# Eulerian Modeling of a Circulating Fluidized Bed, including Mean Particle Diameter

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## Abstract

*This paper presents a computational study of the flow behavior in a cold-flow scale circulating fluidized bed. A multi-fluid Computational Fluid Dynamics (CFD) model has been developed and verified against experimental data reported in the literature. The flow model is based on an Eulerian description of the phase where the kinetic theory of granular flow forms the basis for the turbulence modeling in the solid phase. The model is generalized for one gas phase and mean particle size of solids phase to enable a realistic description of the particle size distribution in gas - solids flow systems. Solid phase is characterized by a mean diameter, density and restitution coefficient. Most emphasis is given to study the effects of mean particle size distributions to study the fluctuating behavior of the dilute gas-solids flow system. Altogether, the simulation results are in good agreement with experimental data. Both mean diameters, axial and radial mean and turbulent velocities are calculated successfully.*

**Keywords**: Eulerian CFD model; Gas–particle flow; Circulating fluidized bed

## Introduction

Fluidized bed reactors are an important unit operation in many processes in the chemical, petrochemical and metallurgical industries, and in combustion and environmental remediation. They are well known as excellent reactors for their superior rates of heat and mass transfer between the gas and the solid particles. In most cases, the scale-up to large commercial applications involves a costly stepwise process of building and testing several sizes of pilot-scale and cold model operations. Frequently the final design can take months or years to reach full production capacity. The main reason for these design problems is that the flow behavior of a fluidized bed of solids is complex and highly sensitive to scale, and specific operating conditions. [1]

Mathematical models can improve the fundamental understanding of the flow inside fluidized beds. Such models can vary in their level of complexity from strictly empirical to those based on the fundamental conservation equations of fluid dynamics .The latter modeling approach, known as Computational Fluid Dynamics (CFD).

Multiphase flow and complex geometries make the solution more difficult. However, CFD in multiphase flow has during the recent years become a well accepted and useful tool in modeling of gas/solids flow systems, and much progress has been made toward developing computer codes for describing fluidized beds. [2]

As is well known, most of today's CFD calculations for gas–solid flows are based on the assumption of a mono-dispersed solid phase (e.g., all particles have the same characteristic size) or on the assumption of a constant particle size distribution (PSD) (e.g. particles may be represented by a few different size classes but no changes in the PSD are accounted for). [3]

Recent research efforts have been directed towards the investigation of the effect of the PSD on the fluid dynamics of fluidized bed reactors. However, most work has focused on the segregation of binary mixtures.

Numerous studies have focused on segregation in gas–solid fluidized beds with binary mixtures (i.e., two particle types differing in size and/or material density). In particular, a number of investigators have performed experiments involving binary mixtures where the particles differ in

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size only and observed that the larger particles segregate preferentially toward the bottom of the bed (near the distributor plate), while the smaller particles accumulate near the free surface. For example, Huilin et al. [4] used a kinetic theory for binary particle mixtures in their Eulerian–Eulerian simulations of a gas-fluidized bed (where the particles differed in size only). The simulations indicate that large particles concentrate near the distributor plate, as was also observed in corresponding experiments. [5]

But in these simulations, much volume of calculation must used and in most cases spend too long time. So, in some case, used mean particle diameter for describe particle size distribution.

Most models are based on a two-phase description, one gas and one solid phase, where all the particles are assumed to be identical, characterized by a diameter, form factor, density and a coefficient of restitution. In gas/solids systems, particle segregation due to different size and/or density will play a significant role determining the flow behavior. To describe such phenomena, an extension to multiple particle phases is essential. The basic assumption was equal turbulent kinetic energy with a small correction for the individual phase temperatures. A model of mathiesen was developed and performed simulations with one gas and three solids phases [6]. The model predicted segregation effects fairly well and good agreement with experimental data was demonstrated. The model was generalized and made consistent for one gas phase and mean particle size of solids phases to enable description of realistic particle size distribution. The model is briefly presented here.

In this work, the model is used to simulate a cold flow circulating fluidized bed. Mean particle size of solids phase is used to enable a realistic description of the particle size distribution. Computational results are discussed and compared against experimental data from the literature [7].

**Experimental setup**

A detailed description of the circulating fluidized bed system is given by Ref. [7]. Fig. 1 gives a sketch of the experimental setup.

The riser has a cross-sectional area of 0.2*0.2 $m^2$ and is 2.0 m high. The primary air inlet is located at the bottom of the riser where the gas is passing through an air distributor to provide a uniform flow of air. The air has ambient temperature and pressure. At the top of the riser the suspended glass particles enter a cyclone and are recycled via a return loop. In experimental setup, laser Doppler anemometry (LDA) is used. Phase Doppler anemometry (PDA) measurement techniques to measure flow parameters, such as mean diameter, particle flux and mean and fluctuating velocities.

The experiments were performed with two different particle size distributions with a constant mean particle diameter at 120 mm. The measurements were conducted with two different superficial gas velocities, 0.7 and 1.0 m/s, respectively. The initial solids concentration in the gas/solid system was dilute with a solids concentration of 1% and 3% of the riser volume.

