

# Thermophoretic deposition efficiency of particles in flow along an annular cross-section

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#### Abstract

Thermophoresis is an important mechanism of micro-particle transport due to a temperature gradient in the surrounding medium and has found numerous applications, especially in the field of aerosol technology. This study investigates the thermophoretic deposition efficiency of particles in laminar gas flow in a concentric annulus using the critical trajectory method numerically. The governing equations include the momentum and energy equations for the gas and the particle equations of motion. The effect of the annulus size and the ratio of inner to outer tube wall temperature on the deposition efficiency were studied. Simulation results suggest that thermophoretic deposition increases with increasing thermal gradient, deposition distance, and the ratio of inner to outer radius.

Keywords: Thermophoresis, thermophoretic deposition efficiency, particle deposition, annulus

#### **1. Introduction**

Thermophoresis is a physical phenomenon in which aerosol particles suspended in a gas acquire a velocity in the direction of decreasing temperature due to collisions with the surrounding gas molecules.

Thermophoresis causes the aerosol particles to move toward and deposit on the relatively cool walls in non-isothermal systems. [6]

Particle deposition due to thermophoresis has been identified as a working principal of many industrial processes such as fabricating optical wave guide in the chemical vapor deposition (CVD), designing of thermal precipitators to remove micro-particles from gas stream, and also micro-contamination controlling in the semi-conductor's industry.

On the other hand, because of unfavorable particle surface deposition, thermophoresis may reduces thermal conductivity of heat-exchanger pipes, production yield of specialty powders which manufactured in high temperature aerosol reactors, and also the safety assessment of radioactive aerosol's transfer, generated in a nuclear reactor accident. [2]

Four types of approaches have been made to derive the theory of thermophoresis. Among them, theories derived by *Epstin (1929)* and *Brock (1962)* were more widely used because of their simplicity of calculation thermal force or thermophoretic velocity. [4]

The expression for thermal force near continuum limit, from *Brock*, is,

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$$F_{th} = \frac{-12pmnd_{p}C_{s}(\frac{K_{g}}{K_{p}} + C_{t}Kn)\frac{\nabla T}{T}}{(1 + 3C_{m}Kn)(1 + 2\frac{K_{g}}{K_{p}} + 2C_{t}Kn)}$$
(1)

Where Kn is Knudsen number, or the ratio of gas mean free pass (1) to the radius of particle.

The thermophoretic velocity is defined as the terminal velocity at which the particle moves in a gas at dynamically equilibrium, i.e., when the drag and thermal forces on the particle balance each other, and generally can be expressed as,

$$v_t = -n \frac{K_T}{T} \nabla T \tag{2}$$

Where  $K_T$  is called thermophoretic coefficient and defined by Brock (1962) as,

$$K_{T} = 2C_{c}C_{s} \frac{(\frac{K_{g}}{K_{p}} + C_{t}Kn)}{(1 + 3C_{m}Kn)(1 + 2\frac{K_{g}}{K_{p}} + 2C_{t}Kn)}$$
(3)

For complete accommodation, the reasonable values for  $C_m$ ,  $C_s$  and  $C_t$  are 1.14, 1.17 and 2.14 respectively. [4]

The Cunningham slip correction factor,  $C_c$ , is given by,

$$C_c = 1 + Kn[1.257 + 0.4\exp(\frac{-1.1}{Kn})]$$
(4)

Theoretical studies about thermophoretic deposition of aerosols in tube and pipe flow has been attempted by several groups in the past. [1]

Thermophoretic deposition of particles in an annular, has been studied theoretically by *Weinberg*(1983) and *Fiebig et al.* (1988) considering particle deposition only by thermophoresis and using Eulerian method. [2,3]

*Chang et al.* (1985), also studied thermophoretic deposition in an annular with both numerical simulation (*Eulerian* method) and experiment. [2]

In this work, the thermophoretic deposition efficiency was calculated for a gas flow in a concentric annulus by critical particle trajectory method, assuming the particle diffusion in the bulk of gas is negligible since the typical particle size in about 1 micrometer.

A particle starting from critical radial position at the entrance, will deposit just at the end of the annulus, which is name the critical particle trajectory. [1]



The analytical velocity distribution, presented by *Bird (1960)* is used to numerically calculate the developing temperature profile. Then the critical particle trajectory and deposition efficiency are obtained by solving the particle equations of motion.

## 2. Governing Equations

### 2.1. Velocity and Temperature Profiles

The system under investigation, as depicted in fig. 1, involves fully developed and laminar flowing of a hot gas containing suspended aerosol particles, in an annulus with the two concentric cylinders at constant wall temperature.



