

CORRELATION OF BUBBLE GROWTH RATE IN SUPERHEATED METHANOL

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Abstract

Boiling heat transfer is used most commonly in chemical and petrochemical industries either to generate vapor or because of its high efficiency in cooling. The high efficiency of boiling is contributed to three mechanisms namely the natural convection to the liquid, latent heat flow by micro-layer evaporation between bubbles and heat transfer surface, and the intense micro-convection flow adjacent to the heat transfer surface which induced by growth and detachment bubbles. All these mechanisms are strong function of bubble growth rate and its accurate prediction is crucial for the theoretical prediction of boiling heat transfer coefficient. Unfortunately, the proposed correlations are either simple with a limited range of validity or complicated. In the present work a unified model is presented by solving equation of energy which is expressed by

$$R = m\sqrt{t} \operatorname{erf}(n\sqrt{t}) + R_o$$

In this article, first the fundamental approach of developing of presented model is discussed. Then the prediction of this model is compared with available experimental data for methanol and with those of better performing correlation from the literature. The absolute average error between the predicated and the experimental data is less than 20% for methanol, which confirms the applicability of the suggested model.

$$R = m\sqrt{t} \operatorname{erf}(n\sqrt{t}) + R_o$$

Keywords: Superheated liquid, Pool boiling, Bubble growth, Heat transfer coefficient

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Introduction

Boiling is a physical phase change process of great significance and has been the subject of intensive research for the past several decades. Nuclear power vapor generator design and its safety that refers to loss of coolant accidents (Plesset et al. [1]) and the sharp rise in energy costs have spurred many of the research initiatives.

Whereas the determination of heat transfer coefficient is the main goal of boiling process investigation, many of researchers have attempted to find it. One of these investigators is Gorenflo [2] who has been presented his correlation for pure liquid. There is a good agreement between the measured pool boiling heat transfer coefficients and the values predicted by his correlation, but his correlation is purely empirical and does not provide any information on the actual mechanism of boiling heat transfer. Also, attempts have

been made to model the boiling process with the realistic physical phenomena underlying this process. Recently, Jamialahamdi, Blochl and Bier [2] developed a semi empirical model for predicting saturated nucleate boiling heat transfer rates. They divided nucleate boiling heat transfer into two mechanisms, namely, bulk convection due to bubble growth and departure, and natural convection. As it can be seen from this definition, once the boiling process is theoretically going to be studied, great important role of bubble growth and its dynamics as departure radius, bubble frequency, nucleate site density and many other parameters will come out.

Rayleigh [3] was the first to formulate the equation of motion for a spherical bubble growth, which was later to be known as the inertia controlled growth. Plesset and Zwick [4] considered the heat diffusion controlled growth neglecting liquid inertia, and provided a solution for the bubble assumption of a thin thermal boundary layer, necessary to make the problem tractable. The analysis of Forster and Zuber [5] was basically in agreement with that of Plesset and Zwick [4].

Mathematical modeling

The bubble growth takes place as a result of the influx of vapor from the phase interface. When time goes on, the temperature of the vapor inside the bubble decreases, so the temperature of bubble surface decreases while the temperature of liquid around the bubble is constant and greater than the bubble wall temperature so the heat transfer occurs at the bubble wall. This effect is defined as cooling effect that was reported by the pioneer researchers in this field e.g. Plesset and Zwick [4], Forster and Zuber [5] and Mikic and Rohsenow and Griffith [6].