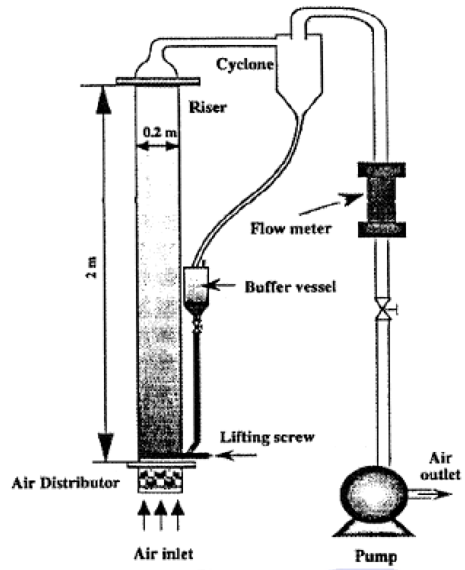



**Figure 1-The experimental setup**

## CFD model

Based on an Eulerian description of the phases, a multiphase CFD model for turbulent gas/solid flow is presented. The Eulerian approach considers the gas as well as the solid phase as continuum. In our model we used one gas phase and one solid phase with mean particle diameter.
The laws of conservation of mass, momentum and granular temperature are satisfied for each phase individually. The dependent variables, i.e. the volume fraction and the momentum, are solved for each phase.
The governing equation of this model is described below.
The continuity equation for phase m is given by:

$$\frac{\partial}{\partial t}(b_v a r)_m + \frac{\partial}{\partial x_i}(b_i a r U_i)_m = 0 \qquad (1)$$

Where α, ρ and U are the phase volume fractions, densities and the i-th direction velocity components, respectively. $\beta_V$ is the volume porosity and $\beta_i$ is the area porosity in i-th direction. The volume and area porosities have values between zero and unity, where zero is a totally blocked and unity is totally open. No mass transfer is allowed between the phases.
The momentum equations in the j-direction for phase m may be expressed as:

$$\frac{\partial}{\partial t}(b_v a r U_j)_m + \frac{\partial}{\partial x_i}(b_i a r U_i U_j)_m = -(b_v a)_m \frac{\partial P}{\partial x_j} + b_v a_m r_m g_j + \frac{\partial}{\partial x_i}(b_i \Pi_{ij,m}) + b_v \sum_{n=1}^{N} b_{mn}(U_{j,n} - U_{j,m}) \quad (2)$$

P and $g_j$ are fluid pressure and j-direction component of gravity, respectively .$\beta_{mn}$ is drag coefficient between the phases m and n.
Hence, the terms on the right side represent pressure forces, mass forces, viscous forces, and drag forces, respectively. Both gas–particle and particle–particle drag are included in the total drag.
We consider each phase as an incompressible fluid. But due to the extension to multiphase flow, the divergence term is included. Hence the stress tensor $\prod_{ij,g}$ for the gas phase g is given by:

$$\Pi_{ij,g} = m_{eff,g}\left[\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right) - \frac{2}{3} d_{ij} \frac{\partial U_k}{\partial x_k}\right]_g \quad (3)$$

Where $\delta_{ij}$ is the Kroenecker delta. And use k-ε turbulence model.
The total stress tensor $\Pi_{i\,j,s}$ for the solid phase s is the sum of a collisional and a kinetic part:

$$\Pi_{ij,s} = -P_s d_{ij} + m_s \left[\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right) - \frac{2}{3} d_{ij} \frac{\partial U_k}{\partial x_k}\right]_s + x_s d_{ij} \frac{\partial U_{k,s}}{\partial x_k} \quad (4)$$

Whereas the bulk viscosity is approximated to be zero for most gases, as done here the bulk viscosity $\xi_s$ should be included when considering granular materials. The solid phase pressure $P_s$, which includes a collisional and a kinetic term, is given by:

$$P_s = a_s r_s [1 + 2(1+e) a_s g_0] q_s \quad (5)$$

Where e is the restitution coefficient, which is unity for fully elastic and zero for plastic collisions. $\theta_s$ is granular temperature. The radial distribution function $g_0$ is an expression for the probability of particle collisions and is given by:

$$g_0 = \left[\left(1 - \frac{a_s}{a_{s,\max}}\right)^{\frac{1}{3}}\right]^{-1} \quad (6)$$

Where $\varepsilon_{s,max}$ is the maximum total volume fraction of solid, in this work set to 0.63.
The solid phase bulk viscosity or volume viscosity, $\xi_s$ , is given by:

$$x_s = \frac{4}{3} a_s^2 r_s d_s g_0 (1+e) \sqrt{\frac{q_s}{p}} \quad (7)$$

Where $d_s$ is particle diameter.
The solid phases shear viscosity, $\mu_s$ , is given by:

$$m_s = \frac{2 m_{s,dil}}{(1+e) g_0}\left[1 + \frac{4}{5}(1+e) g_0 a_s\right]^2 + \frac{4}{5} a_s^2 r_s d_s g_0 (1+e) \sqrt{\frac{q_s}{p}} \quad (8)$$

$$m_{s,dil} = \frac{5}{96} r_s d_s \sqrt{p q_s}$$

The gas–particle drag coefficient is given by Gidaspow:

$$b_{sg} = \begin{cases} \frac{3}{4} C_D \frac{a_s a_g r_g U_{slip}}{d_s} a_g^{-2.65} & a_g \geq 0.8 \\ 150 \frac{a_s^2 m_{lam,g}}{a_g d_s^2} + 1.75 \frac{a_s r_g U_{slip}}{d_s} & a_g < 0.8 \end{cases} \quad (9)$$

