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Thermophoretic deposition efficiency of particles in flow along an annular cross-section

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Abstract

Thermophoresis is an important mechanism of micro-particle transport due to a temperature gradient in the surrounding medium and has found numerous applications, especially in the field of aerosol technology. This study investigates the thermophoretic deposition efficiency of particles in laminar gas flow in a concentric annulus using the critical trajectory method numerically. The governing equations include the momentum and energy equations for the gas and the particle equations of motion. The effect of the annulus size and the ratio of inner to outer tube wall temperature on the deposition efficiency were studied. Simulation results suggest that thermophoretic deposition increases with increasing thermal gradient, deposition distance, and the ratio of inner to outer radius.

Keywords: Thermophoresis, thermophoretic deposition efficiency, particle deposition, annulus

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1. Introduction

Thermophoresis is a physical phenomenon in which aerosol particles suspended in a gas acquire a velocity in the direction of decreasing temperature due to collisions with the surrounding gas molecules.

Thermophoresis causes the aerosol particles to move toward and deposit on the relatively cool walls in non-isothermal systems. [6]

Particle deposition due to thermophoresis has been identified as a working principal of many industrial processes such as fabricating optical wave guide in the chemical vapor deposition (CVD), designing of thermal precipitators to remove micro-particles from gas stream, and also micro-contamination controlling in the semi-conductor's industry.

On the other hand, because of unfavorable particle surface deposition, thermophoresis may reduces thermal conductivity of heat-exchanger pipes, production yield of specialty powders which manufactured in high temperature aerosol reactors, and also the safety assessment of radioactive aerosol's transfer, generated in a nuclear reactor accident. [2]

Four types of approaches have been made to derive the theory of thermophoresis. Among them, theories derived by *Epstin (1929)* and *Brock (1962)* were more widely used because of their simplicity of calculation thermal force or thermophoretic velocity. [4]

The expression for thermal force near continuum limit, from *Brock*, is,

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$$F_{th} = \frac{-12pmnd_p C_s (\frac{K_g}{K_p} + C_t Kn)\frac{\nabla T}{T}}{(1+3C_m Kn)(1+2\frac{K_g}{K_p} + 2C_t Kn)} \quad (1)$$

Where Kn is Knudsen number, or the ratio of gas mean free pass *(l)* to the radius of particle.

The thermophoretic velocity is defined as the terminal velocity at which the particle moves in a gas at dynamically equilibrium, i.e., when the drag and thermal forces on the particle balance each other, and generally can be expressed as,

$$v_t = -n\frac{K_T}{T}\nabla T \quad (2)$$

Where K_T is called thermophoretic coefficient and defined by *Brock (1962)* as,

$$K_T = 2C_c C_s \frac{(\frac{K_g}{K_p} + C_t Kn)}{(1+3C_m Kn)(1+2\frac{K_g}{K_p} + 2C_t Kn)} \quad (3)$$

For complete accommodation, the reasonable values for C_m, C_s and C_t are 1.14, 1.17 and 2.14 respectively. [4]

The Cunningham slip correction factor, C_c, is given by,

$$C_c = 1 + Kn[1.257 + 0.4\exp(\frac{-1.1}{Kn})] \quad (4)$$

Theoretical studies about thermophoretic deposition of aerosols in tube and pipe flow has been attempted by several groups in the past. [1]

Thermophoretic deposition of particles in an annular, has been studied theoretically by *Weinberg(1983)* and *Fiebig et al. (1988)* considering particle deposition only by thermophoresis and using Eulerian method. [2,3]

Chang et al. (1985), also studied thermophoretic deposition in an annular with both numerical simulation (*Eulerian* method) and experiment. [2]

In this work, the thermophoretic deposition efficiency was calculated for a gas flow in a concentric annulus by critical particle trajectory method, assuming the particle diffusion in the bulk of gas is negligible since the typical particle size in about 1 micrometer.

