:

- ***Method I***: Two fixed probes, measuring two wave heights and one phase angle (Goda and Suzuki, 1976).
- ***Method II:*** Three fixed probes, measuring three wave heights and two phase angles (Mansard and Funke, 1980).
- ***Method III:*** Three fixed probes, measuring three wave heights (Isaacson, 1991).

The first two methods were developed for irregular waves, but can also be used for regular waves. Method II provides more knowns than unknowns, so involves the application of the least square fit method to the results. All three methods become inaccurate for conditions close to those at which they fail. Some errors may arise from inaccurate measurement and may also be associated with those differences from the assumed linear wave theory.

2-2 Reflection Analysis of Irregular Waves

Using the following instrument deployments, the incident and reflected irregular waves can be recorded:

- **Spatially-separated Wave Gauges:** In this method a series of fixed gauges set on a line, parallel to the wave propagation are used for recording the water level and subsequent reflection analysis.
- **Vertical Array:** In this method a current meter and wave gauge mounted in a vertical line, are used to record the horizontal velocity and water surface level.
- **Co-located Velocities:** In this method using a current meter, the time series record of horizontal and vertical water velocity components is obtained.

The main assumption behind these methods is that the irregular waves can be described as a linear superposition of an infinite number of discrete components, each with their own frequency, amplitude and phase. Thus, the non-linear wave interactions are not represented in the analyses. Furthermore, all methods are restricted to the two-dimensional cases, as in the waves generated in the wave tank.

3 The Method of Goda and Suzuki

Because of the simplicity and acceptable accuracy of using two or three separated gauges for estimation of reflection coefficient, in most laboratory techniques the methods based on spatially-separated wave gauges are employed. Considering earlier work of Kajima (1969) and Thornton and Calhoun (1972), Goda and Suzuki (1976) using a method of spatially-separated wave gauges, proposed a technique of analysis for estimation of incident and reflected spectra. Using time series of sea surface elevation and the Fourier analysis for two wave gauges, set up at a short distance from each other parallel to the direction of wave travel, the incident and reflected amplitudes were obtained. The two measured time series of surface waves are given by:

$$\eta_1(x,t)=\sum_{i=1}^{\infty}[a_{I_i}\cos(\Phi_{I_i}-\omega_i t)+a_{R_i}\cos(\Phi_{R_i}+\omega_i t)]$$

$$\eta_2(x,t)=\sum_{i=1}^{\infty}[a_{I_i}\cos(\Phi_{I_i}-\omega_i t+k_i\Delta l)+a_{R_i}\cos(\Phi_{R_i}+\omega_i t+k_i\Delta l)] \tag{3}$$

where a_I , a_R , Φ_I , Φ_R are the incident and reflected amplitudes and phases and Δl is the horizontal distance between two probes. Passing the time series records of surface wave elevation through a FFT and solving the system of four Equations with four unknown parameters $a_{I_i}, a_{R_i}, \Phi_{I_i}$ and Φ_{R_i}, for each component of frequency. The results of solving these Equations, give the spectral components at each discrete Fourier coefficient as follows:

$$a_{I_i}=\frac{1}{2|\sin k_i\Delta l|}\left[(A_{2_i}-A_{1_i}\cos k_i\Delta l-B_{1_i}\sin k_i\Delta l)^2+(B_{2_i}-B_{1_i}\cos k_i\Delta l+A_{1_i}\sin k_i\Delta l)^2\right]^{1/2}$$

$$a_{R_i}=\frac{1}{2|\sin k_i\Delta l|}\left[(A_{2_i}-A_{1_i}\cos k_i\Delta l+B_{1_i}\sin k_i\Delta l)^2+(B_{2_i}-B_{1_i}\cos k_i\Delta l-A_{1_i}\sin k_i\Delta l)^2\right]^{1/2} \tag{6}$$

$$\Phi_{I_i}=\tan^{-1}\left[\frac{-(A_{2_i}-A_{1_i}\cos k_i\Delta l-B_{1_i}\sin k_i\Delta l)}{(B_{2_i}-B_{1_i}\cos k_i\Delta l+A_{1_i}\sin k_i\Delta l)}\right]$$

$$\Phi_{R_i}=\tan^{-1}\left[\frac{(A_{2_i}-A_{1_i}\cos k_i\Delta l+B_{1_i}\sin k_i\Delta l)}{(B_{2_i}-B_{1_i}\cos k_i\Delta l-A_{1_i}\sin k_i\Delta l)}\right] \tag{7}$$