Whereas:

$$U_{slip} = \left|\vec{U}_g - \vec{U}_s\right|$$

$$C_D = \frac{24}{a_g \mathrm{Re}_s}\left(1 + 0.15 (a_g \mathrm{Re}_s)^{0.687}\right)$$

$$\mathrm{Re}_s = \frac{d_s r_g U_{slip}}{m_{lam,g}}$$

Conservation equation for granular temperature is solved for solid phase:

$$\frac{3}{2}\left[\frac{\partial}{\partial t}\left(b_v a_s r_s q_s\right)+\frac{\partial}{\partial x_i}\left(b_i a_s r_s U_{i,s} q_s\right)\right]=b_v\left(\Pi_{ij,s}\frac{\partial U_{j,s}}{\partial x_i}\right)+\frac{\partial}{\partial x_i}\left(b_i k_s\frac{\partial q_s}{\partial x_i}\right)-b_v g_s-3b_v b_{sg} q_s \qquad (10)$$

The terms on the right side of the equation represent production due to shear, diffusive transport of granular temperature, dissipation due to inelastic collisions and dissipation due to fluid friction. A corresponding production term due to fluctuations in drag has been assumed negligible.
The conductivity of granular temperature, $\kappa_s$ , and the dissipation due to inelastic collisions $\gamma_s$ are determined from the kinetic theory for granular flow. The granular conductivity is given by a dilute and a dense part as:

$$k_s=\frac{2k_{s,dil}}{(1+e)g_0}\left[1+\frac{6}{5}(1+e)g_0 a_s\right]^2+2a_s^2 r_s d_s g_0(1+e)\sqrt{\frac{q_s}{p}} \qquad (11)$$

$$k_{s,dil}=\frac{75}{384} r_s d_s\sqrt{p q_s}$$

The dissipation of the turbulent kinetic energy due to particle collisions is given by:

$$g_s=3\left(1-e^2\right)a_s^2 r_s g_0 q_s\left[\frac{4}{d_s}\sqrt{\frac{q_s}{p}}-\frac{\partial U_{k,s}}{\partial x_k}\right] \qquad (12)$$

The governing equations are solved by a finite volume method, where the calculation domain is divided into a finite number of non-overlapping control volumes. At main grid points placed in the center of the control volume, volume fraction, density, and turbulent kinetic energy are stored. The conservation equations are integrated in space and time. This integration is performed using first order upwind differencing in space and fully implicit in time. For a first-order upwind solution, the value at the center of a cell is assumed to be an average throughout the cell. The SIMPLE algorithm is used by Fluent to relate the velocity and pressure corrections to recast the continuity equation in terms of a pressure correction calculation.
The simulation geometry with grid nodes is shown in Fig. 2. The 2D calculation domain is divided into 80*200 grid nodes, in the radial and axial directions, respectively. The grid is chosen to be uniform in the axial direction, whereas a non-uniform grid is used in radial direction. Also the mesh size in the axial and radial direction is different. We use quad grid in riser section and Tri-pave grid for other section of system.