A particle starting from critical radial position at the entrance, will deposit just at the end of the annulus, which is name the critical particle trajectory. [1]

The analytical velocity distribution, presented by *Bird (1960)* is used to numerically calculate the developing temperature profile. Then the critical particle trajectory and deposition efficiency are obtained by solving the particle equations of motion.

2. Governing Equations

2.1. Velocity and Temperature Profiles

The system under investigation, as depicted in fig. 1, involves fully developed and laminar flowing of a hot gas containing suspended aerosol particles, in an annulus with the two concentric cylinders at constant wall temperature.

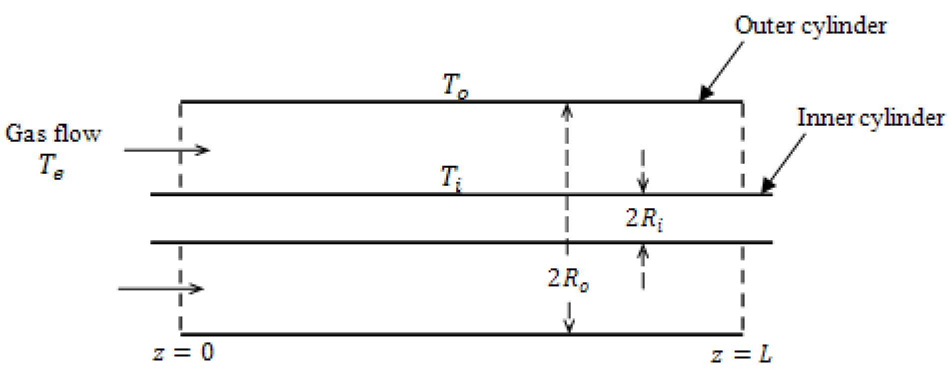

Fig. 1. Schematic of the physical configuration of the investigated system

The steady state momentum balance for a fully developed, laminar and incompressible fluid flowing through the annular region between two coaxial circular cylinders has been solved and presented by *Bird et. al. (1960)*. The solution for velocity is,

$$v = \frac{2u_m}{1+k^2+\dfrac{1-k^2}{\ln k}}\left[1-\left(\frac{r}{R_o}\right)^2-\frac{1-k^2}{\ln k}\ln\left(\frac{r}{R_o}\right)\right] \tag{5}$$

Where, k is the ratio of inner to outer radius of the cylinders and u_m is the bulk mean velocity.

When the flow is fully developed, the temperature could still be developing when there is a sudden temperature jump in the annulus walls.

The fully developed velocity profile was used to calculate the developing temperature field numerically by solving the 2-D cylindrical energy equation,

$$u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}=a\left[\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}\right] \tag{6}$$

Since v is equal to 0 in fully developed flow and also, the axial conduction of heat is negligible relies on the fact that Peclet number is taken to be large, equation (6) reduced to form,

$$u\frac{\partial T}{\partial z} = a\left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right] \tag{7}$$

With boundary conditions,

$$T(r,0) = T_e\ ,\ T(R_i, z) = T_i\ ,\ T(R_o, z) = T_o \tag{8}$$

Equation (7) can be non-dimensionalized as below, assuming all relevant physical properties are constant,

$$u^*\frac{\partial q}{\partial z^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial q}{\partial r^*}\right) \tag{9}$$

Where

$$q = \frac{T - T_e}{T_i - T_o}\ ,\ r^* = \frac{r}{D_H}\ ,\ z^* = \frac{z/D_H}{\mathrm{Re.Pr}}\ \text{ and }\ u^* = \frac{u}{u_m}$$

The governing equation was discretized using a finite volume method with non uniform grid spacing, i.e. the grid spacing is finer near the walls and inlet region where temperature gradients in radial directions are expected to be larger. The convective terms were discretized using hybrid scheme, also.

