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Evaluating efficiency of different responses in structural damage detection by sensitivity-based analyses

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Abstract

Different methods are used to estimate the location and intensity of damage in structures and these methods use different static and dynamic responses. Even though many methods have been proposed for damage detection in the last two decades, sensitivity-based analyses are still popular. In this research, the effects of different responses in sensitivity-based methods, for two cases of additive and multiplicative noises in a truss and a two layers dramatic dome are studied. First, by imposing noise in various responses and implementing Monte Carlo sampling, an index was devised to evaluate the sensitivity of each response to sensor location. Then, different responses are used to detect damage in structures, but since it is an ill-posed problem, Tikhonov regularization scheme is used to evaluate exact results of each response, through an updating problem. By comparing the results, it is concluded that sensitivity of static responses to sensors location is less and static response is the most effective response with fewer errors in damage detection of the cases.

Keywords: damage detection; response; sensitivity analysis; noise; regularization

1. INTRODUCTION

Determining primary damages in a structure is very important, but small damages may not be detected easily while their effects are important in the stability of structures. Hence, it is necessary to determine the exact and more sensitive characteristics in relation to small damages. Presence of damages in a structure causes change in the static and dynamic characteristics of structures of damage detection (Doebling et al., 1996; Zou, 2000). Different responses which are used in damage detecting methods, such as static and dynamic transformation, frequencies, mode shapes, and frequency response function, have errors in measuring sensors, as a result of inaccessibility to accurate measurement schemes and equipments set up, which may cause serious obstacles in damage detection if the intensity of damage were small or noise level is high. Hence, choosing an appropriate response in damage detection to minimize errors and finding the damage location better, caught the attention of many researchers in recent years. For example Jenkin et al., (1999) concluded that static deformations in determining the position of damage, show more sensitivity to frequency in most cases. Zhao and Dewolf (1999) studied sensitivity of natural frequency, mode shape and modal softness in damage detection and concluded that softness modal detects the damage with more probability in comparison to other responses. Oh and Jung (1998) showed that combination of static displacement data and slope or curvature of mode shape, represent the most trustable results for damage detection. Yam and Wong (2002) studied systematically the sensitivity of static and dynamic parameters to damage of plain structures and proposed a failure index for checking their capacity in identifying damage. Cao et al., (2010) studied the sensitivity of mode shape and static deformations for damage detection in cantilever beams. Wahab and Roeck (1999) and Sampaio et al., (1999) concluded that differences of Curvatures between Mode Shape (CMS) and curvature of frequency response function were more sensitive to displacements and are applicable to determine intensity and location of damage better. Wang and Hu (2001) represented a structural damage detection algorithm by static laboratory data and changes in natural frequencies and showed that the proposed algorithm for damage detection was effective. Hajela and Soeiro (1989) used both static transformations and mode shapes for damage detection. Hence, there is no comprehensive study to show that which response contains fewer errors in damage detection.

In this research, the effects of these responses in sensitivity based analyses for both additive and multiplicative noises were studied. In this method, system of linear equations of damage detection was determined and sensitivity matrix of structural response to damages in members was derived as a matrix of coefficient of linear equation. By using Monte Carlo simulation and by considering noise for different responses and implementing regularization, an index was extracted for different numbers of sensor putting and by this index, a response which has less sensitivity to sensor putting was determined. Then, different responses were used to detect damage in structures, but since it was an ill-posed problem, Tikhonov regularization was used to derive exact results for each response, through an updating problem.

2. DEFINING DIFFERENT RESPONSES

To measure different responses, which may be used in damage detection, sensors are used. Generally, the data measured by sensors, consist of static and dynamic parameters. In measuring static responses, inputs are applied and system is allowed to answer and be stabilized, and then the output is measured. Static responses have benefits and limitations, which influence their application. Simplicity in experimental setup, cheapness of laboratory equipments, convenience of performing the tests and elimination of dynamic parameters (structural mass etc) in analyses, are positive points of static responses. However, in measuring dynamic parameters, the rate of response of structure to changes in reaction input is important. One of the static parameters whose measuring is easier than dynamic parameters is static transformation response and is arrived by analyzing structures under resident outside loads that are imposed to joints. Natural frequency response, mode shape, dynamic response of time history and frequency response function are some of dynamic parameters. Frequency response depends on physical characteristics of system and response of mode shape is eigenvectors of dynamic system. Response of transformation under dynamic analysis of time history by considering earth acceleration through a time function in building base level and using standard calculations of dynamics of structures are done and deformations of structures are determined by a function of time (in any time period). In this research, two structures are analyzed using finite element software "Opensees". Frequency Response Function (FRF) is another dynamic characteristic that is used in structural damage detection. Frequency response function is a function of natural frequency of structures and by checking it, behavior of structure under dynamic loads can be studied. To calculate this response, transfer functions matrix which is denoted by H and whose norm is freedom degree of system is derived from motion equation by n degree freedom (Esfandiari et al., 2009).

3. DIFFERENT TYPES OF NOISES

Errors in measurements, cause noise in responses. Noises that are made through random processes have Gaussian or normal distribution (Baringhaus and Henze, 1988). Since the responses are in a multi-dimensional space, this normal distribution is a multivariate normal distribution. These noises can be imposed in additive or multiplicative case. A normal distribution will be written as:

$$\boldsymbol{Y} \sim N\,(\boldsymbol{\mu}, \textstyle\sum) \tag{1}$$

Where $\boldsymbol{Y}$ is random variable vector, $\boldsymbol{\mu}$ is mean vector and $\sum$ is covariance matrix (Matrices are denoted by bold letters). If A were the structural response:

$$\boldsymbol{Z} \sim \boldsymbol{N}\,(\boldsymbol{A}+\boldsymbol{\mu}, \textstyle\sum) \tag{2}$$

When normal distribution is as above equation, it is called additive noise. In other words, in additive noise, corresponding response is added to the noise. If the normal distribution is as Equation (3), the noise is multiplicative.

$$\boldsymbol{Z} \sim \boldsymbol{N}\,(\boldsymbol{A}\,\mu, \boldsymbol{A}\textstyle\sum\boldsymbol{A}^T) \tag{3}$$

4. DAMAGE DETECTION BASED ON SENSITIVITY ANALYSES

To find the damage, natural frequencies of structure, mode shapes, static deformations or a combination of them is used. Damage reduces the stiffness of structure, while its mass is still constant. Hence, its transformation increases because of external loading, its natural frequencies reduce and mode shapes change. If the structure is differently damaged, variation of deflections, natural frequencies and mode shapes will occur in different ways. Hence, to find the damage, one must find the damage area analytically in the same response as the real damaged structure. Therefore, the damage detection is a problem of finding a system of damaged variables which analytical responses of structure match coincidentally with what is measured in a different path. Mathematical expression of a problem can be written as bellow (Naseralavi et al., 2010):

$$\boldsymbol{R}_d=\boldsymbol{R}(\boldsymbol{X}) \Rightarrow \boldsymbol{X}=? \tag{4}$$

$$\boldsymbol{X}=\{ x_1, x_2, \ldots, x_n\}^T, 0\leq x_i\leq 1 \tag{5}$$

In the above equations, $\boldsymbol{X}$ is called the damage vector and x_i is damage ratio of ith element and n is the number of structural elements. The values $x_i=0$ and $x_i=1$ represent the intact and completely damaged cases. $\boldsymbol{R}_d= \{r_{d1},r_{d2} ,\ldots, r_{dm}\}^T$ is vector of m structural response. Responses can be natural frequencies, complete or incomplete mode shapes, structural deformations because of static or dynamic loads or a combination of them. $\boldsymbol{R}(\boldsymbol{X}) = \{ r_1(\boldsymbol{X}), r_2(\boldsymbol{X}), \ldots, r_m(\boldsymbol{X})\}^T$ is vector of m structural response for the structure which is assumed to be damaged and may be evaluated by analytical model. Hence, the damage detection problem is considered as solving a system of non-linear equations where the number of unknowns (x_i) and number of equations may be equal or unequal. To make the system linear, first order approximation is used and higher orders of Taylor series are ignored. Then, Equation (4) is expressed as Equation (6) (Naseralavi et al., 2010):

$$\boldsymbol{R}_d=\boldsymbol{R}_h+\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{X}} \Delta\boldsymbol{X} \Rightarrow \boldsymbol{R}_d - \boldsymbol{R}_h = \Delta\boldsymbol{R} \cong \boldsymbol{S}\, \Delta\boldsymbol{X} \tag{6}$$

Where $\mathbf{S} = \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{X}}$ is sensitivity matrix. $\boldsymbol{R}_h$ is response vector of intact structure and $\Delta\boldsymbol{X}$ is damage vector change.

5. SENSITIVITY OF STRUCTURAL RESPONSES

5.1. Sensitivity of eigenvalues to damage

A method for calculating derivatives of eigenvalues with respect to damage ratio (or any other arbitrary design variable) of an element, is using stiffness and mass matrices of the represented structure (Wittrick, 1962):

$$\frac{\partial \lambda^{(i)}}{\partial x_j} = \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{K}}{\partial x_j} \boldsymbol{\varphi}_i - \lambda^{(i)} \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{M}}{\partial x_j} \boldsymbol{\varphi}_i \tag{7}$$

Where $\boldsymbol{\varphi_i}$ is the ith eigenvector and λ_i is ith eigenvalue. In addition, $\boldsymbol{K}$ and $\boldsymbol{M}$ are stiffness and mass matrices of structure respectively. In this research, since the damage is considered as a reduction in elasticity module of elements, mass matrix would not change. Therefore, second term in the right side of Equation (7) disappears.