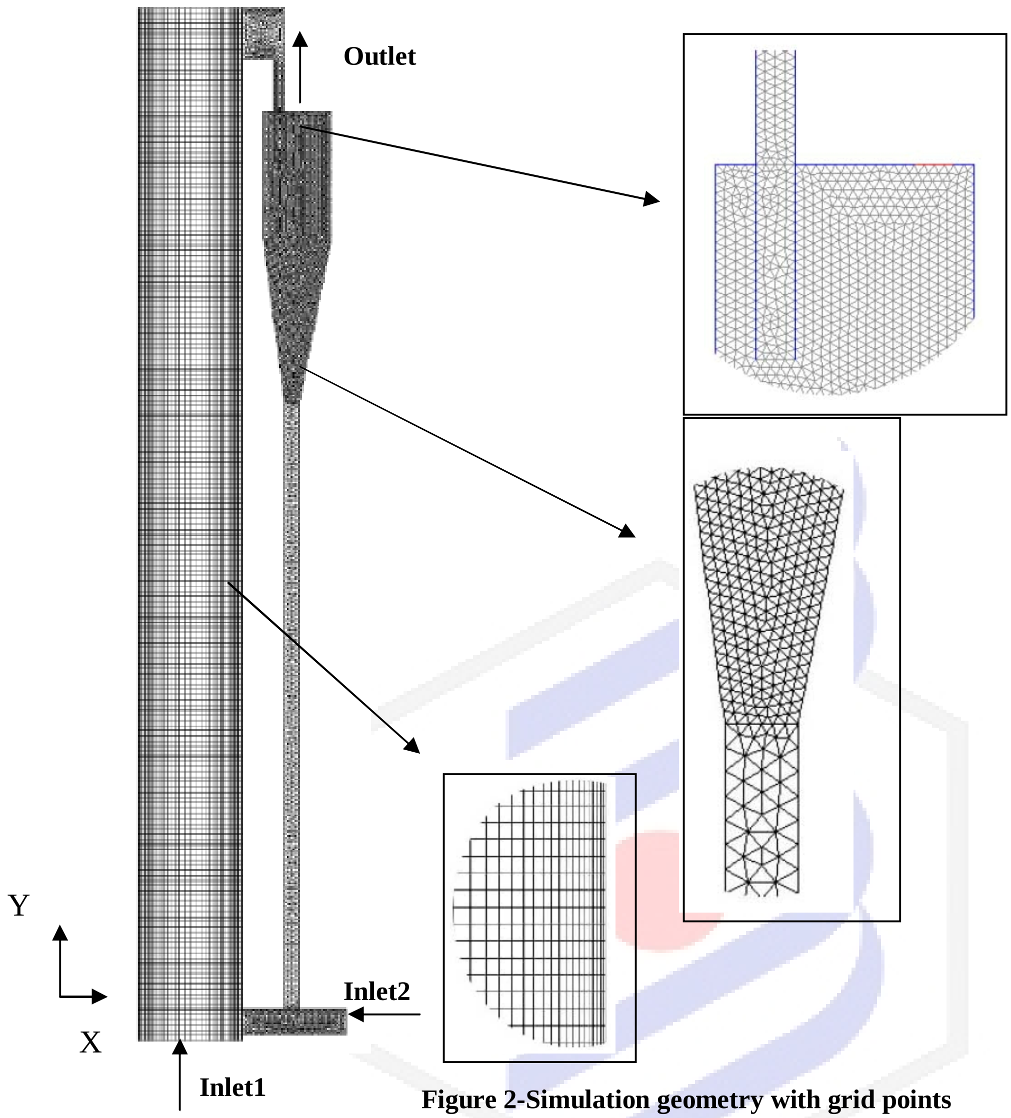



**Figure 2-Simulation geometry with grid points**

To enable a realistic particle size distribution, Mathiesen etal. used three solid phases for their simulation but in this work we used the solid phases with mean particle diameter for simulation to decrease the volume of calculation. The mean particle diameter is 120 μm. The results compare against experimental data and model of Mathiesen.

Fig. 3 shows the numerical of model of Mathiesen and real particle size distribution. In Fig.3 (B), we can see at 120 μm for diameter the figure is symmetric and at two sides of this line we have the same profile.

Also, Figure 3(A) gives a volume – surface diameter (or mean diameter), $d_{vs}$, calculated as:

$$d_{vs} = \left( \frac{\sum_i d_i^3 N_i}{\sum_i d_i^2 N_i} \right) \qquad (13)$$

For numerical simulations of dilute flow Peirano and Leckner [8], recommend the volume-length diameter, $d_{vl}$, calculated as:

$$d_{vl} = \left( \frac{\sum_i d_i^3 N_i}{\sum_i d_i N_i} \right)^{\frac{1}{2}} \qquad (14)$$

For this distribution (Fig. 3(A)), we calculated $d_{vl}$ as:

**Table 1-The required data for calculated volume-length diameter**

| | Number of particle,N(%) | Diameter,d(μm) | d^3 | (d^3)*N | d*N |
|---|---|---|---|---|---|
| 1 | 0.084 | 44.8 | 89915.392 | 7552.892928 | 3.7632 |
| 2 | 0.155 | 51.1 | 133432.831 | 20682.08881 | 7.9205 |
| 3 | 0.141 | 55.9 | 174676.879 | 24629.43994 | 7.8819 |
| 4 | 0.309 | 60.2 | 218167.208 | 67413.66727 | 18.6018 |
| 5 | 0.549 | 64.9 | 273359.449 | 150074.3375 | 35.6301 |
| 6 | 0.661 | 69.5 | 335702.375 | 221899.2699 | 45.9395 |
| 7 | 1.069 | 73.4 | 395446.904 | 422732.7404 | 78.4646 |
| 8 | 1.913 | 78.3 | 480048.687 | 918333.1382 | 149.7879 |
| 9 | 2.068 | 82.6 | 563559.976 | 1165442.03 | 170.8168 |
| 10 | 2.814 | 87.3 | 665338.617 | 1872262.868 | 245.6622 |
| 11 | 3.925 | 91.4 | 763551.944 | 2996941.38 | 358.745 |
| 12 | 5.261 | 95.7 | 876467.493 | 4611095.481 | 503.4777 |
| 13 | 6.274 | 100.4 | 1012048.064 | 6349589.554 | 629.9096 |
| 14 | 8.427 | 104.6 | 1144445.336 | 9644240.846 | 881.4642 |
| 15 | 9.271 | 108.9 | 1291467.969 | 11973199.54 | 1009.6119 |
| 16 | 9.538 | 113.6 | 1466003.456 | 13982740.96 | 1083.5168 |
| 17 | 11.17 | 117.7 | 1630532.233 | 18213045.04 | 1314.709 |
| 18 | 11.072 | 121.9 | 1811386.459 | 20055670.87 | 1349.6768 |
| 19 | 9.9 | 127.1 | 2053225.511 | 20326932.56 | 1258.29 |
| 20 | 7.203 | 131.5 | 2273930.875 | 16379124.09 | 947.1945 |
| 21 | 4.671 | 136.6 | 2548895.896 | 11905892.73 | 638.0586 |
| 22 | 2.307 | 141.25 | 2818158.203 | 6501490.975 | 325.86375 |
| 23 | 1.069 | 145.65 | 3089807.812 | 3303004.551 | 155.69985 |
| 24 | 0.324 | 151.22 | 3458021.596 | 1120398.997 | 48.99528 |
| 25 | 0.155 | 161.49 | 4211500.956 | 652782.6482 | 25.03095 |
| sum | ------------------------ | --------------------- | ---------------- | 152887172.7 | 11294.71243 |