The reflection coefficient can be estimated by obtaining the energy spectra of incident and reflected waves and using the following Equation:

$$K_R=\sqrt{m_{0R}/m_{0I}} \tag{8}$$

where m_{0I} and m_{0R} are the representative values of the total energy of incident and reflected waves, respectively. An assumption in the above analysis is that the waves are linear and the dispersion relation between k (wave number) and f (frequency) is satisfied. However, when a wave maker is moving in simple harmonic motion it also creates non-linear higher harmonic wave trains that can not satisfy the dispersion relation. Furthermore, Eqs 6 and 7 contain singularities that occur when:

$$\sin(k_i \Delta l) = 0 \quad \text{or} \quad k_i \Delta l = n\pi \quad \text{for} \quad n = 0,1,2,\ldots \qquad (9)$$

To avoid this problem, Goda and Suzuki (1976) recommended a guideline for the effective frequency range of resolution:

$$0.05 \langle \Delta l / \lambda \langle\ 0.45 \qquad (10)$$

4 Results and conclusions

An extensive series of experiments in a laboratory study was performed covering different wave conditions to find the percentage of reflected waves using the linear method of Goda and Suzuki (1976). If the reflection coefficient is considerable, by taking the wave reflection characteristics into account, new expressions should be used for the wave characteristics. Three wave gauges were mounted at a suitable distance from the beach (see Fig. 1). Then two sets of experiments were performed as follows:

- Experiment 1: Measurement of reflection coefficient for regular waves
- Experiment 2: Measurement of reflection coefficient for groupy waves

For all of the experiments a constant nominal wave height of 8 cm for regular and groupy waves were selected. Figs.2 shows time histories of a typical groupy wave generated in this study . Some typical wave energy spectra and reflection wave characteristics in the frequency domain for $\Delta l = 20$ cm are shown in Figs 3 and 4. It is evident from these figures that the energy spectra of regular and groupy waves show the existence of some energy at higher harmonics. Also, these figures show that the reflection coefficient is highly contaminated by some large unreal spikes. This problem can be clearly seen for the frequencies where the energy spectral density is very low. Therefore, a good estimation of the reflection coefficient can not be obtained in such areas. This means that utilisation of the Goda and Suzuki method required some modification before calculating the reflection coefficient. Another reason for these spikes is the singularities in the sine of the denominator of the Goda's Equation (Eqs. 6). After calculating the incident and reflected amplitudes for each pair of Fourier transform components, two methods were employed to obtain the reflection coefficients:

- ***Method 1***. Averaging the reflection coefficient components for a suitable range of frequency.
- ***Method 2.*** Using the power spectra of incident and reflected waves (Eq. 8) in a suitable range of frequency.

The results showed that in the first method the reflection coefficient versus frequency is highly contaminated for both regular and groupy waves by the singularities. In the second method, the spectra of incident and reflected regular waves are less contaminated with unreal spikes. However, for groupy waves the unreal results have to be removed. Therefore, using both of these methods requires recognition of the unreal spikes and their sources. It would seem that there could be a routine way of removing the unreal results that is common to all of the sample waves in different ranges of frequency. For example according to Eqs 6 , for singularities due to $\sin(k_i \Delta l) \to 0$, both amplitudes and phases of incident and reflected wave components simplify such that $a_I = a_R$ and $\Phi_I = -\Phi_R$. This may suggest a method for identifying these spikes. In these methods for each sample wave, after considering the results of superposed spectral density and discrete reflection coefficients, a high pass filter based on power spectra, was selected, so that just the frequencies related to high amplitudes were used for determining the reflection coefficient. Also, a low pass filter for reflection coefficient was used to remove the unreal results, i.e. the reflection coefficients larger than unity. By selecting the appropriate filters the results of two methods in most cases were very close to each other and at most 10% different (for very high frequency waves).