5.2. Sensitivity of displacements to damage

The static equilibrium equation of the structure is:

$$\boldsymbol{Kd}=\boldsymbol{F} \tag{8}$$

Where $\boldsymbol{K}$ is stiffness matrix, $\boldsymbol{d}$ is displacement vector and $\boldsymbol{F}$ is applied load vector. By differentiating Equation (8) with respect to an elemental damage ratio (or any other design variable), the i^{th} column of deformation sensitivity matrix is obtained. By differentiating from Equation (8) with respect to damage intensity of element (or any other design variable), ith column of deformation sensitivity matrix arrive:

$$\frac{\partial \boldsymbol{d}}{\partial x_i} = - \boldsymbol{K}^{-1} \frac{\partial \boldsymbol{K}}{\partial x_i} \boldsymbol{d} \tag{9}$$

It should be noted that for calculating different columns of sensitivity matrix of displacements by using Equation (9), only $\frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}_i}$ changes and calculation of $\boldsymbol{K}^{-1}$, which involves the most computational effort, is executed once. Hence, the computational effort of this scheme is much lower than conventional finite difference methods. In this problem, to evaluate each column of sensitivity matrix by finite difference method, bellow processes is done.

$$\boldsymbol{S} = \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{X}} = [\, S_1\ S_2\ S_3 \ldots S_n] \tag{10}$$

$$S_i = \frac{R_h - R_{di}}{0.001} \tag{11}$$

Where R_{di} is response from the damage of ith element while the damage is simulated by reduction in elasticity module of element. To calculate sensitivity matrix, elasticity module (E) of each member must be reduced 0.001times (E-0.001E) and response of it must be measured. Then sensitivity matrix is calculated from Equation (10) and Equation (11).

5.3. Sensitivity matrix of frequency response function

Sensitivity matrix of frequency response function arrives from Equation (12).

$$\boldsymbol{S} = \frac{\partial \boldsymbol{H}}{\partial b} = -\boldsymbol{H}_d \frac{\partial \boldsymbol{K}}{\partial b}\boldsymbol{H} \tag{12}$$

Where $\boldsymbol{H}$ (ω) is n×n transformation matrix and ω is frequency of excitation load, b is structure design variable (damage of a member), $\boldsymbol{H}_d$ is transfer function of the damaged structure and $\boldsymbol{S}$ is sensitivity matrix for frequency response matrix (Esfandiari et al., 2009)

6. NOISE IMPLEMENTATION AND CALCULATING STANDARD DEVIATION

As mentioned in section 3, there are errors in measuring responses of a real damaged structure. These errors have considerable effects on the results of damage detection, so the errors should be considered in the analyses. In this study, noise effects are implemented using the equation (6) as follows:

$$\boldsymbol{S}.\Delta\boldsymbol{X} = \Delta\boldsymbol{R} + \varepsilon \Longrightarrow \boldsymbol{S}.\,(\boldsymbol{X}\text{-}\boldsymbol{X}_0) = \Delta\boldsymbol{R} + \varepsilon \tag{13}$$

$\boldsymbol{X}_0$ in Equation (13) is the response of intact structure and its magnitude is zero. Then, this relation is written as:

$$\boldsymbol{S}.\boldsymbol{X} = \Delta\boldsymbol{R} + \boldsymbol{\varepsilon} \tag{14}$$

$\boldsymbol{\varepsilon}$ is noise value by a normal distribution:

$$\varepsilon \sim N(0,\sum) \tag{15}$$

Where $\sum$ is covariance and mean of distribution is zero. In this case, if noise is considered to be additive, $\sum$ is a diagonal matrix which all of its elements in the main diagonal are the same. In fact, noise in response change space is assumed to be spherical, but if noise is multiplicative, the elements of the main diagonals are not the same and noise in the space is ellipsoid. Equation (14) may be written as:

$$\boldsymbol{X} = \boldsymbol{S}^{+}\Delta\boldsymbol{R} + \boldsymbol{S}^{+}\boldsymbol{\varepsilon} \tag{16}$$

In the above equation, normal distribution of $\mathbf{S}^{+}\boldsymbol{\varepsilon}$ is:

$$\boldsymbol{S}^{+}\varepsilon \sim \boldsymbol{N}(0,\boldsymbol{S}^{+}\sum \boldsymbol{S}^{+T}) \tag{17}$$

In fact, $\boldsymbol{S}^{+} \sum \boldsymbol{S}^{+T}$ is new covariance in damaged space. Standard deviation is evaluated by:

$$\beta = \sqrt{\gamma_1^2 + \gamma_2^2 \dots + \gamma_n^2} \tag{18}$$

Where γ_i are eigenvalues of covariance matrix $\boldsymbol{S}^{+}\sum \boldsymbol{S}^{+T}$ and diagonal of ellipsoid and β is standard deviation. When a mapping of response change space to damage space is done, noise in damage space is ellipsoid which values of γ_i is size of diagonals of ellipsoid. In the second section, regularization is also implemented and standard deviation is arrived by implementing filter coefficient.

7. IMPLEMENTING REGULARIZATION IN CALCULATING EXTRACTED STANDARD DEVIATION

Most of the methods, which are used in the last two decays for damage detection, cannot suffer the effects of measured errors that make the problem ill-posed as it is shown in Friswell et al., (2001) and Humar et al., (2006). This limitation makes the existence and unity of solutions unsure and probable numerical instability occurs during solution (Natke, 1993 and Tikhonov, 1995). Our case also is ill-posed problem and during mapping from response change space to damage space, some of diagonals of ellipsoid in damage space are large. Appearance of these large values is because of the existence of errors, which affect the calculation of standard deviation. Hence, damage detection must be performed after regularization is done on these diagonals. In a regularization technique, the most important thing is to keep the effects of the regularization in parameter estimation stable. Regularization factor, which is a function, controls these effects. Tikhonov regularization was used in this research. Filter factor f_i based on regularization parameter λ is (Li and Law, 2010):

$$f_i = \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \tag{19}$$

Where σ_i is the square root of eigenvalues after singular value decomposition of sensitivity matrix and λ is regularization parameter. In calculating standard deviation, big errors in calculating eigenvalues of matrix $S^+ \sum S^{+T}$, made some diagonals of ellipsoid very big. In other words, here the diagonal affected the problem solution greatly and caused unreliable results. So, to regularize these diagonals, a filter coefficient used which had great effects on big diagonals, but small diagonals did not changed a lot. Then, by using filter coefficient of Equation (19), a new filter coefficient is proposed:

$$h_i = \frac{\lambda^2}{\gamma_i^2 + \lambda^2} \tag{20}$$

Where h_i is filter coefficient of each diagonal in calculating standard deviation. If λ is very big, the problem become limited to the same ill-posed problem, but if λ is very small, then the problem deviate a lot from main problem. Hence, the value of parameter λ is limited to $\lambda_1 \leq \lambda \leq \lambda_m$. Which λ_1 and λ_m are smallest and biggest diagonal respectively. When filter coefficient was multiplied to each diagonal of ellipsoid in addition to change the volume of ellipsoid, its center is also displaced and the effects of this displacement were considered. To calculate this displacement, $\boldsymbol{X}$ which is damage vector, by imposing filter coefficient was transformed to $\boldsymbol{Y}$, and displacement of $\boldsymbol{d}$ is calculated:

$$\boldsymbol{d} = \boldsymbol{Y} - h.\boldsymbol{X} \tag{21}$$

To implement this transformation, theory of parallel axis was used. To evaluate standard deviation and effects of this transformation on it, in this research an innovative equation is proposed and have been used (Equation 22). Since the ellipsoid consists of multivariable normal distribution, effects of cross section were not considered.

$$\beta(\lambda) = \sqrt{\left(\left(\frac{\lambda^2}{\gamma_1^2+\lambda^2}\right)\times\gamma_1\right)^2 + \left(\left(\frac{\lambda^2}{\gamma_1^2+\lambda^2}\right)\times\gamma_2\right)^2 + \cdots + \left(\left(\frac{\lambda^2}{\gamma_m^2+\lambda^2}\right)\times\gamma_m\right)^2 + |\boldsymbol{d}|^2} \tag{22}$$

Since the minimum of the above equation must be calculated based on parameter λ, golden section method was used for minimizing λ function. In following, the way of comparing different responses by Equation (22) which are arrived for different types of sensor putting is explained.

8. DETERMINATION OF A STANDARD TO COMPARE THE RESPONSES

To achieve the responses, sensors may be placed in different locations and different loads can be applied. Monte Carlo method was used to compare the responses of this research. Sensors were putted in 10000 locations and responses of each one to different loads were calculated, and standard deviation of 10000 seeds of Monte Carlo was calculated. Then for each response, box plot for arrived standard deviation (β) was created. To compare the responses from these box plots, one can use the extracted index as follows.

- CALCULATING M INDEX

In this work, standard of comparing the responses was their sensitivity to sensor putting. As much as the values in box plots have bigger distances to each other, it means that desired response is more sensitive to sensor putting and noise of that response is bigger. Since the dimensions of different responses are not the same, to compare the responses, a dimensionless index for each box plot is proposed (box plot shows the minimum, maximum, upper and lower three quartiles and median).

$$m = \frac{\max - min}{median} \tag{23}$$

Where max is maximum and min is minimum and median is median value in the box plot. As much as m is bigger, standard deviation values are more far from each other and sensitivity to sensor putting becomes greater. As a result, desired response will have more errors in damage detection steps.

9. TIKHONOV REGULARIZATION IN AN UPDATING PROBLEM

Existence of errors in measured responses, cause some difficulties in damage detection. Most of the methods which are used for damage detection in the last two decays cannot sustain the effects of measurement errors and lead to ill-conditioned problems

(Friswell et al., 2001 and Humar et al.,2006). On the other side, updating problems are sensitive to noises and have conditions of ill-posed. This drawback, limit the existence and unity of results unsure and probable numerical instability happens in the solving process (Natke, 1993 and Tikhonov, 1995). Hence, the small measuring errors lead to big deviation in parameters of model. A classic method to deal with such problems is using regularization methods. It is recognized that the conventional output errors, which are usually the vector of differences between the computed and measured responses, can be made arbitrarily small if the process of damage identification is allowed to behave ''badly'', such that the variable has unrealistically large deviations from the true set of parameter change, or there may be infinite sets of solutions (ill-posed). Regularization methods are implemented by knowing that damage happens in limited number of members and damage intensity is small. As it is pointed out, equation of damage detection is as follows (Li and Low, 2010):

$$\boldsymbol{S}_k \Delta \boldsymbol{X}_{K+1} = \Delta \boldsymbol{R}_k\,, \qquad k=(0,1,2) \tag{24}$$

Equation (24) may be expressed by method of least square, by minimizing cost function as:

$$J(\Delta \boldsymbol{X}_{k+1}) = \left\| S_k \Delta \boldsymbol{X}_{k+1} - \Delta \boldsymbol{R}_k \right\|_2^2 \tag{25}$$

In regularization methods, by adding a penalty function to Equation (25), Equation (26) is proposed as follows:

$$J(\Delta \boldsymbol{X}_{k+1},\lambda) = \left\| \boldsymbol{S}_k \, \Delta \boldsymbol{X}_{k+1} - \Delta \boldsymbol{R}_k \right\|_2^2 + \lambda^2 \left\| \Delta \boldsymbol{X}_{k+1} \right\|_2^2 \tag{26}$$

λ in Equation (26) is regularization parameter and its calculation is explained later. Big damages cause outward instability, hence, our damage vector is full of zero elements and its norm is near to zero. A regularized solution of Equation (26) is written as follows:

$$\Delta X_{k+1} = \sum_{i=1}^{m} f_i \frac{U_i^T \Delta R_k}{\sigma_i} V_i \tag{27}$$

Where m is the number of structural parameters, columns U_i and V_i are left and right singular vectors, respectively. Filter coefficient in Equation (27) is showed by f_i which its calculations are according to Equation (19).Small eigenvalues σ_i has little influence on values of sensitivity matrix but they have great effects on solution process. In the presence of measurements errors, eigenvalues have impressive rules in Equation (27) and will deviate it remarkably from exact results.