By considering cooling effect, the mass-flow rate is determined by the heat input from the liquid passing through thermal boundary layer around the bubble to the bubble surface. This concept can be explained by the differential equation of transient heat conduction for the liquid around the bubble in the spherical symmetry that was expressed in equation (1)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} = \frac{a}{r} \frac{\partial^2}{\partial r^2} (rT) \quad (1)$$

Since the convection effects are very small at onset of bubble growth, eliminating of convection effects in equation (1) is suggested. So,

$$\frac{\partial T}{\partial t} = \frac{a}{r} \frac{\partial^2}{\partial r^2} (rT) \quad (2)$$

The boundary value problem equation (2) needs a set of boundary conditions and an initial condition. For this problem the initial condition as expressed earlier is

$$t = 0 \quad , \quad T = T_{\infty} \quad (3)$$

And Boundary condition at infinity is

$$r \rightarrow \infty \quad , \quad T = T_{\infty} \quad (4)$$

And the boundary condition at the bubble surface after simplification is

$$k \left(\frac{\partial T}{\partial r} \right)_{r=R} = Lr_v \frac{dR}{dt} \quad (5)$$

Equation (5) means that all of heat supplied to the interface from the liquid is used for vaporization of the liquid into the bubble.

Also it is assumed that the mixing in the bubble is complete, so the temperature of the bubble content is uniform and equals to the temperature at the bubble surface. Therefore one can obtain

$$r = R \quad , \quad T = T_v \quad (6)$$

The problem of transient heat conduction equation in polar coordinate is readily transformed into a problem of that equation in Cartesian coordinate by defining proper variables.

It is noted that the vapor temperature T_v and the bubble radius R are function of time. However, to obtain the solution of equation (1), it is assumed that T_v and R are independent of time. But when the boundary condition is solved at the bubble surface, i.e. equation (5), T_v and R are assumed to be a function of time. This was also done by Kang and Bartsch [7].

After some extended and complicated mathematical solution and some simplification that is expressed in reference [8] one can obtain

$$\frac{dR}{dt} = \sqrt{\frac{a}{p}} \frac{(T_\infty - T_v) C_{PL} r_L}{Lr_v} \frac{1}{\sqrt{t}} \quad (7)$$

Equation (7) is multiplied to $\left(\frac{T_\infty - T_{sat}}{T_\infty - T_v} \right) = 1$

$$\frac{dR}{dt} = \sqrt{\frac{a}{p}} \frac{C_{PL} r_L}{Lr_v} \frac{(T_\infty - T_{sat})}{\sqrt{t}} \times \frac{T_\infty - T_v}{T_\infty - T_{sat}} \quad (8)$$

The bubble temperature T_v begins from T_∞ at initial stage to T_{sat} at the final stage, so $T_{sat} \leq T_v \leq T_\infty$. So one can write

$$\left. \frac{T_\infty - T_v}{T_\infty - T_{sat}} \right|_{t \rightarrow 0} = 0 \quad (9)$$

$$\left. \frac{T_\infty - T_v}{T_\infty - T_{sat}} \right|_{t \rightarrow \infty} = 1 \quad (10)$$

In the entire range of bubble growth it can be supposed that $(T_\infty - T_v)$ has the following approximation form

$$\frac{T_\infty - T_v}{T_\infty - T_{sat}} = \text{erf}(a\sqrt{t}) \quad (11)$$

In the Figure 1 variation of bubble temperature is illustrated for various values of parameter a . This figure shows that if a is small, the bubble temperature, i.e. T_v , converges to saturation temperature, i.e. T_{sat} , very slowly. But if a has a large value, the bubble temperature converges very rapidly to saturation temperature.