$$d_{vl} = \left( \frac{152887172.7}{11294.71243} \right)^{\frac{1}{2}} = 116.345 \quad mm$$

The volume-length diameter is calculated 116.345, which can approximation as120 μm.

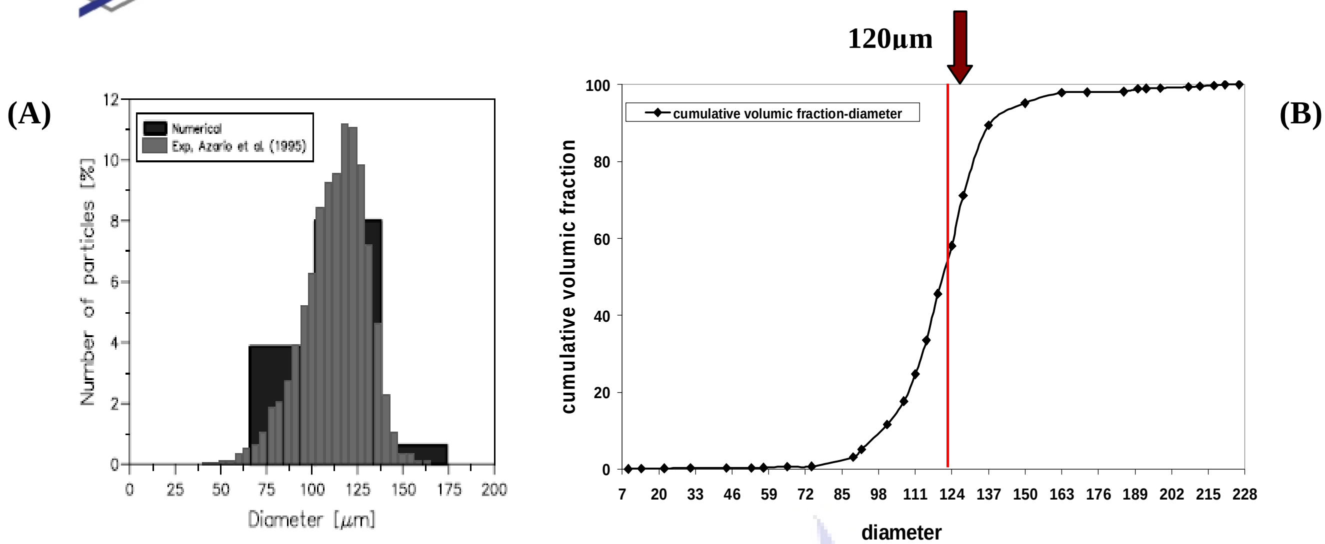



**Figure 3-particle size distribution: (A) Experimental data and model of mathiesen, (B) cumulative volume fraction versus diameter**

We calculated again with two other diameter, 33% less than 120µm (80µm) and 25% higher than one (150µm), the result is showed at below figure. It can see the 120µm is better than others.

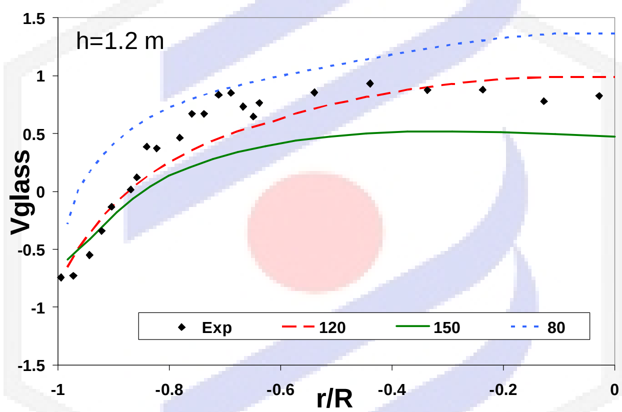



**Figure 4-Axial glass velocity at 1.2m height with three different diameters**

The particle density is 2400 kg/m$^3$ and the restitution coefficient for all the solid phases are set to 1.0 in simulation. In order to get reasonable computational results to compare against the experimental data, time averaged results are obtained between 7 and 20 s of real simulation time.
The outlet is located at the top of the cyclone. No particles are allowed to leave the circulating fluidized bed system. At the walls, no-slip conditions are used for the solid phases as well as the gas phase. A zero-flux boundary condition is applied to the energy fluxes.
The initial solids concentration in system was dilute with a solids concentration of 3% of the riser volume. The superficial gas velocity is equal 1.0 m/s. There are two inlets for gas flow, one of them is at bottom of riser (u at y direction) and another is at the side pipe (0.1*u, vice versa x direction) and there is one outlet above semi-cyclone section. (See fig.2)