Filter coefficient removes this harmful σ_i or make them small. By controlling these two norms, optimized regularization parameter (λ_{opt}) is arrived and according to this regularization parameter, filter coefficient is arrived. Side norm and residual norm are written according to Equation (28) and Equation (29) respectively.

$$\eta^2 = \left\| \Delta X_{k+1} \right\|_2^2 = \sum_{i=1}^{m} \left(\frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \frac{U_i^T \Delta R_k}{\sigma_i} \right)^2 \tag{28}$$

$$\rho^2 = \left\| S_k \Delta X_{k+1} - \Delta R_k \right\|_2^2 = \sum_{i=1}^{m} \left(\frac{\lambda^2}{\sigma_i^2 + \lambda^2} U_i^T \Delta R_k \right)^2 \tag{29}$$

The two above parameters, show the smoothness and chance of equality of solution and they must be regulated by optimized regularization parameter. In Tikhonov regularization, Optimum regularization parameter can be selected by different methods. L-curve methods and generalized cross-validation are two of these methods. In L-curve method, regularization parameter is found by maximizing bellow equation.

$$\frac{d\theta}{ds} = \frac{\rho' \eta'' - \eta' \rho''}{((\rho')^2 + (\eta')^2)^{1.5}} \tag{30}$$

Where $\rho' = \frac{d\rho}{d\lambda}$, $\eta' = \frac{d\eta}{d\lambda}$, $\rho'' = \frac{d^2\theta}{d\lambda^2}$ and $\eta'' = \frac{d^2\theta}{d\lambda^2}$. Maximizing Equation (30) is done by this condition that selected regularization parameter have a singular value between min and max values or $\sigma_1 < \lambda < \sigma_m$. In this method, different λ between $\sigma_1$and σ_m were assumed and depending on the assumed λ, a chart for η^2 toward $\rho^2$is plotted. The sharp corner of this chart was considered to be selected λ. Convergence standard which can be used in Tikhonov regularization is minimum angle between A^k and r^k which is defined (Li and Low, 2010) as:

$$A^k = \begin{bmatrix} S^k \\ \lambda^k I \end{bmatrix} \tag{31}$$

$$r^k = \begin{bmatrix} \Delta R^k \\ -\lambda^k \ (\sum_{i=1}^{k} \Delta X^i) \end{bmatrix} \tag{32}$$

The angle between mentioned vectors is as follows.

$$\cos\theta = \frac{(r^k)^T.(A^k((A^k)^T A^k)^{-1}(A^k)^T r^k)}{\left\|r^k\right\|_2.\left\|(A^k((A^k)^T A^k)^{-1}(A^k)^T r^k)\right\|_2} \tag{33}$$

While using regularization parameter, one may use Newton-Raphson convergence standard by $cos\theta$. θ is angle between ΔR_k and subspace of sensitivity matrix and minimum value of $cos\theta$ can be taken as an criteria to stop the iteration. Hence, if there is no improvement in the value of angle after some iteration, damage vector of min value of $cos\theta$ is taken as converged value.

10. CASE STUDIES

10.1. Evaluating responses by extracted index (m)

10.1.1. Two-dimensional truss

A truss model example by 12 joints and 25 elements was used as the first case study (Figure 1). Elasticity module and special mass of steel were assumed to be 2010000 kg/cm^2 and 7850 kg/cm^2 respectively. Cross-section area of down and up members are 45.9 cm^2 and cross section of vertical and oblique members are 28.5 cm^2 and 39.1cm^2.

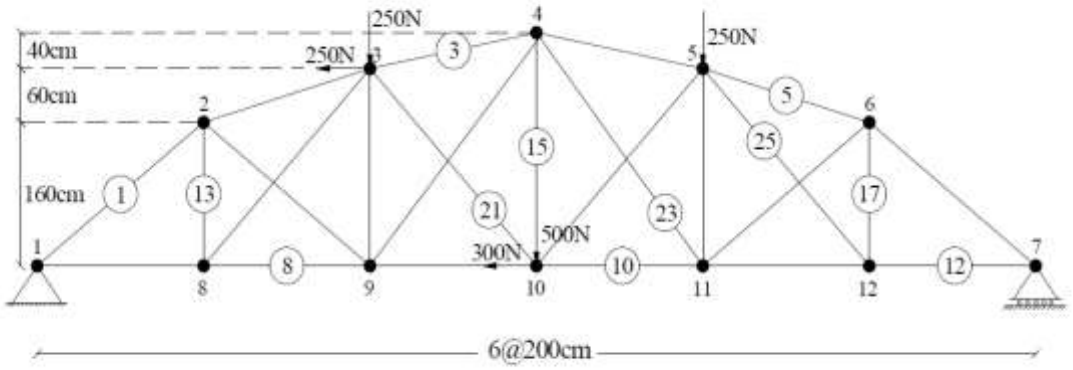

Figure 1. Geometry of two dimensional truss

First, an example of calculating standard deviation for static response and multiplicative noise case is represented. The structure were loaded according to loads shown in Figure 1 and according to Table 1, four sensors were putted in joints 1, 3, 8 and 10 and 4 elements were assumed to be damaged.

Table 1. Damaged elements of structure

Perceagent of damage	Elements place	Elements number
40	3-2	2
10	12-7	12
30	10-4	15
50	8-3	18

After calculating static response of the structure and sensitivity matrix, by using Equation (10) eigenvalues of covariance matrix $S^+ \sum S^{+T}$ is arrived. Since there were 4 sensors and 25 elements, norm of sensitivity matrix is 4×25.Then by considering regularization effects and by Equation (22), standard deviation β according to regularization parameter λ is calculated which for minimizing β toward λ, golden minimization method were used. In Figure 2, different values of β toward λ are plotted.

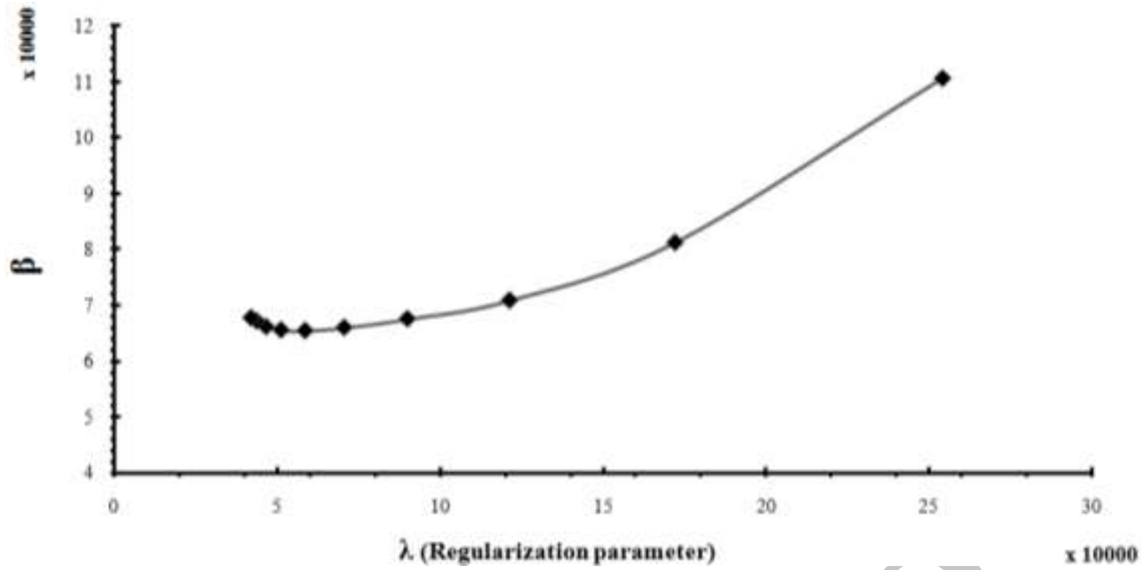

Figure 2. β chart based on λ

In Figure 2, β is minimum in λ=41802.53 and its value is β=64167.67. If regularization effects have not been considered in the problem, β would be 4205200 from Equation (18). The difference between calculated quantities in two cases is because of big effects of filter coefficient on big diagonals of ellipsoid that make them small, but does not have great effects on small diagonals. For evaluating responses and analyzing the effect of sensor location in any of responses, Monte Carlo method was used. 10000 sensors in random locations were putted and static, dynamic, time history frequencies, mode shape responses, and frequency response function were measured.

To derive static responses, different loads were exerted on the structure and results of joints 3, 5 and 10 are presented in tables and figures. For time history response, earthquakes with high and low frequencies were imposed to the structure. The results of the analysis of the two earthquakes are presented later. There were ten modes. Earthquakes which are recorded in Iran in a place near to fault and are taken as high frequency earthquakes are Bam and Tabas earthquakes. Here the results of Bam earthquake are presented and Golbaft was used as low frequency earthquake.

By considering two cases of multiplicative and additive noises, The values of standard deviation β according to Equation (22) for each response is arrived and box plot of each one is created. The values of main diagonal of covariance matrix is one in additive noise case and covariance matrix in multiplicative noise case is a diagonal matrix that values of main diagonal are response of damage structure. In Figure 3, box plots for static response and time history responses with 4, 7 and 10 sensors are given. In Figure 4, box plots for frequency response and FRF with 4, 7 and 10 sensors are created. Box plots for static response and time history with 10 sensors in Figure 5(a) and box plot of mode shape responses with 4, 7 and 10 sensors in Figure 5(b) are presented. LFE and HFE in box plots indicate for low and high frequency earthquakes respectively and F and m are frequency and mode shape.