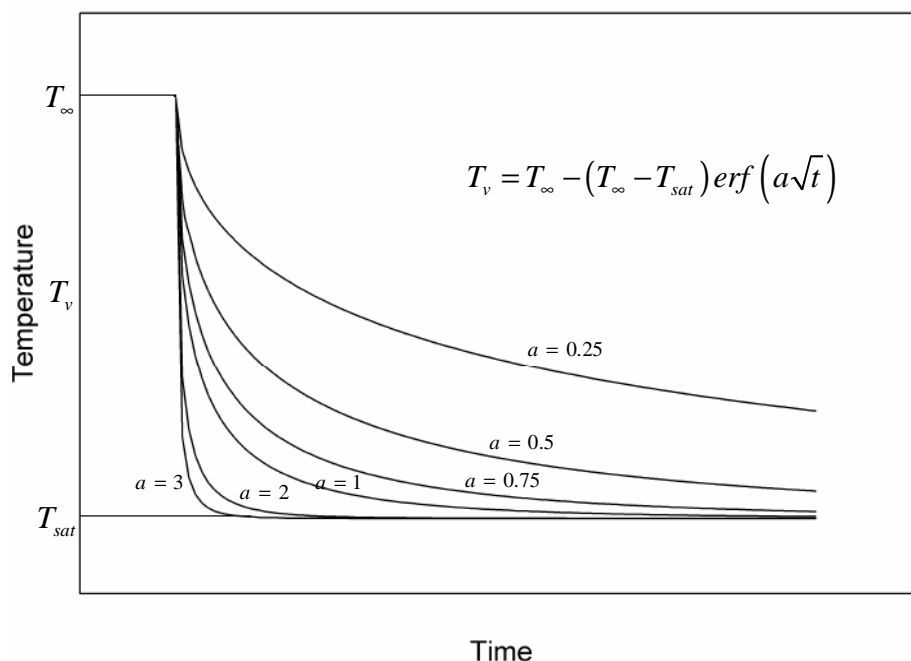


Figure 1. Approximation of variation of bubble temperature in the entire range of bubble growth with a the different values for parameters

By using above approximation the final form of the mathematical solution become

$$R = \sqrt{\frac{a}{p}} Ja \left(\sqrt{t} \text{erf}(a\sqrt{t}) + \frac{2}{a\sqrt{p}} \exp(-a^2 t) \right) + R_o \quad (12)$$

Equation (12) is the analytical solution of bubble growth in superheated liquid. The second term of this equation, $\left(2\sqrt{a}/ap\right) Ja \exp(-a^2 t)$ is very small in comparison with the first term, so no significant error will appear if this term is eliminated. The effect of these terms on bubble growth for liquid methanol was shown graphically in Figures 2 and 3. Nevertheless, the general form of equation (12) after eliminating of second term may still used for prediction of bubble growth as

$$R = m\sqrt{t} \text{erf}(n\sqrt{t}) + R_o \quad (13)$$

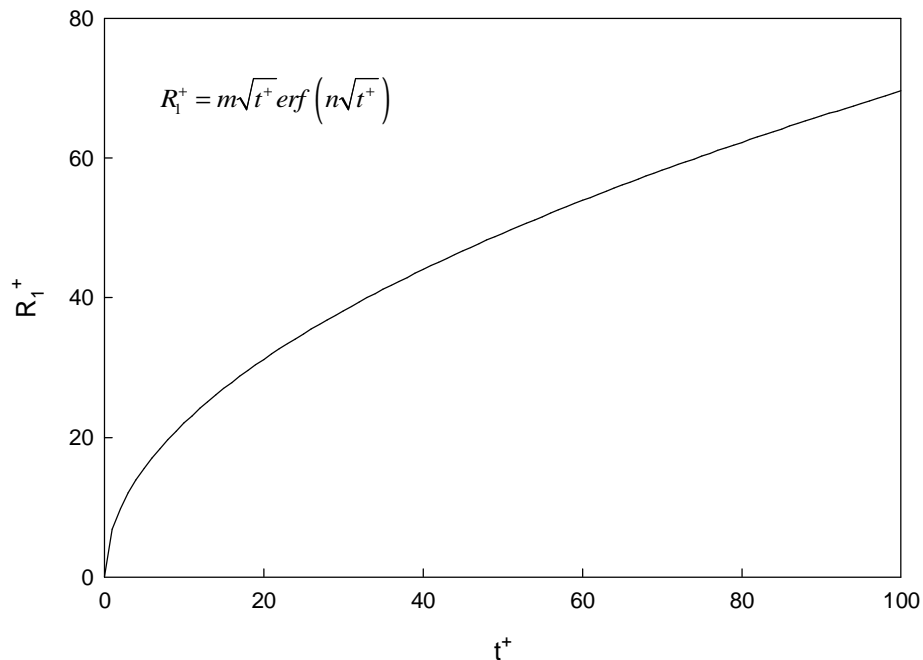


Figure 2. The first term in equation (12)