2.2. Particle Trajectory and Thermophoretic Deposition Efficiency

The Lagrangian approach is used for predicting the trajectory of a single particle in the fluid. The particle equations of motion in z (axial) and r (radial) directions, in a cylindrical coordinates, are:

$$\frac{d^2 z}{dt^2} = C_d \frac{\mathrm{Re}_p}{24}\frac{(u - u_p)}{t} \tag{10}$$

$$\frac{d^2 r}{dt^2} = C_d \frac{\mathrm{Re}_p}{24}\frac{(v - v_p)}{t} + \frac{v_t}{B.m_p} \tag{11}$$

Where C_d is the drag coefficient and was described by *Rader and Marple (1985)* as,

$$C_d = \begin{cases} \frac{24}{\mathrm{Re}_p}(1+0.0916\,\mathrm{Re}_p) & \mathrm{Re}_p < 5 \\ \frac{24}{\mathrm{Re}_p}(1+0.158\,\mathrm{Re}_p^{\;2/3}) & 5 < \mathrm{Re}_p < 1000 \end{cases} \tag{12}$$

The first term on the right hand side of equations (10) and (11) is the drag force due to relation motion between particle and fluid. The second term in equation (11) is the thermophoretic force.

The particle equations of motion were integrated numerically through the domain of interest by mean of Fourth-order Rung-Kutta method. It is assumed that the initial velocity of particles at the entrance is the same as the gas flow velocity at that point. The new particle position and velocity after a small time increment is calculated by numerical integration. The procedure is repeated until the particle hits the walls or leaves the calculation domain, when the critical radial position, r_c, will be obtained.

The thermophoretic deposition efficiency is defined as the ratio of the particle's rate of deposition on the wall to the particle's rate of entering the annulus, assuming particle concentration is uniform at the inlet of annulus.

For fully developed flow, the velocity profile is known by equation (5) and the equation of deposition efficiency, Eq. (13), can be solved by means of numerical integration.

$$h = 1 - \frac{\int_{r_{c_i}}^{r_{c_o}} u(r)\, 2prdr}{u_m p R_o^{\,2}(1-k^2)} \tag{13}$$

Where r_{c_o} and r_{c_i} in the above equation, are critical radial positions due to outer and inner cylinders, respectively.

3. Results and discussion

The fluid and particle properties used in the calculation were estimated at the averaged temperature of inlet gas and annulus walls.

The validity of the numerical solution was checked by comparing simulation results with that previously published by *Lin* and *Tsai (2003)* for a laminar tube flow case as shown in Fig. 2. The results were shown to be in good agreement with that simulation since the deviation is about 15%.

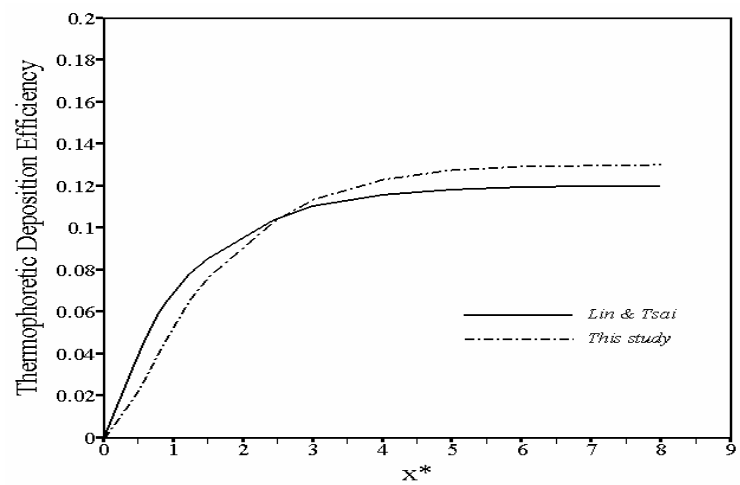

Fig. 4. Comparison of present numerical thermophoretic deposition efficiency with theoretical prediction of Lin & Tsai in laminar tube flow vs. x^*,for $\theta^* = 2.7$ and $Pr.K_T = 0.4$

The simulation for an annular flow system was modified by incorporating suitable boundary condition and velocity expression.

In order to make sure the simulated temperature field is correct, the numerically solution of developing temperature is compared with analytical solution presented by *Kakac et al.* (1987).

Kakac et al. Estimated the fluid bulk mean temperature or "mixing cup", T_m , for a finite number of k and dimensionless axial distance, Z .