Fig. 1. Schematic of the physical configuration of the investigated system

The steady state momentum balance for a fully developed, laminar and incompressible fluid flowing through the annular region between two coaxial circular cylinders has been solved and presented by *Bird et. al. (1960)*. The solution for velocity is,

$$v = \frac{2u_m}{1 + k^2 + \frac{1 - k^2}{\ln k}} \left[ 1 - \left(\frac{r}{R_o}\right)^2 - \frac{1 - k^2}{\ln k} \ln\left(\frac{r}{R_o}\right) \right]$$
(5)

Where, k is the ratio of inner to outer radius of the cylinders and  $u_m$  is the bulk mean velocity.

When the flow is fully developed, the temperature could still be developing when there is a sudden temperature jump in the annulus walls.

The fully developed velocity profile was used to calculate the developing temperature field numerically by solving the 2-D cylindrical energy equation,

$$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = a\left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
(6)



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Since v is equal to 0 in fully developed flow and also, the axial conduction of heat is negligible relies on the fact that Peclet number is taken to be large, equation (6) reduced to form,

$$u\frac{\partial T}{\partial z} = a\left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right]$$
(7)

With boundary conditions,

$$T(r,0) = T_e, \ T(R_i, z) = T_i, \ T(R_o, z) = T_o$$
(8)

Equation (7) can be non-dimensionalized as below, assuming all relevant physical properties are constant,

$$u^* \frac{\partial q}{\partial z^*} = \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial q}{\partial r^*} \right)$$
(9)

Where

$$q = \frac{T - T_e}{T_i - T_o}, r^* = \frac{r}{D_H}, z^* = \frac{z/D_H}{\text{Re.Pr}} \text{ and } u^* = \frac{u}{u_m}$$

The governing equation was discretized using a finite volume method with non uniform grid spacing, i.e. the grid spacing is finer near the walls and inlet region where temperature gradients in radial directions are expected to be larger. The convective terms were discretized using hybrid scheme, also.

#### 2.2. Particle Trajectory and Thermophoretic Deposition Efficiency

The Lagrangian approach is used for predicting the trajectory of a single particle in the fluid. The particle equations of motion in z (axial) and r (radial) directions, in a cylindrical coordinates, are:

$$\frac{d^2 z}{dt^2} = C_d \frac{\operatorname{Re}_p \left(u - u_p\right)}{24} \tag{10}$$

$$\frac{d^2 r}{dt^2} = C_d \frac{\text{Re}_p}{24} \frac{(v - v_p)}{t} + \frac{v_t}{B.m_p}$$
(11)

Where  $C_d$  is the drag coefficient and was described by *Rader and Marple (1985)* as,

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$$C_{d} = \begin{cases} \frac{24}{\text{Re}_{p}} (1 + 0.0916 \,\text{Re}_{p}) & \text{Re}_{p} < 5\\ \frac{24}{\text{Re}_{p}} (1 + 0.158 \,\text{Re}_{p}^{2/3}) & 5 < \text{Re}_{p} < 1000 \end{cases}$$
(12)

The first term on the right hand side of equations (10) and (11) is the drag force due to relation motion between particle and fluid. The second term in equation (11) is the thermophoretic force.

The particle equations of motion were integrated numerically through the domain of interest by mean of Fourth-order Rung-Kutta method. It is assumed that the initial velocity of particles at the entrance is the same as the gas flow velocity at that point. The new particle position and velocity after a small time increment is calculated by numerical integration. The procedure is repeated until the particle hits the walls or leaves the calculation domain, when the critical radial position,  $r_c$ , will be obtained.

The thermophoretic deposition efficiency is defined as the ratio of the particle's rate of deposition on the wall to the particle's rate of entering the annulus, assuming particle concentration is uniform at the inlet of annulus.

For fully developed flow, the velocity profile is known by equation (5) and the equation of deposition efficiency, Eq. (13), can be solved by means of numerical integration.

$$h = 1 - \frac{\int_{r_{c_i}}^{r_{c_o}} u(r) \, 2pr dr}{u_m p R_o^2 (1 - k^2)} \tag{13}$$

Where  $r_{c_o}$  and  $r_{c_i}$  in the above equation, are critical radial positions due to outer and inner cylinders, respectively.

#### 3. Results and discussion

The fluid and particle properties used in the calculation were estimated at the averaged temperature of inlet gas and annulus walls.

The validity of the numerical solution was checked by comparing simulation results with that previously published by *Lin* and *Tsai* (2003) for a laminar tube flow case as shown in Fig. 2. The results were shown to be in good agreement with that simulation since the deviation is about 15%.



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Fig. 4. Comparison of present numerical thermophoretic deposition efficiency with theoretical prediction of Lin & Tsai in laminar tube flow vs.  $x^*$ , for  $\theta^* = 2.7$  and  $Pr. K_T = 0.4$ 

The simulation for an annular flow system was modified by incorporating suitable boundary condition and velocity expression.

In order to make sure the simulated temperature field is correct, the numerically solution of developing temperature is compared with analytical solution presented by Kakac et al. (1987).