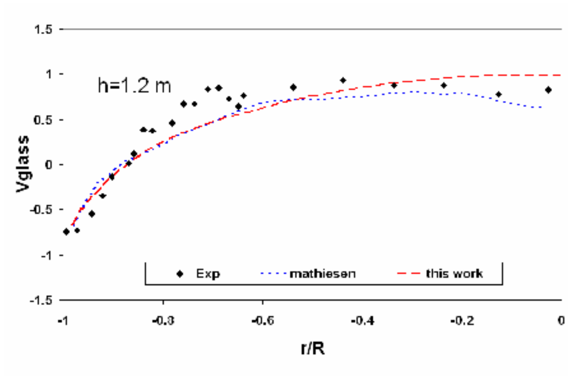
**Results and discussion**

Mathiesen et al [2] performed simulation of this system for one gas phase and three solid phases to enable a realistic description of the particle size distributions in gas/solids flow systems. Each solid phase is characterized by a diameter, density and restitution coefficient. We performed simulation this system with mean particle diameter, 120µm, and compare their results with the Mathiesen and experimental results. So we consider the effect of mean particle diameter on some parameter such as glass velocity**.**

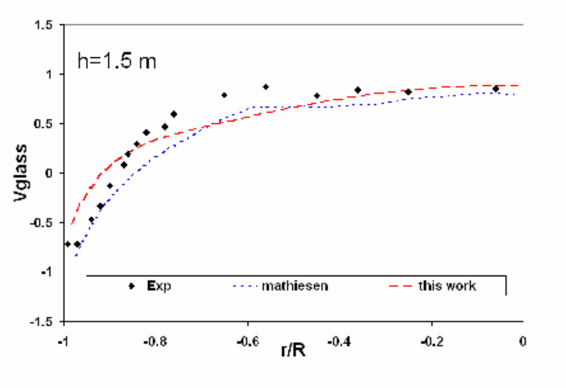
The fluctuations are studied at the center of riser. The circulating fluidized beds never reach a normal steady state condition, but exhibit a strong fluctuating behavior. The x-component and y-component of computed velocity fluctuations of the solid phase are showed this behavior. But after the first 7 s the fluctuations have a constant mean value. So we averaged between 7 and 20 second for compare against experimental data. A constant time step of 0.001 second was used, and the simulation is run on a computer with 1.83 GHZ, 1.00GB of RAM for 20 second of real fluidization time corresponding to 65 hours of computational time. It noted that this computational time is for this model (with one solid phase with mean particle diameter).It is obvious that if used three solid phases, we should spend more computational time for 20 second of real time.

In Figure 5(A), (B), (C) the glass velocity profiles are compared against experimental data. The mean particle velocity profiles are obtained 1.2, 1.5 m and 1.9 m above the primary gas inlet. In this work simulation with one solid phase with mean diameter is performed whereas in mathiesen's simulation three solid phases for description particle size distribution are used. This simulation is in good agreement with the measurements, whereas have good agreement with mathiesen's simulation. It can see there is 21% error at 1.2m height in this simulation whereas there is 32% error in mathiesen's simulation. Also because of use of one solid phase with mean diameter; we applied less calculation in this simulation and good agreement as mathiesen's simulation with experimental data. As in the experiments, the velocity profiles become flatter as the height above the inlet increases, which can see in below Figures.

(A)




(B)




(C)

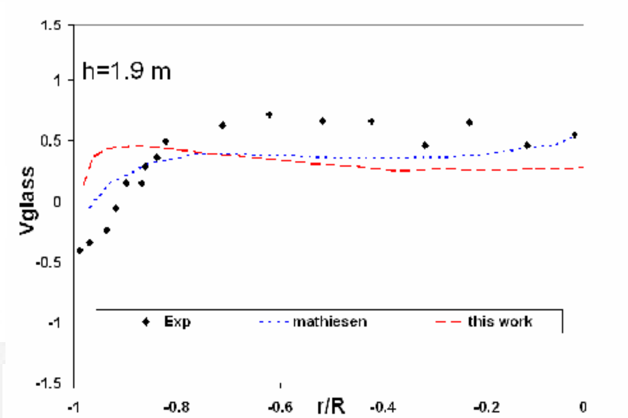



**Figure5-Axial glass velocity at (A) h=1.2m, (B) h=1.5 m, (C) h=1.9m height**

In figure6, show radial glass velocity at 1.2m height. It can see there is good agreement at center of riser but near the wall not good result. It can see the velocity profiles became flatter when the height above the inlet increases.