In this study the fluid bulk's mean temperature can be calculated by numerical integration of equation (14).

$$T_m = \frac{1}{A_c u_m} \int_{A_c} uT\, dA_c \tag{14}$$

Fig. 3 shows good agreement in the comparison of T_m between present study and analytical solution described by *Kakac et al.* .

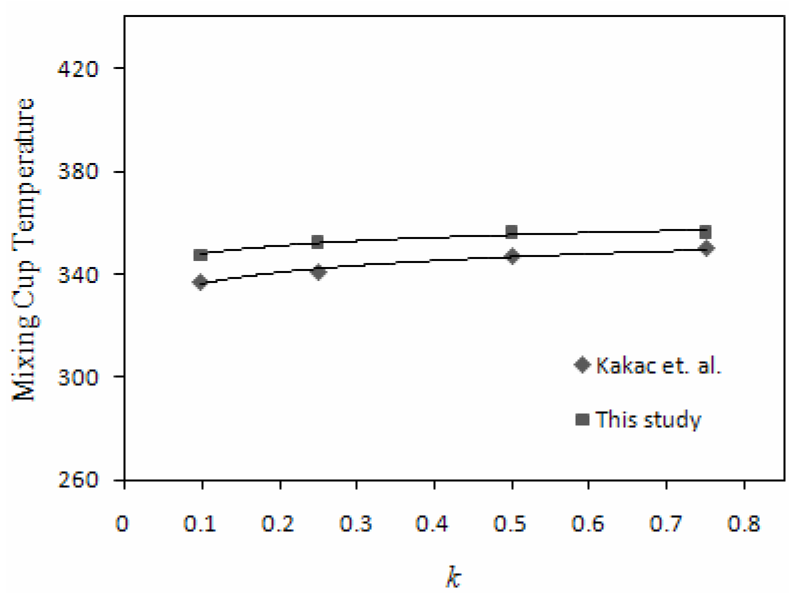

Fig. 3. Comparison of T_m between present study and analytical solution described by *Kakac* vs. κ, for $Z = 0.01$

This study was carried out to examine the effects of axial distance, thermal gradient, and the tube radius ratio on thermophoretic deposition efficiency.

Fig. 4 illustrate the thermophoretic deposition efficiency as a function of Z for $k = 0.1$ and $q = 0.1, 0.5$ and 0.92 .

It is apparent from fig. 4 that thermophoretic deposition increases with increasing thermal gradient at the same axial distance and with increasing axial distance at the same thermal gradient.

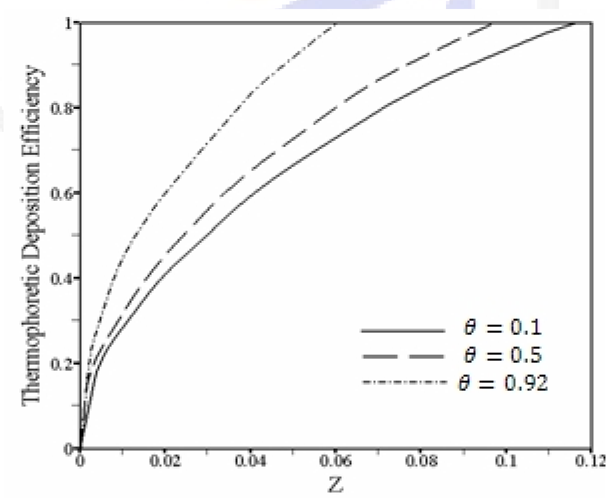

Fig. 4. Thermophoretic deposition efficiency vs. Z, for $\kappa = 0.1$ and $\theta = 0.1$, 0.5 and 0.92

We have also investigated the thermophoretic deposition efficiency for various values of the tube radius ratio, k . In fig. 5 deposition efficiency is plotted as a function of Z for $q = 0.5$ and

$k = 0.1,\ 0.3$ and 0.5. At all axial positions the tube configuration with the smaller annular region produces a larger efficiency, since the temperature gradients become greater.