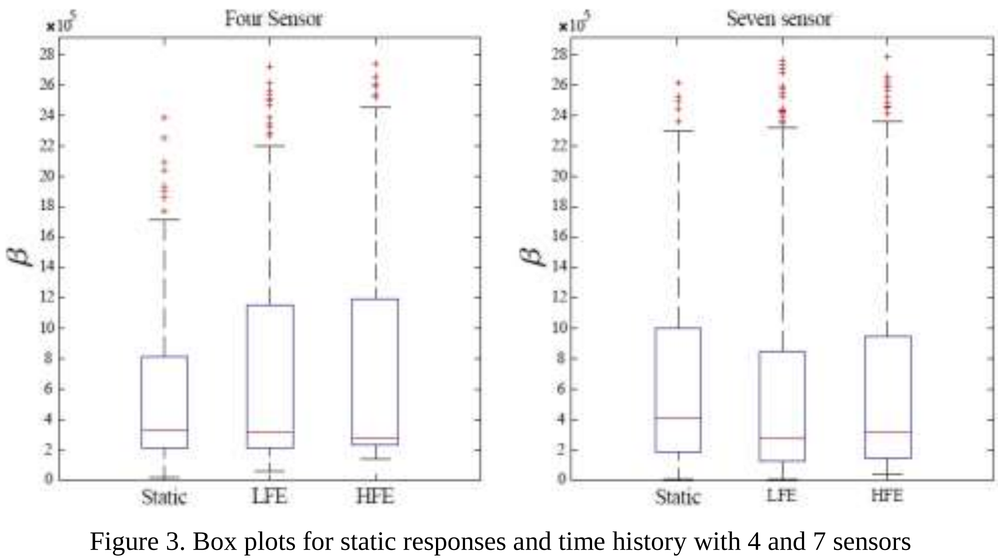

Figure 3. Box plots for static responses and time history with 4 and 7 sensors

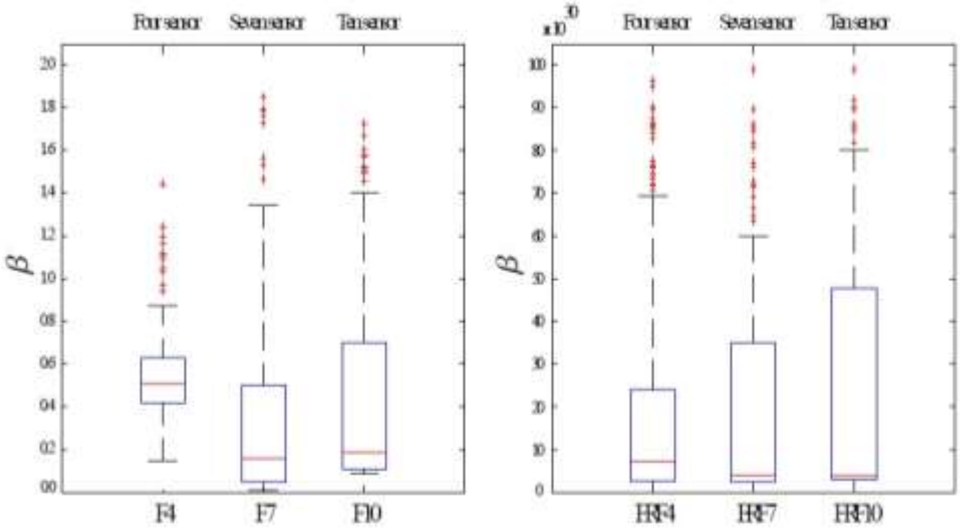

Figure 4. Box plots for frequency response and FRF with 4,7 and 10 sensors

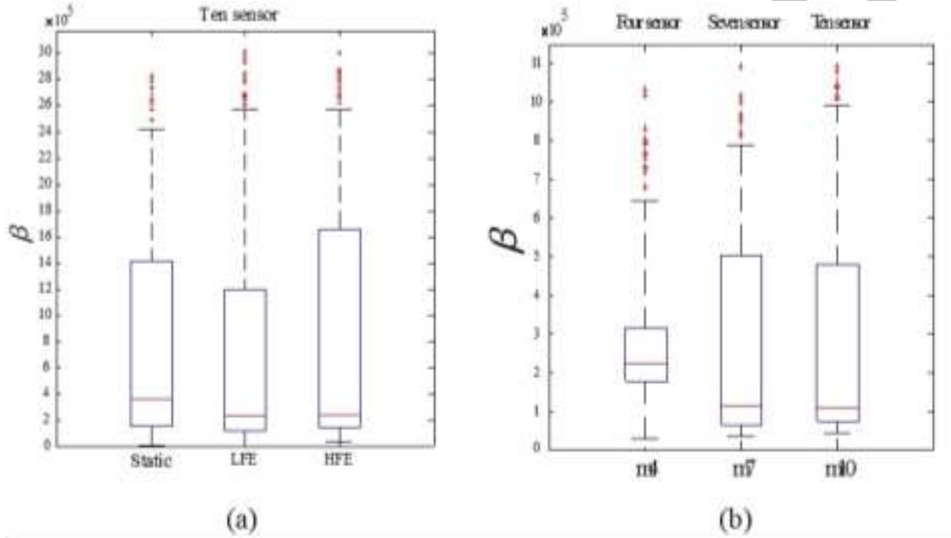

Figure 5(a). Box plots for static response and time history with 10 sensors and b) Box plot for mode shape response with 4,7 and 10 sensors

From box plots, values of m for 4, 7 and 10 sensor putting of different responses for multiplicative and additive noise cases are arrived and presented in Table 2 and 3.

Table 2. m values for different responses in multiplicative noise case

Low Frequency Earthquake	**High Frequency Earthquake**	**Frequency**	**Mode shape**	**FRF**	**Static**	**Number of sensors**
7.2139	8.2534	1.6459	2.9134	7.8162	5.2141	4
8.8831	9.3210	7.4285	7.8361	14.7368	5.5101	7
11.0146	11.1943	9.4651	9.4214	22.8823	6.5200	10

Table 3. m value for different responses in additive noise

Low Frequency Earthquake	High Frequency Earthquake	Frequency	Mode shape	FRF	Static	Number of sensors
8.3431	8.6120	6.7346	8.5875	12.0375	6.6694	4
12.7102	12.3325	7.1189	10.1190	15.5133	6.8163	7
12.5320	11.8821	11.8421	12.3879	21.4111	8.3384	10

It is recognizable from tables that in multiplicative noise case with 4sensors, m has a lower value in frequency response in comparison to other responses. This index in mode shapes was lower toward static case and for responses of time history analyses, in low frequency earthquake had lower value toward high frequency earthquake. In 7 and 10 sensors in static responses, value of m was a little bigger than 4 sensors and had the minimum value between responses. Comparing m values in responses of high and low frequency earthquakes, showed no difference and m in FRF was very big. For frequency response with 7 sensors, this value was much bigger in measuring responses with 4 sensors. There were no differences between the values of m for static responses and frequency response which were less than other responses in additive noise with 4,7 and 10 sensors. Hence, with using 4 sensors, frequency response and with 7 and 10 sensors, response of static response had less sensitivity to sensor putting and had fewer errors in damage detection of this truss.

10.1.2. Two layers diamatic dome

A two layers diamatic dome with 85 joints and 286 members was analyzed as second case study (Figure 6). Elasticity modulus and special gravity of each element are respectively 2010000 kg/cm^{2}and7850 Kg/cm^2. Length, width and height of this structure are 20m, 20m and 6m respectively. This structure in joints 1,7,43 and 49 has supports, which in Figure 6 these joints are shown. Cross section area of the elements is 50 cm^2.

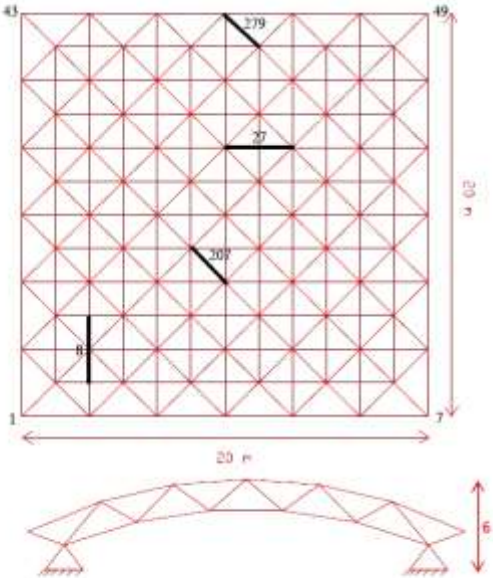

Figure 6. Two layers dome with section

Figure 7 shows the applied loads to the structure and a sample of standard deviation in case of using static response and in multiplicative noise case is shown for this structure. Four sensors in joints 8, 54, 136 and 174 were putted and four elements were supposed to be damaged which are defined in Table 4 and Figure 6. Magnitude of loads was 100 kg and according to what is shown in Figure 7, loads were applied to structure vertically.

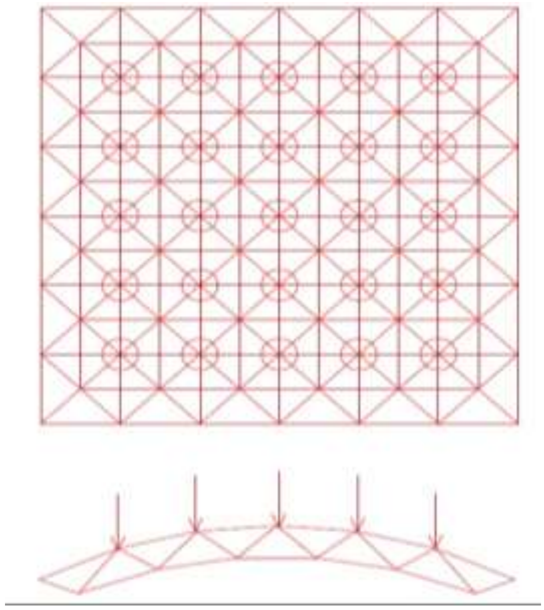

Figure 7. Loads applied to two layers dome

Table 4. Damaged elements of structure

Percentage of damage	Elements place	Elements number
40	51–57	8
10	31–32	27
30	25–65	207
50	46–83	279

After evaluating static response of structure and calculating sensitivity matrix, the eigenvalues of covariance matrix $S^{+}\sum S^{+T}$ are arrived. Then, by considering the effects of regularization and by Equation (22), the magnitude of standard deviation β based on regularization parameter λ is calculated. In Figure 8, the different values of β are plotted toward λ.