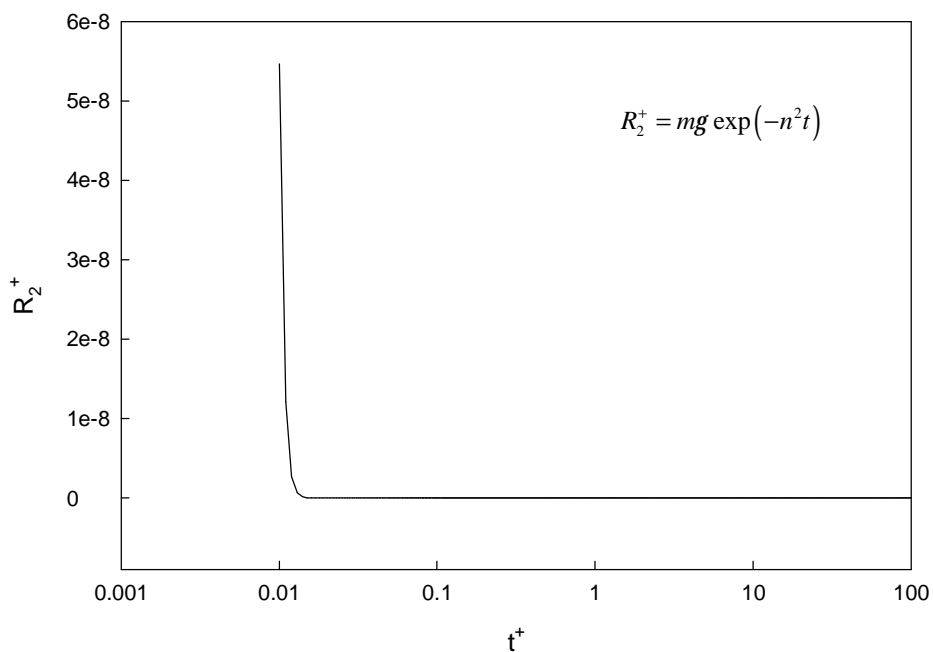


Figure 3. The second term in equation (12)

Dimensional analysis of the bubble growth

Because of many assumptions, the analytical solution is not properly applicable for bubble growth, so dimensional analysis for bubble growth and bubble departure was conducted to create a relation between analytical and experimental aspects.

According to Lee et al. [9] work, the dimensionless bubble radius and time can be expressed as

$$R^+ = \frac{R}{R_c}, \quad t^+ = \frac{t}{t_c}, \quad T^+ = \frac{T}{T_c} \quad (14)$$

The characteristic time scale can be determined from the ratio of the corresponding latent heat transfer and the conduction heat transfer rate through the interface. Thus

$$\frac{q_{latent}}{q_{conduction}} = \frac{r_v L \left(\frac{4}{3}\right) p R^3}{k 4 p R^2 \frac{\partial T}{\partial r}} = \frac{1}{3} \frac{r_v L R_c^3}{k R_c^2 \frac{T_c}{R_c}} \frac{R^{+3}}{R^{+2} \frac{\partial T^+}{\partial r^+}} = t_c \frac{R^+}{\frac{\partial T^+}{\partial r^+}} \quad (15)$$