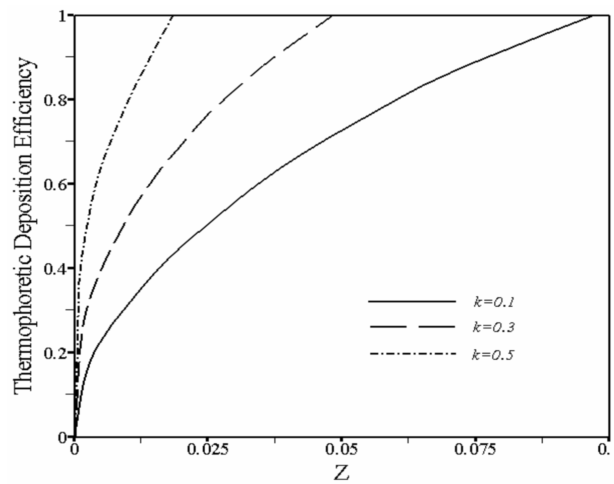

Fig. 5. Thermophoretic deposition efficiency vs. Z, for $\theta = 0.5$ and $\kappa = 0.1$, 0.3 and 0.5

4. Conclusion

We have studied the thermophoretic deposition efficiency for a model system where gas flows in an annular region between tow concentric cylinders.

The governing equations include the momentum and energy equations for the gas and the particle equations of motion. The critical trajectory method is used to investigating the thermophoretic deposition efficiency of particles in laminar gas flow numerically.

It was shown that, by maintaining the temperature gradient in the gas, complete deposition may be readily achieved.

Furthermore, the present work appears to indicate that for increasing the thermophoretic deposition efficiency, one would need a higher thermal gradient, a longer deposition distance or a larger tube radius ratio.

Nomenclature

B	Dynamic mobility, $B = C_c / 3pmd_p$
C_c	Cunningham slip correction factor
C_d	Drag coefficient (defined by Eq. (12))
C_m	Momentum exchange coefficient
C_s	Thermal slip coefficient
C_t	Temperature jump coefficient

D_H	Hydraulic diameter of annulus, $2R_o(1-k)$ $2R_o(1-\kappa)$
d_p	Particle diameter
F_{th}	Thermophoretic force
K_g	Gas thermal conductivity
Kn	Knudsen number
K_p	Particle thermal conductivity
K_T	Thermophoretic coefficient
L	Annular length
m_p	Particle mass
Pe_g	Gas Peclet number, $(u_m R_o)/a.L$
$\Pr$	Gas Prandtl number
r	Radial coordinate
r^*	Dimensionless radial coordinate, r/D_H
r_c	Critical radial position
Re	Gas Reynolds number
Re_p	Particle Reynolds number
R_i	Inner tube radius
R_o	Outer tube radius
T_e	Inlet gas temperature
T_i	Inner tube gas temperature
T_m	Mixing cup temperature, (defined by Eq. (14))
T_o	Outer tube gas temperature
∇T	Temperature gradient
u	Gas velocity (z-component)
u^*	Dimensionless gas velocity (z -component), u/u_m
u_m	Average gas velocity
v	Gas velocity (r-component)
u_p	Particle velocity (z -component)
v_p	Particle velocity (r -component)
v_t	Thermophoretic velocity
x^*	Dimensionless distance from the entry for tube calculation, $x^* = z/0.1R_o Pe_g$
z	Axial coordinate
z^*	Dimensionless axial coordinate, $(\frac{z}{D_H})/\mathrm{Re}.\Pr$
Z	Dimensionless distance from the entry, $Z = z/R_o Pe_g$

Greek letters

a	Gas thermal diffusivity

h	Thermophoretic deposition efficiency
q	Dimensionless temperature, $(T - T_e)/(T_i - T_o)$
q^*	Dimensionless temperature parameter, $T_w/(T_e - T_w)$
k	Inner to outer radius ratio, R_i/R_o
m	Gas dynamic viscosity
n	Gas kinematic viscosity
t	Particle relation time, $t = r_p d_p^2 C_c / 18m$

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