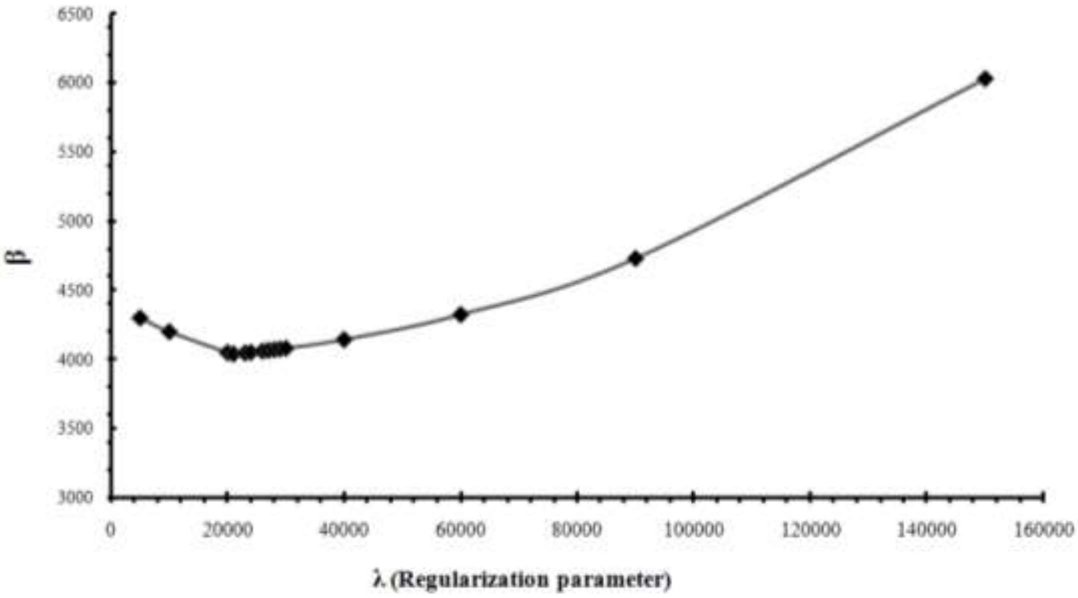

Figure 8. β Chart based on parameter λ

From Figure 8, in λ=21000.71, β= 4039.69 is minimum. If regularization effects were not considered in problem, the value of β would be 15409. For comparing the responses and studding the effects of sensor putting location in each response, Monte Carlo method was used. In 10000 random locations for 4,7 and 10 sensors, different static, dynamic, time history, frequency, mode shape and frequency response function were obtained. To calculate static response, structure was loaded under different loads and results of different joints are shown in Figure 6. To compare the time history response, the structure was subjected to low and high frequency earthquakes. By considering additive and multiplicative noise cases, standard deviation β according to Equation (22) for each response was arrived and box plot of each one is plotted. Figure 9 shows the box plot of 4 and 7 sensors for static response and time history. In Figure 10, box plots of 4 and 7 sensors for FRF response and frequency in multiplicative noise case is represented.

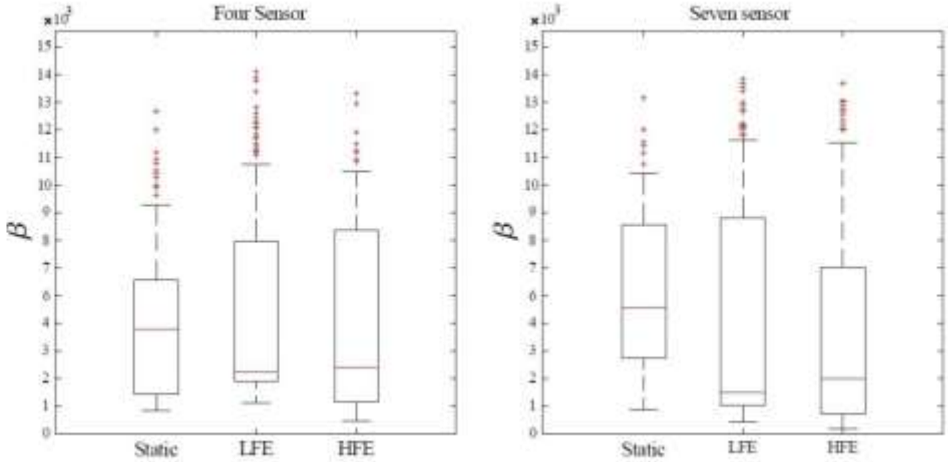

Figure 9. Box plots for static responses and time history with 4 and 7 sensors

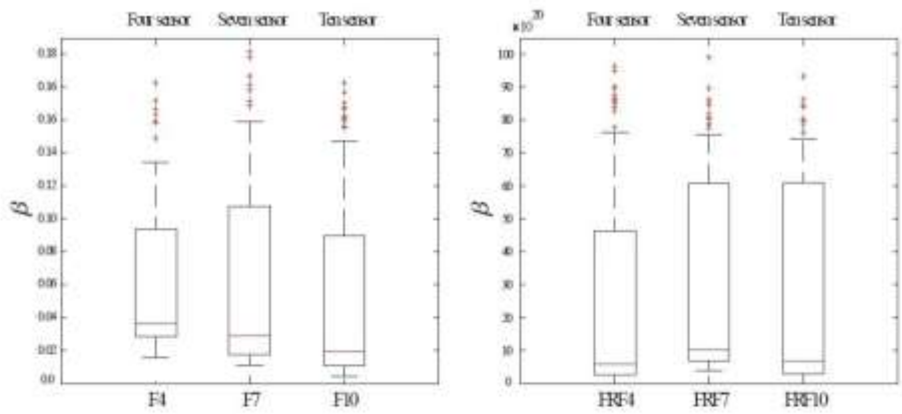

Figure 10. Box plots for frequency response and FRF with 4,7 and 10 sensors

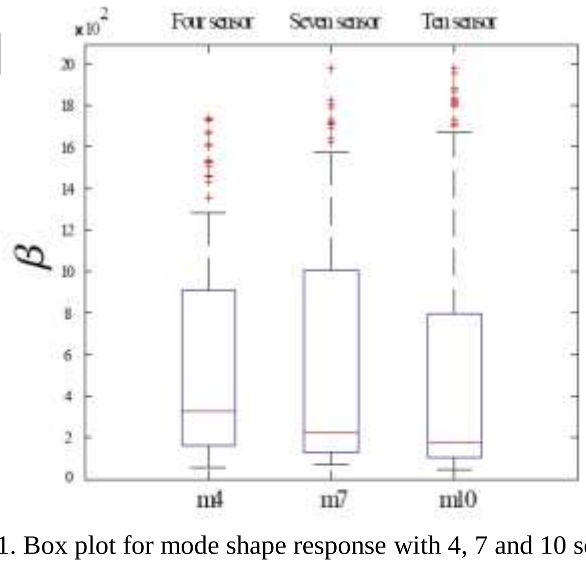

Figure 11. Box plot for mode shape response with 4, 7 and 10 sensors

From box plots shown in Figures 9 to 11, the value of m for 4, 7 and 10 sensor putting in different sensor putting and for two cases of additive and multiplicative noises are arrived according to Table 5 and 6.

Table 5. The values of m index for different responses in multiplicative noise case

Low Frequency Earthquake	High Frequency Earthquake	Frequency	Mode shape	FRF	Static	Number of sensors
4.4044	4.3912	5.4521	4.9136	6.5520	2.2275	4
8.4651	9.3210	6.1284	7.1720	13.7516	2.1327	7
9.6580	10.2574	8.4651	8.4214	14.7231	3.5438	10

Table 6. The values of m index for different responses in additive noise case

Low Frequency Earthquake	High Frequency Earthquake	Frequency	Mode shape	FRF	Static	Number of sensors
6.3812	5.9641	3.6178	3.5131	10.3381	3.8561	4
9.7102	8.0189	4.1189	4.254	14.3671	3.9644	7
8.9137	8.3974	6.8421	8.9130	19.3451	4.3384	10

As it is shown in tables and figures, in multiplicative noise case, response of static deformation for case of using 4 sensors, has lowest value of m compared with other responses. In mode shapes and frequencies, m was bigger than static response and for responses of time history analyses; m was not different between the two cases of using low and high frequency earthquakes. Also in cases of using 7 and 10 sensors to measure static response, the value of m was a little bigger than case of using 4 sensors and had minimum value between other responses. For response of earthquakes, this value was not different a lot between high or low frequency earthquakes and this value was very high in FRF response. This value in frequency response was less than case of using 4 sensors. In additive noise case with 4 sensors, m value was not different in static and frequency response and also mode shapes, while they were less than other responses. However, in using 7 and 10 sensors, this value for static response was less than other responses. Hence, frequency response by using 4 sensors and static response by using 7 and 10 sensors had less sensitivity to sensor locations and less errors in damage detection.

10.2. Evaluating responses by updating sensitivity matrix and applying Tikhonov regularization methods

In this section, by using different responses and Tikhonov Regularization in defined structures, damage identification is performed.

10.2.1. T*wo dimensional truss*

A two dimensional truss of Figure 1 was analyzed in this research as second case study. 4 different cases of damage were studied (Table 7).

Table 7. Different case of damage in structure

Damage case	Damaged members	Percentage of damage of damaged member	Cross section of damaged member (cm^2)
1	Num 4	30	45.9
	Num 12	30	45.9
2	Num 6	20	45.9
	Num 15	30	28.5
3	Num 2	10	45.9
	Num 17	40	28.5
	Num 22	35	39.1
4	Num 4	20	45.9
	Num 9	30	45.9
	Num 15	30	28.5
	Num 20	30	39.1

First, for different application of static response, Damage detection for 0%, 1% and 2 % noise levels in measured responses of structure were done. Damage case 1 was considered and by using Newton-Raphson iteration and updating sensitivity matrix in iterations, optimized regularization parameter was arrived. After some steps, damage became closer to solution. Static loads, which were applied to structure, are as Figure 1. First, the structure was analyzed and measured responses were considered noiseless. In first iteration after arriving values of $\eta^2$and ρ^2 from Equations (28) and (29) and by assuming different λ between minimum and maximum eigenvalues σ_i, L-curve was plotted and regularization parameter of the edge of curve was selected as optimum regularization parameter in this stage of iteration. L- curve of the first iteration is shown in Figure 12.