Kakac et al. Estimated the fluid bulk mean temperature or "mixing cup",  $T_m$ , for a finite number of k and dimensionless axial distance, Z.

In this study the fluid bulk's mean temperature can be calculated by numerical integration of equation (14).

$$T_m = \frac{1}{A_c u_m} \int_{A_c} u T \, dA_c \tag{14}$$

Fig. 3 shows good agreement in the comparison of  $T_m$  between present study and analytical solution described by Kakac et al.





Fig. 3. Comparison of  $T_m$  between present study and analytical solution described by Kakac vs.  $\kappa$ , for Z = 0.01

This study was carried out to examine the effects of axial distance, thermal gradient, and the tube radius ratio on thermophoretic deposition efficiency.

Fig. 4 illustrate the thermophoretic deposition efficiency as a function of Z for k = 0.1 and q = 0.1, 0.5 and 0.92.

It is apparent from fig. 4 that thermophoretic deposition increases with increasing thermal gradient at the same axial distance and with increasing axial distance at the same thermal gradient.



Fig. 4. Thermophoretic deposition efficiency vs. Z, for  $\kappa = 0.1$  and  $\theta = 0.1$ , 0.5 and 0.92

We have also investigated the thermophoretic deposition efficiency for various values of the tube radius ratio, k. In fig. 5 deposition efficiency is plotted as a function of Z for q = 0.5 and



k = 0.1, 0.3 and 0.5. At all axial positions the tube configuration with the smaller annular region produces a larger efficiency, since the temperature gradients become greater.



Fig. 5. Thermophoretic deposition efficiency vs. Z, for  $\theta = 0.5$  and  $\kappa = 0.1$ , 0.3 and 0.5

#### 4. Conclusion

We have studied the thermophoretic deposition efficiency for a model system where gas flows in an annular region between tow concentric cylinders.

The governing equations include the momentum and energy equations for the gas and the particle equations of motion. The critical trajectory method is used to investigating the thermophoretic deposition efficiency of particles in laminar gas flow numerically.

It was shown that, by maintaining the temperature gradient in the gas, complete deposition may be readily achieved.

Furthermore, the present work appears to indicate that for increasing the thermophoretic deposition efficiency, one would need a higher thermal gradient, a longer deposition distance or a larger tube radius ratio.

#### Nomenclature

В	Dynamic mobility, $B = C_c / 3pm d_p$
$C_{c}$	Cunningham slip correction factor
$C_{d}$	Drag coefficient (defined by Eq. (12))
$C_m$	Momentum exchange coefficient
$C_{s}$	Thermal slip coefficient
$C_t$	Temperature jump coefficient



$D_{_H}$	Hydraulic diameter of annulus, $2R_o(1-k) 2R_o(1-\kappa)$
$d_p$	Particle diameter
$F_{th}$	Thermophoretic force
$K_{g}$	Gas thermal conductivity
Kn	Knudsen number
$K_p$	Particle thermal conductivity
$K_T$	Thermophoretic coefficient
L	Annular length
$m_p$	Particle mass
$Pe_{g}$	Gas Peclet number, $(u_m R_o)/a.L$
Pr	Gas Prandtl number
<i>r</i>	Radial coordinate
r	Dimensionless radial coordinate, $r/D_H$
$r_c$	Critical radial position
Re	Particle Reynolds number
Λ <sub>i</sub> D	Outer tube radius
$\mathbf{K}_{o}$	
I <sub>e</sub> T	Inlet gas temperature
I <sub>i</sub>	Inner tube gas temperature
$T_m$	Mixing cup temperature, (defined by Eq. (14))
	Outer tube gas temperature
VT	Temperature gradient Gas velocity (z-component)
и И <sup>*</sup>	Dimensionless gas velocity (z -component) $\mu/\mu$
u	Average gas velocity
V	Gas velocity (r-component)
<i>u</i> <sub>p</sub>	Particle velocity (z -component)
$v_p$	Particle velocity (r -component)
V <sub>t</sub>	Thermophoretic velocity
$x^*$	Dimensionless distance from the entry for tube calculation, $x^* = z/0.1R_a Pe_a$
Z.	Axial coordinate
$z^*$	Dimensionless axial coordinate, $\left(\frac{z}{D_{H}}\right)/\text{Re}.\text{Pr}$
Ζ	Dimensionless distance from the entry, $Z = z/R_o P e_g$
	,

Greek letters

*a* Gas thermal diffusivity

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h	Thermophoretic deposition efficiency
q	Dimensionless temperature, $(T - T_e)/(T_i - T_o)$
$q^*$	Dimensionless temperature parameter, $T_w/(T_e - T_w)$
k	Inner to outer radius ratio, $R_i/R_o$
m	Gas dynamic viscosity
n	Gas kinematic viscosity
t	Particle relation time, $t = r_p d_p^2 C_c / 18m$

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