After considering all phenomena that may affect the results the reflection coefficient was calculated and plotted versus frequency for regular and groupy waves (see Fig. 5). The increasing rate of reflection coefficient with the increase of regular wave frequency and increase in fundamental groupy wave frequency is evident. In addition the reflection coefficient is larger for groupy waves with longer duration. These Figures also show that the reflection coefficient is less than 10% in the range of wave frequencies is usually used for wave generation in the wave flume. It should be noted that for very low frequency waves ($F < 0.4$ Hz) because of the high contamination of the main wave with locked and free higher harmonic waves, the results were unreliable and, therefore, such results were disregarded. The following factors may lead to errors in the estimation technique:

1. Deviation from the linear dispersion relationship due to non-linear effects.
2. Presence of second order harmonics in the incident wave train.
3. Generation of non-linear interactions in the standing wave field.
4. Presence of transverse waves and other disturbances in the wave flume.
5. High levels signal noise in the measured time series.

Consideration of the results of these series of experiments, shows that the reflection coefficient for the sloping beach of this wave flume is on average less than 10%. This means that most of the waves, break rather than reflect. However, this percentage of reflection may not be neglected. Therefore, the effects of reflection should be included in the wave characteristics in all subsequent analysis wherever necessary.

5 References

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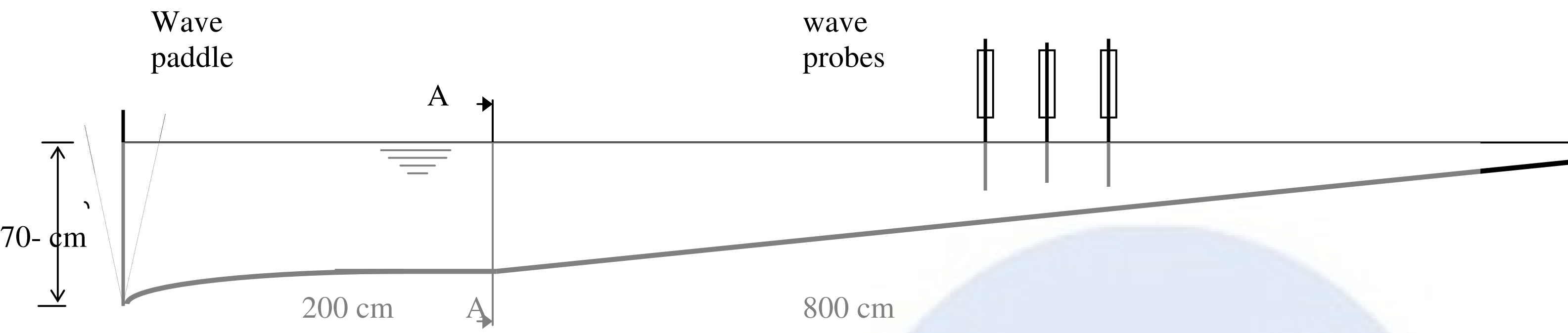