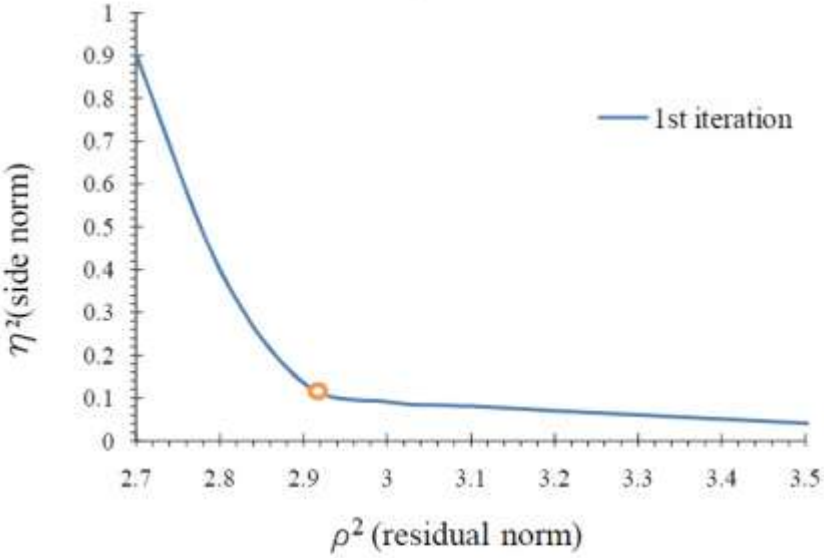

Figure 12. L-curve and optimized regularization parameters in first iteration

According to Figure 12, the values of optimized regularization parameter (λ_{opt}) in first iteration are 22.984. After 9 iterations, exact solution was achieved and $cos\theta$ in this step was minimum (Figure 13).

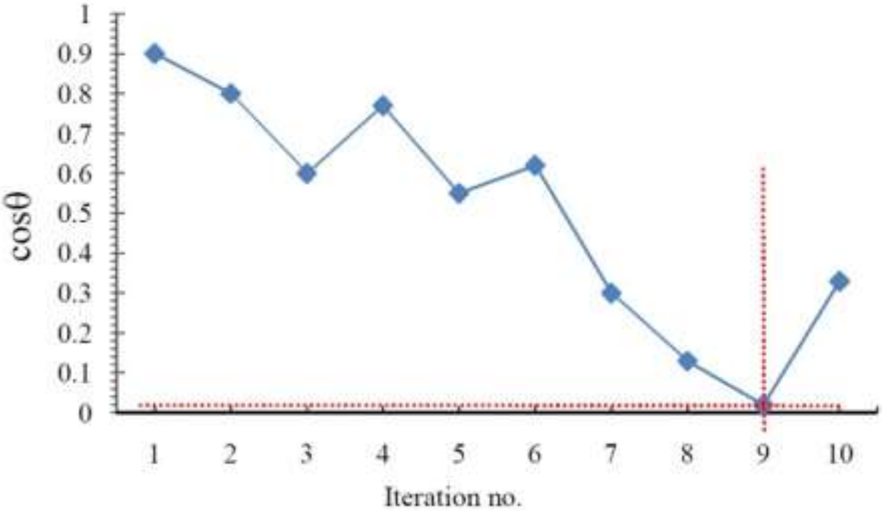

Figure 13. Convergence Standard ($cos\ \theta$) by Tikhonov regularization

Percentage of damage in members 4 and 12 after 9 steps became near to real damage percentage and percentage of damage of other members became near to zero. When noise was implemented to measured response, like noiseless case, damage vector after some iteration converged to real damage, but since the data were noisy, some errors were observed. When there were 1% and 2 % noise levels in static responses, damage vector converged after 13 and 12 steps respectively. Figure 14 show the damage vector after mentioned iteration steps, for 0%, 1% and 2% noise levels.

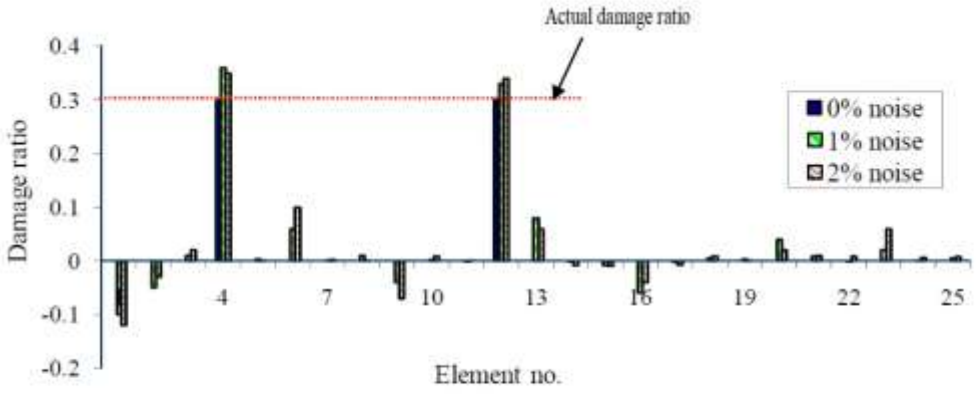

Figure 14. Real and identified damage amounts for 0%, 1% and 2% noise levels in stage of static response (damage case 1)

As it is shown in Figure 14, when there are noises in measured static response, damage vector has some errors and percentage of damage of undamaged members is not completely zero. For other responses, like static response, damage detection is done by updating sensitivity matrix and using Tikhonov regularization. Number of iterations in each response, for different percentages of noise is presented in Table 8.

Table 8. Number of iterations in Newton-Raphson procedure for different responses

Low Frequency Earthquake	**High Frequency Earthquake**	**Frequency**	**Mode shape**	**FRF**	**Static**	**Noise percent**	**Damage case**
17	32	10	12	37	9	0%	
21	28	12	14	33	13	1%	1
21	35	13	14	39	12	2%	
19	29	12	15	35	8	0%	
23	31	15	13	39	11	1%	2
26	35	16	17	41	12	2%	
18	36	13	16	36	12	0%	
25	33	17	18	38	14	1%	3
30	36	20	19	41	17	2%	
29	43	21	26	48	19	0%	
34	44	24	27	50	23	1%	4
33	48	25	27	57	23	2%	

In Figures 15 to 23, the value of real and identified damages for 0%, 1% and 2% noise levels in case of using different responses and for two damage cases 1 and 4 are shown.

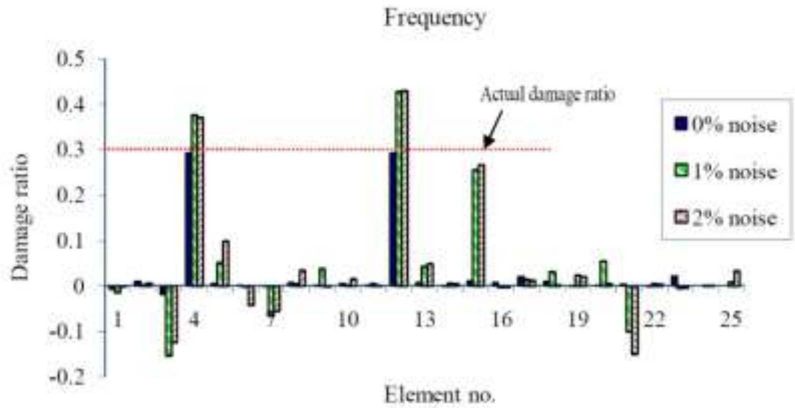

Figure 15. The magnitude of real and identified damages for 0%, 1% and 2% noise levels in case of using frequency response (damage case 1)

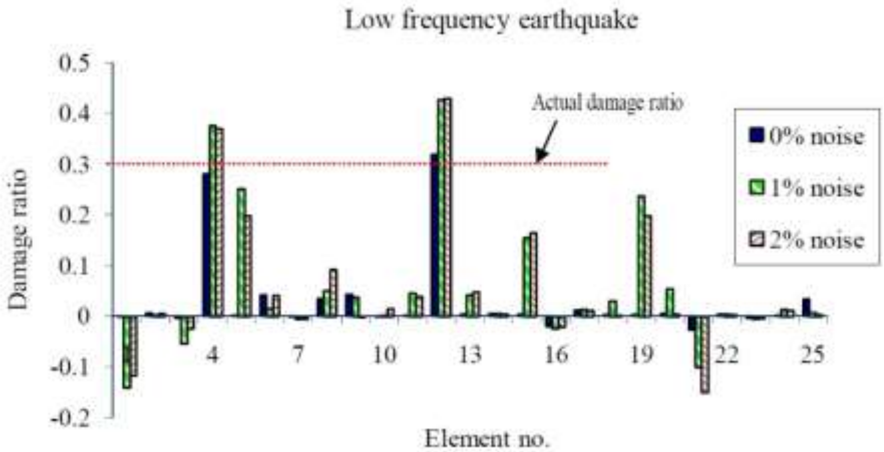

Figure 16. The magnitudes of real and identified damage for 0%, 1% and 2% noise levels in case of low frequency earthquake (damage case 1)

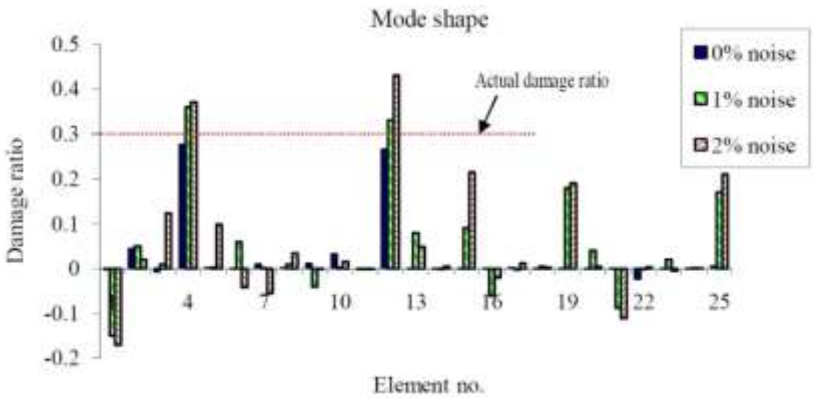

Figure 17. The magnitude of real and identified damages for 0%, 1% and 2% noise levels in case of using mode shapes (damage case 1)