Therefore the characteristic time scale is found to be

$$t_c = \frac{1}{3} \frac{r_v L R_c^2}{k T_c} \quad (16)$$

Where T_c can be defined as $(T_\infty - T_{sat})$, when the bubble is surrounded by liquid that has no contact with the wall. But Cole and Shulman [10] have reported that using of the difference between wall temperature and liquid saturation temperature is more suitable. However $\Delta T = T_{wall} - T_{sat}$ might be a good approximation if the thermal layer in which the bubble grows was uniformly superheated to the temperature at the wall. In fact, since the liquid superheat approaches zero far from the wall, an average driving force of $\Delta T = (T_{wall} - T_{sat})/2$ might be a better approximation. Also, Dalle Donne and Ferranti [11] have suggested the same approximation. Thus

$$t_c = \frac{1}{3} \frac{r_v L R_c^2}{k \Delta T} = \frac{1}{3} \frac{1}{Ja} R_c^2 \quad (17)$$

Suppose that bubble growth can be characterized by the pressure difference between the vapor and the bulk liquid pressures, i.e. $P_v - P_\infty$, then the characteristic velocity scale v_c can be determined from the solution of the equation of motion by Rayleigh [3].

$$v_c = \frac{R_c}{t_c} = \sqrt{\frac{2}{3} \frac{P_v - P_\infty}{r_L}} \quad (18)$$

From equations (17) and (18), the characteristic radius and time scale can be expressed as

$$R_c = \sqrt{\frac{27}{2}} Ja a \sqrt{\frac{r_L}{P_v - P_\infty}} \quad (19)$$

$$t_c = \frac{9}{2} Ja a \frac{r_L}{P_v - P_\infty} \quad (20)$$

By relating pressure difference in equations (19) and (20) with the departing radius and considering that the radial acceleration and the radial velocity close to bubble departure radius are negligible, extended Rayleigh equation gives

$$P_v - P_\infty = \frac{2s}{R_d} \quad (21)$$

Therefore, equations (19) and (20) can be rewritten as

$$R_c = \frac{\sqrt{27}}{2} Ja a \sqrt{\frac{r_L R_d}{s}} \quad (22)$$

$$t_c = \frac{9}{4} Ja a \frac{r_L R_d}{s} \quad (23)$$

Considering general form of bubble growth, equation (13), one can introduce the global growth behavior based on analytical solution and dimensionless parameter R^+, t^+