Figure 1 Experimental set up for reflection measurement

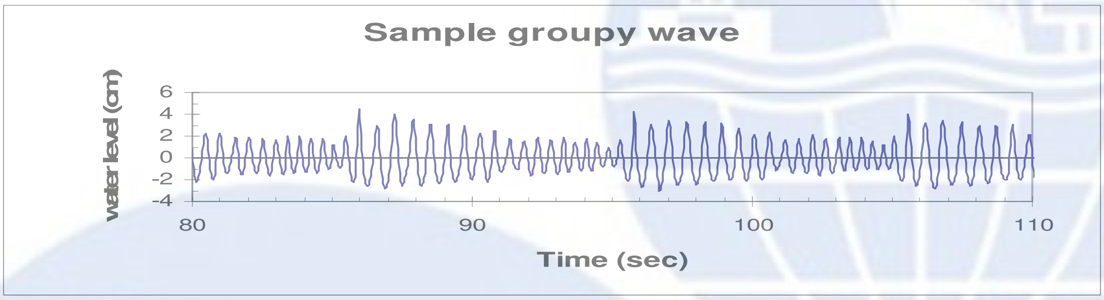


Figure 2 Sample groupy wave generated in the wave flume

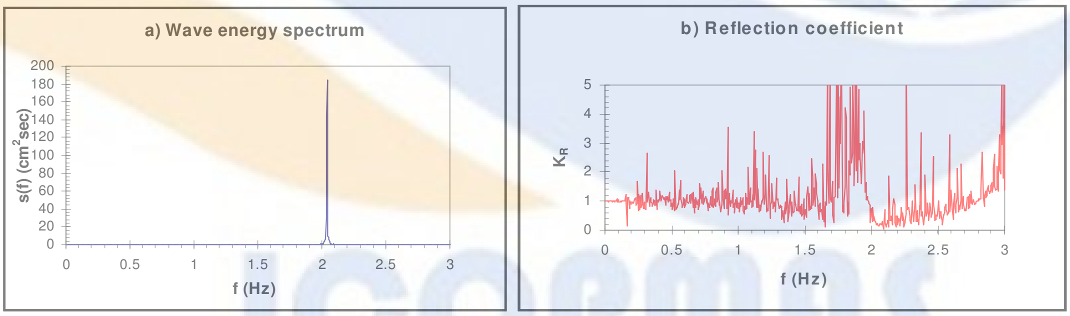


Figure 3 Generated regular wave a) Incident spectral energy b) Frequency domain reflection

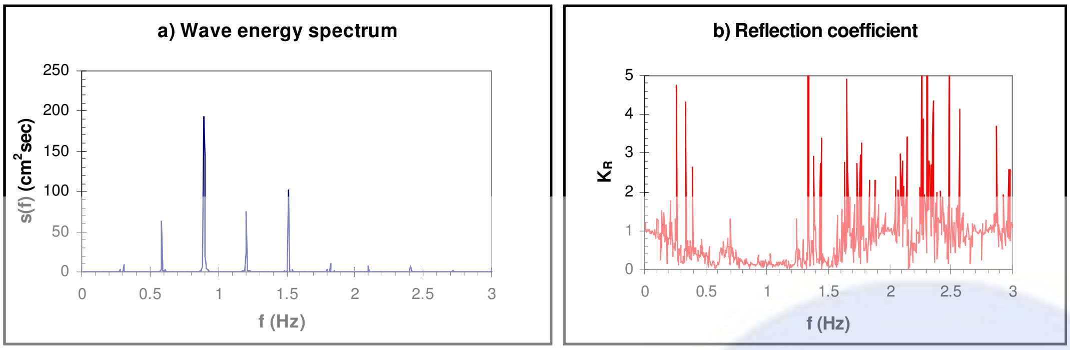
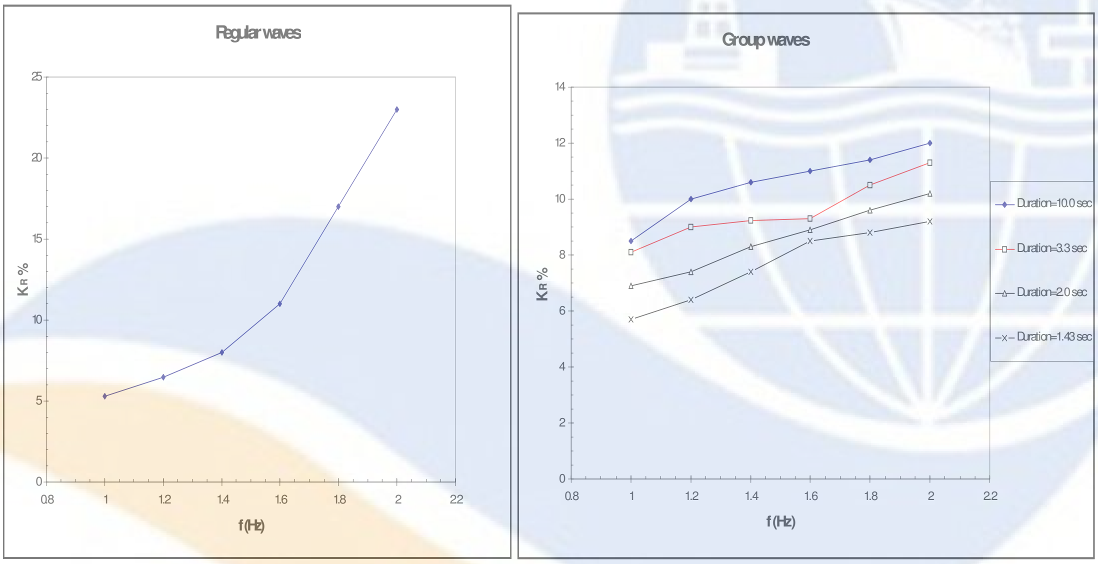


Figure 4 Generated groupy wave a) Incident spectral energy b) Frequency domain reflection



(a) (b)

Figure 5 Reflection coefficient versus wave frequency for a) Regular b) Groupy waves