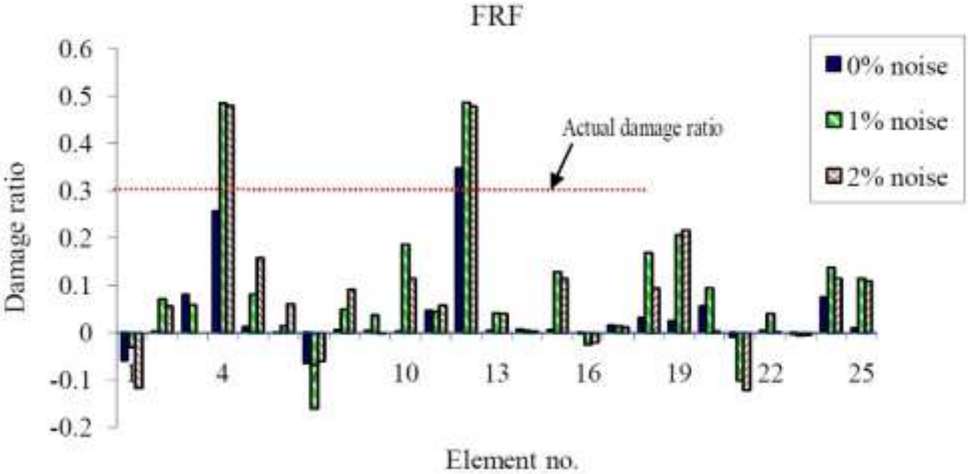

Figure 18. The magnitudes of real and identified damage for 0%, 1% and 2% noise levels in case of using FRF (damage case 1)

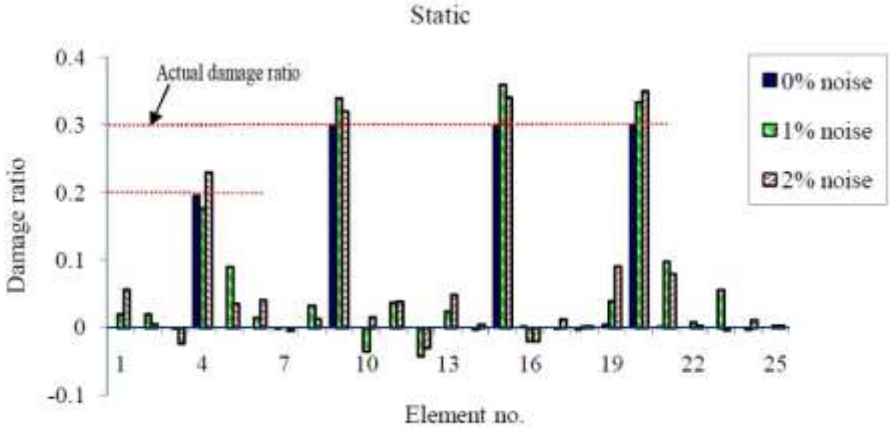

Figure 19. The magnitudes of real and identified damage for 0%, 1% and 2% noise levels in case of using static response (damage case 4)

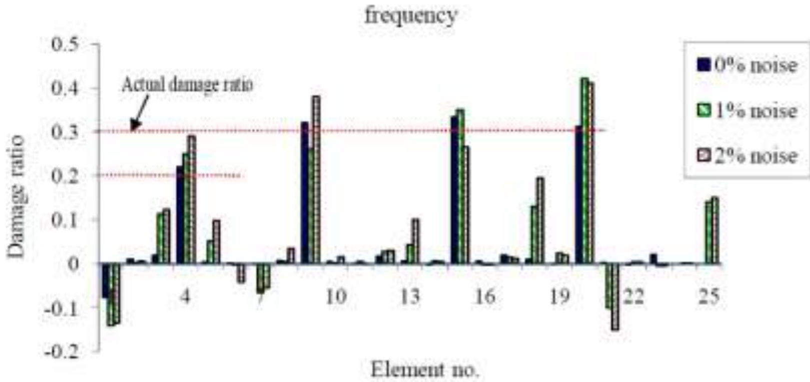

Figure 20. The magnitudes of real and identified damage for 0%, 1% and 2% noise levels in case of using frequency response (damage case 4)

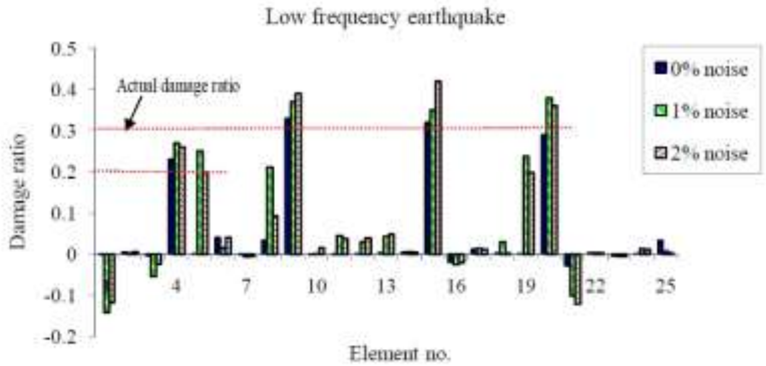

Figure 21.The magnitudes of real and identified damages for 0%, 1% and 2% noise levels in case of using low frequency earthquake (damage case 4)

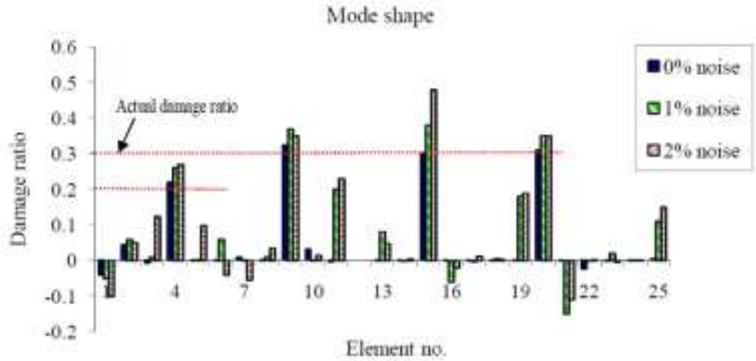

Figure 22. The magnitudes of real and identified damage for 0%, 1% and 2% noise level in case of using mode shapes (damage case 4)

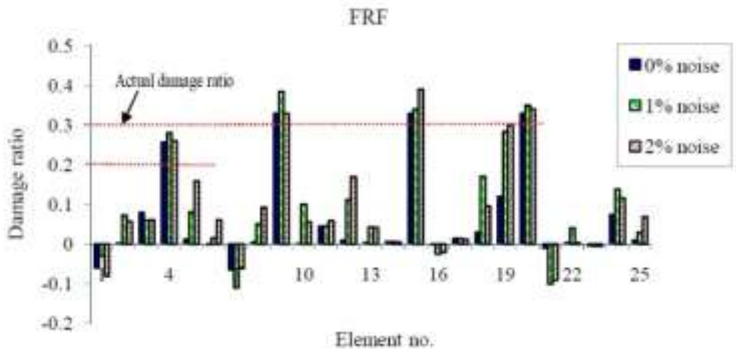

Figure 23. The magnitudes of real and identified damage for 0%, 1% and 2% noise levels in case of using FRF response (damage case 4)

As it is shown in Table 8, the number of iterations in updating sensitivity matrix for convergence, in static response and frequency response for different noises is smaller than other responses and becomes near to real damage earlier.
According to Figures 14 to 23, different responses, does not detect the damage similarly. For example, in Figure 14, in static response of noiseless case, identified damage is completely in agreement with real damage, while in 1% and 2% noise levels is not completely in agreement and has little errors.
According to Figure 15 which shows frequency response in noiseless case, beside identified damage had errors, but damage was identified perfectly and in addition to damaged members 4 and 12, member 15 were also reported to be damaged.
Also in Figure 16, for the response of time history of low frequency earthquake in noiseless case, some errors in damage detection exist and for 1% and 2% noise levels, in addition to real damaged members, members 15, 19 and 22 are also reported as damaged members. In Figure17, also for mode shape responses and noiseless case, damaged members are not shown properly and for noise polluted cases, some intact members are reported as damaged members. In Figure18, in FRF for 0%, 1% and 2% noise levels, identified damage is not coincident to real damage and has many errors. In Figures19 to 23, for 4th case of damage, identified damage value for different responses is shown. As it is recognizable from figures, static response of this structure toward other responses is more effective in damage detecting and has less error.

10.2.2 Two layers diamatic dome

Second case study was a two layers diamatic dome of Figure 6, which its characteristics are given in section 10.1.2. Identifying the location and intensity of damage in this structure was done by updating sensitivity matrix. Two different cases of damage were considered according to Table 9.

Table 9. Different cases of damage in structure

Damage case	Damaged members	Percentage of damage in members	Label of members by joints
1	63	30	21-28
	211	30	26-66
4	8	20	51-57
	27	30	31-32
	207	30	25-65
	279	30	46-83

For static response, damage detection for 0%, 1% and 2% noise levels in measured responses of structure were done and static loads were applied to structure as Figure 7. First, damage case 1 was considered, structure was analyzed and measured responses were considered noiseless. In Figure 24, L-Curve of first iteration is shown.

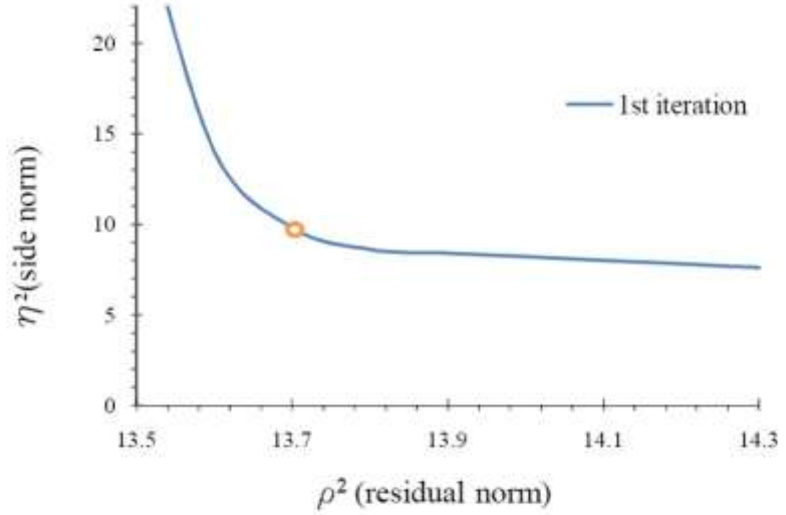

Figure 24. L-curve and optimized regularization parameters in first iteration

According to Figure 24, the value of optimized regularization parameter (λ_{opt}) in first iteration is 72.547. After 80 iterations, damage became near to real damage and $cos\theta$ in this case was minimum (Figure 25).