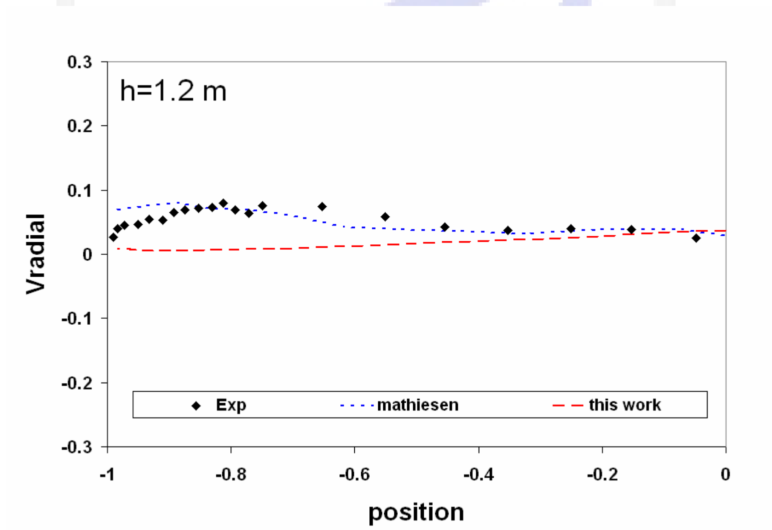
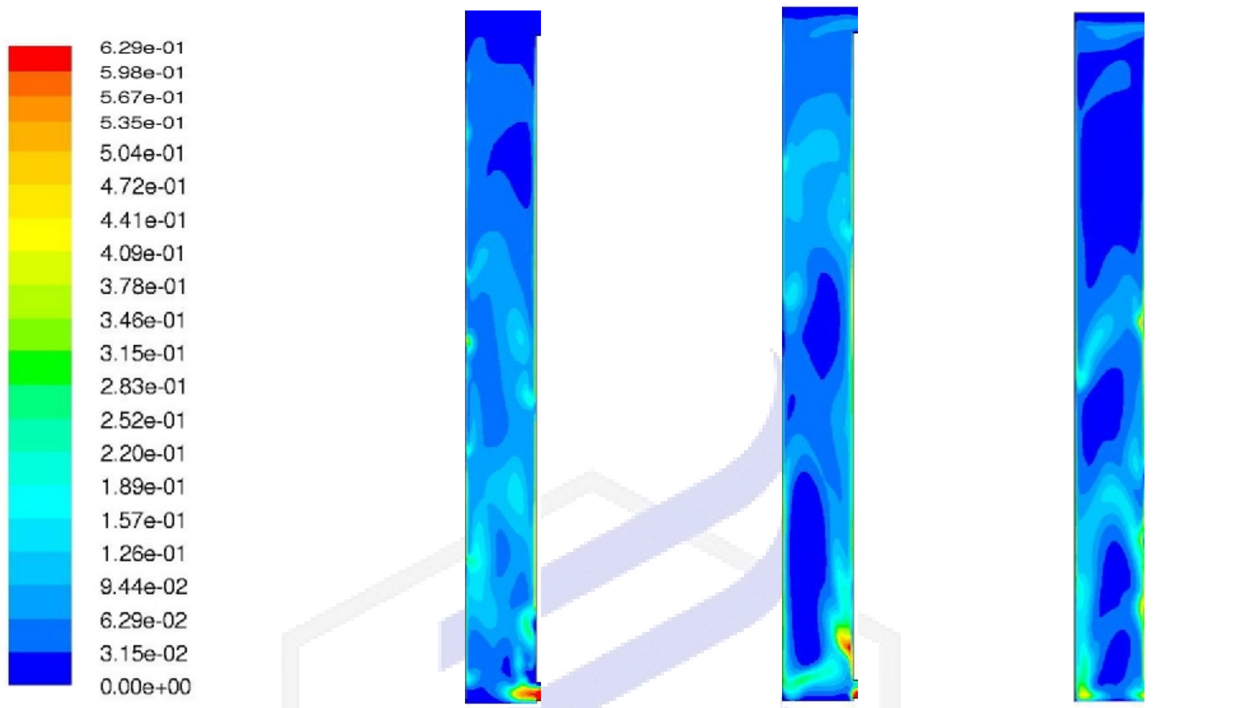