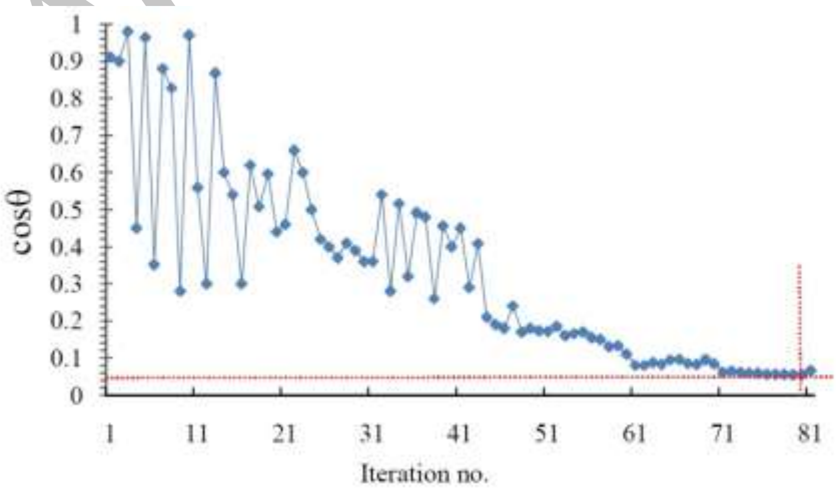

Figure 25. The convergence standard ($cos\theta$) by Tikhonov regularization

When noise was implemented to measured responses like noiseless case, damage vector after some iteration became near to real damage, but since there were some noises, calculations were not exact. For other responses, just like static response, damage detection was performed. In table 10, the number of iterations in each response is shown for different numbers of noises.

Table 10. Number of iterations in Newton-Raphson process for different responses

Low Frequency Earthquake	High Frequency Earthquake	Frequency	Mode shape	FRF	Static	Noise percent	Damage case
181	193	134	131	201	80	0%	
201	208	160	141	250	130	1%	1
220	226	144	150	258	135	2%	
154	163	156	160	195	110	0%	
199	170	135	190	221	140	1%	4
206	168	170	175	225	120	2%	

As it is shown in Table 10, the number of iterations for updating sensitivity matrix to converge in case of static response for different noises was smaller than other responses and it converged to real damage earlier. In Figure 26 to 29, the value of identified damage for 0%, 1% and 2% noise levels in case of using different responses and for damage case 1 is shown.

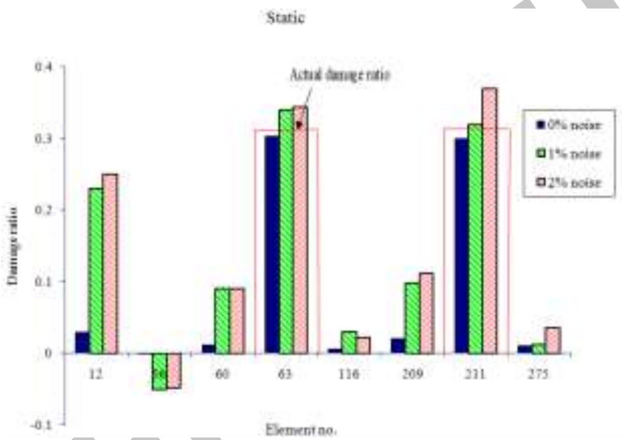

Figure 26. The values of real and identified damages for 0%,1% and 2% noise levels in case of using static response

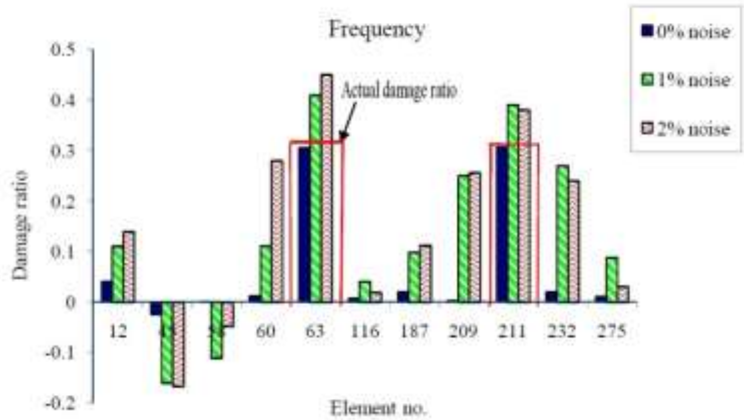

Figure 27. The values of real and identified damages for 0%, 1% and 2% noise levels in case of using frequency response

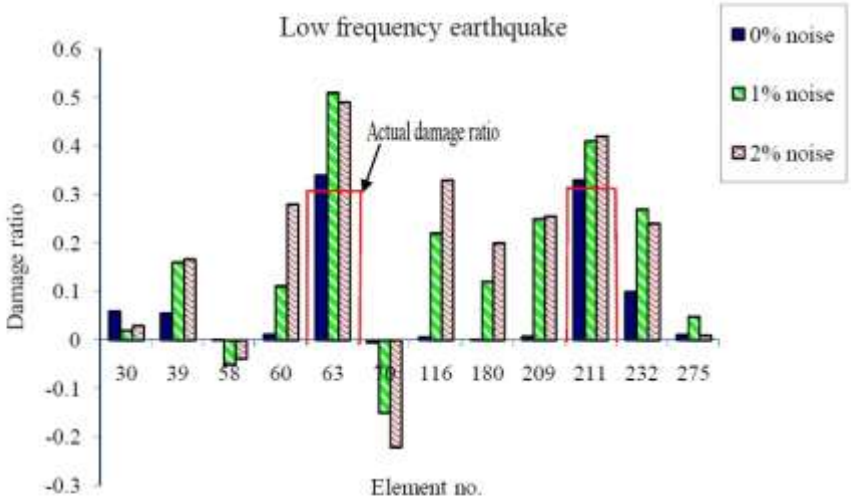

Figure 28. The values of real and identified damages for 0%, 1% and 2% noise levels in case of using low frequency earthquake

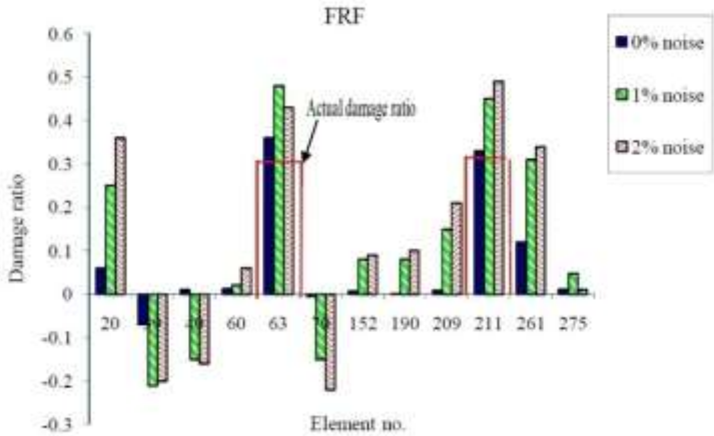

Figure 29. The values of real and identified damages for 0%, 1% and 2% noise levels in case of using FRF response

In Figure 26 to 29, the values of identified damage for different responses are different. In Figure 26, for static response in case of noise free data, identified damage is nearly in agreement with real damage, but for 1% and 2% noise levels, it is not completely in agreement with real damage and has few errors. In Figure 27, for frequency response in case of noiseless data, identified damage has trivial error and damage in members is well identified. When measured response has noise and identified damage has big errors, in addition to showing damaged members 63 and 211, members 45, 60, 209 and 232 were also reported to be damaged. In Figure 28, in response of time history in low frequency earthquakes, in noiseless case, there are more errors compared with static and frequency responses in damage detection and for 1%, 2% noise levels in addition to real damaged members, members 60, 70, 209 and 232 also have damaged. In Figure 29 and in case of using FRF response also for 0%, 1%, 2% noise levels identified damage does not coincidence to real damage and has a lot of errors, so that in addition to real damaged members, members 20, 39, 70, 209 and 261 are reported to be damaged. Therefore, in this structure also, static response for identifying damage, had fewer errors and available noises in this response had fewer effects on damage detection.

11. CONCLUSIONS

In this paper the effect of each response in damage detection by sensitivity-based analyses are studied. In evaluating a two dimensional truss, it was shown that for multiplicative noise case with small numbers of sensor putting, desirable index for frequency response had lower value and it was more sensitive to sensor putting but for big numbers of sensor putting, static response had the lowest value of this index and had minimum sensitivity to sensor putting. Hence, static response is the most trustful response for damage detecting of this structure but in additive noise case for evaluating responses, it is not appropriate. Also in evaluating a two layers dramatic dome, desired index for each response was extracted and it was shown that for different numbers of sensor putting, this index for static deformation response had lower value and this response had less sensitivity toward sensor putting.

In the second section of this research, different responses were used to detect damage in structures, but since it was an ill-posed problem, Tikhonov regularization method was used to derive exact results for each response, through an updating problem. In evaluating a two-dimensional truss and two layers dramatic dome, by assuming different cases of damage, the number of iterations in case of using static and frequency response, for different cases of noise is lower than other responses. Detected damage by static response was not coincident completely to real damage and had a little noise. But for other responses, when measured response was noise polluted, identified damage had a lot of errors and some of undamaged members also had big percentage of damages. Consequently, static response is more efficient in comparison to other responses in damage detecting of the case studies. In damage detection by sensitivity-based methods, static responses are the most effective responses and have fewer errors. Comparing to other analysis, static analyses require fewer information and for solving the static equilibrium, only the stiffness of structure is needed and applied loads must be defined. However, the form of applying loads must be in a way that zero force members do not form in the structure and this is one of the drawbacks of static response which dynamic and time history responses do not.

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