**Figure6 -Radial glass velocity at 1.2m height**

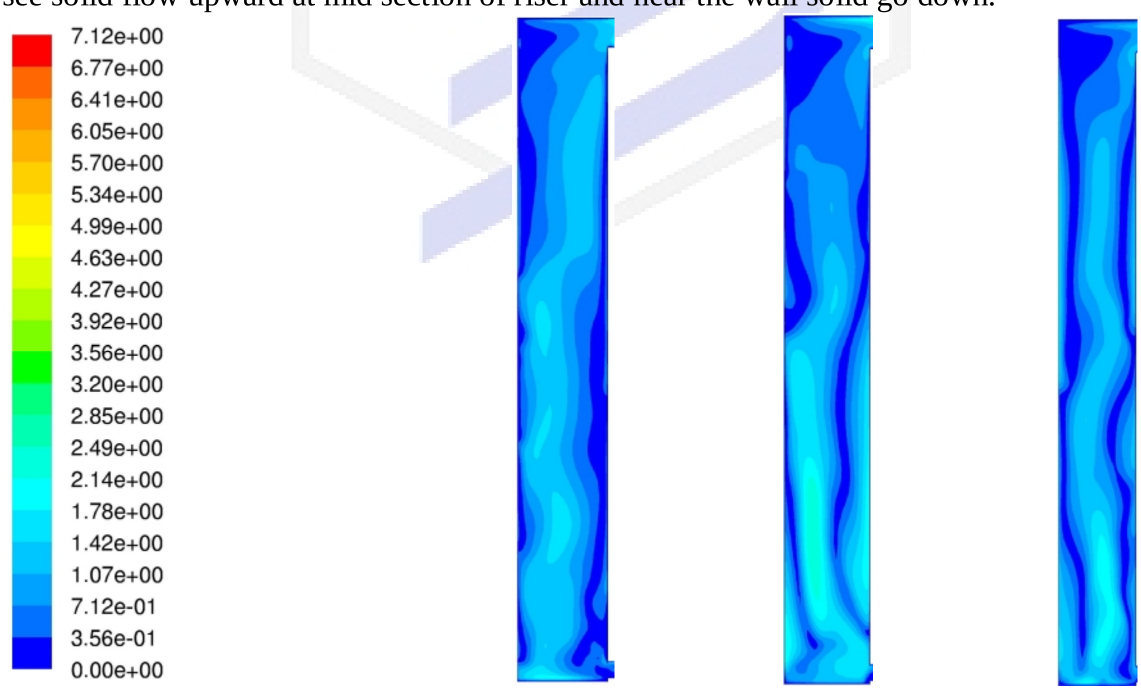
In figure7,can see the contour of glass volume fraction at three time, 11 second, 12 second and 13 second. It can see the surface that have the same solid volume fraction, is the same color. The distribution of solid at riser can see is different and at little different time so no steady state condition at one time so we averaged between 7 , 20 second, which noted above.




**(A)** **(B)** **(C)**

**Figure7 - Contour of solid volume fraction at three times, (A) at 11second, (B) 12second, (C) 13 second**

In figure 8, can see Contour of glass velocity magnitude at three times. It can see solid flow at mid section of riser. But the distance of wall for this upward flow is different. In figure 8(D) can see solid flow upward at mid section of riser and near the wall solid go down.




**(A)** **(B)** **(C)**

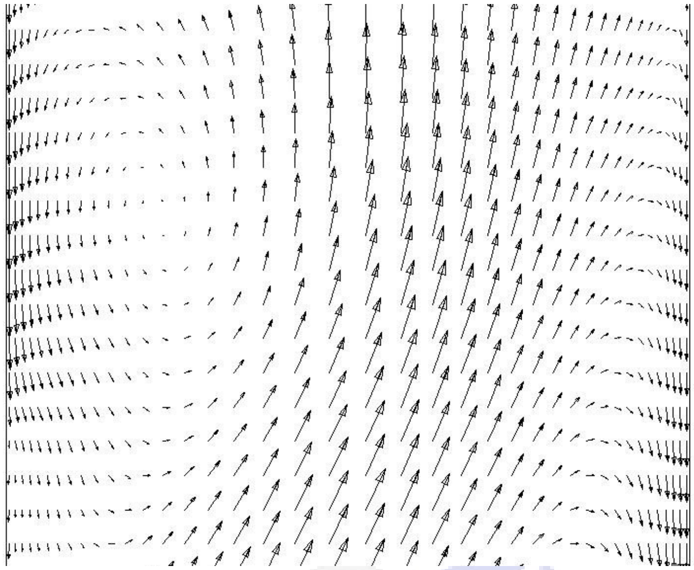

(D)

**Figure8 - Contour of glass velocity magnitude at three times, (A) at 11second, (B) 12second, (C) 13 second,(D) section of riser at 12second**

## Conclusion

The computed mean diameters are in good agreement with the experimental data. The axial velocities along the median axes are calculated for mean particle size distributions as well. The simulations show the same tendency as the experiments, although some discrepancies are observed. Mean particle velocity profiles are obtained at three different heights with only insignificant deviations from the measurements. A typical core–annulus flow is calculated with a nearly constant velocity in the central part of the riser and a down flow of particles in the wall region. As in the experimental data, the velocity profiles became flatter when the height above the inlet increases. It can see less time used for simulation, because of, in this simulation is used one solid phase with mean particle diameter, whereas the value of mean particle diameter should calculate correctly.

**NOMENCLATURE**

$C_D$ *drag coefficient*
$d_s$ *particle diameter*
***e*** *restitution coefficient*
$g_0$ *radial distribution function for solid phase*
***g*** *gravity*
***P*** *fluid pressure*
$P_s$ *solid phase pressure*
$Re_s$ *particle Reynolds number*
$U_{slip}$ *slip velocity*

***Greek symbols***

$\alpha$ *volume fraction*
$\alpha_{s,max}$ *maximum total volume fraction of solid*
$\beta_i$ *area porosity in i-direction*
$\beta_v$ *volume porosity*
$\delta_{i,j}$ *Kroenecker delta*
$\theta$ *granular temperature*
$\mu$ *shear viscosity*
$\xi$ *bulk viscosity*
$\Pi_{i,j}$ *stress tensor, solid phase*
$\rho$ *density*

sss**Reference:**

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