$$R^+(t^+) = m\sqrt{t^+} \operatorname{erf}(n\sqrt{t^+}) + R_o^+ \quad (24)$$

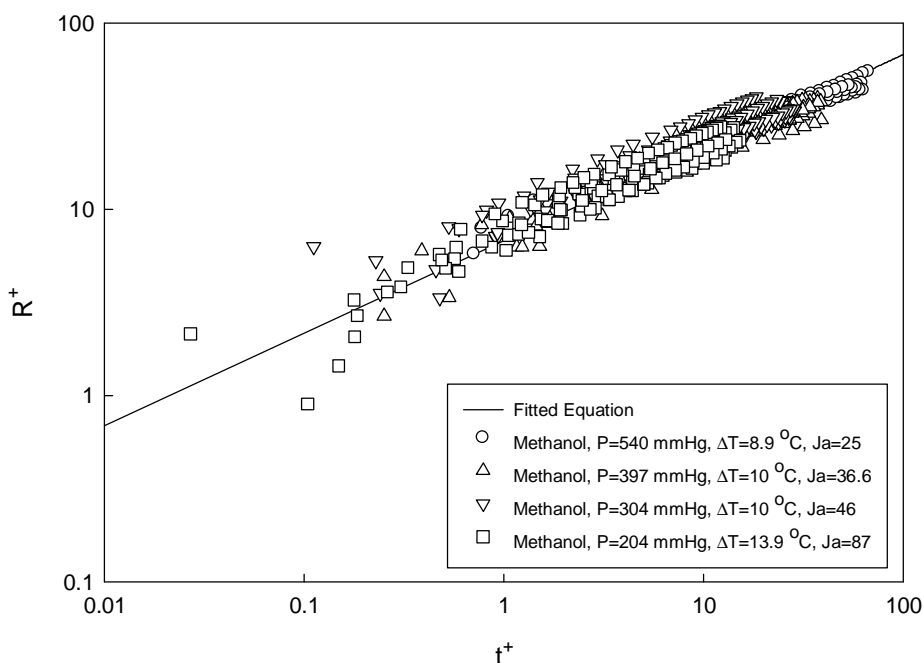


Figure 4. Fitted equation and experimental data of methanol

Results and discussion

The experimental data of bubble growth that were provided by Cole and Shulman [12] for methanol was selected to check this model. Using the present model, the table curve fit Software and about 600 experimental data of methanol, the following correlation was obtained for superheated methanol

$$R^+ = 6.7851\sqrt{t^+} \operatorname{erf}\left(38.856\sqrt{t^+}\right) + 0.02655 \quad (25)$$

The fitted curve was drawn in Figure 4.

This correlation was used to predict the bubble growth of methanol at 135mmHg pressure and $\Delta T = 13.9^\circ\text{C}$ superheat. The results of the present model in comparison with Plesset and Zwick [4] and Forster and Zuber [5] are shown in Figure 5. It shows that the present model predicts bubble growth more accurately. In Figure 6, the error band of 20 % for present model is shown. It shows that the results are within the error band and this model is applicable.

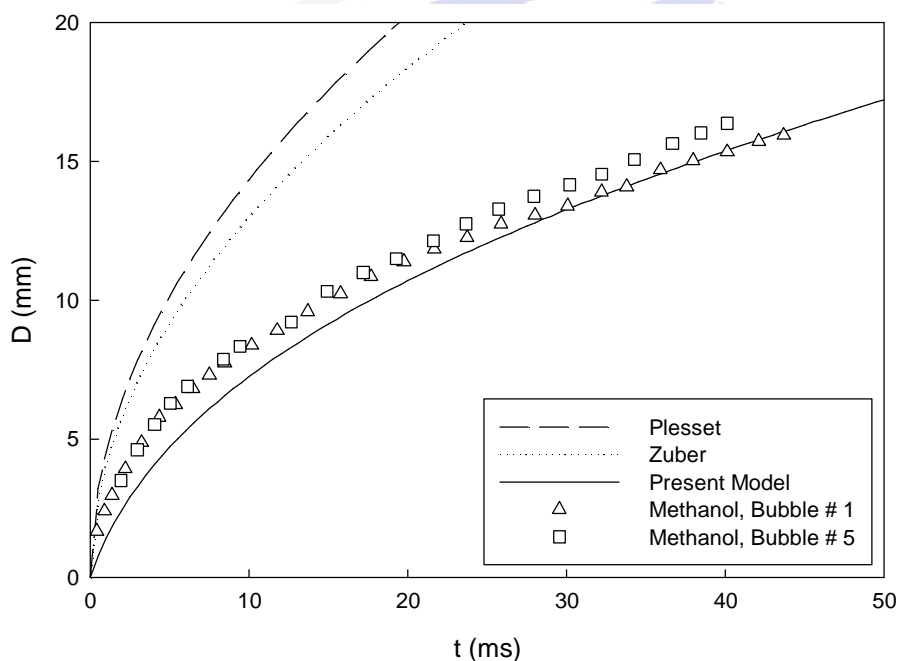


Figure 5. Comparison of the various models with the experimental data at 135 mmHg and $\Delta T = 13.9^\circ\text{C}$

Conclusions

From the solution of transient heat conduction equation and energy balance equation around the single bubble in superheated liquid a new analytical model has been derived for bubble growth. The unknown parameters, m and n from the dimensionless form of the present model, according to Lee et al. [9] dimensional analysis work, were estimated to be, 6.785 and 38.856, respectively for methanol.

The present model shows that bubble radius is proportional to $\sqrt{t} \operatorname{erf}(n\sqrt{t})$. The form of $R \propto \sqrt{t}$ was recommended by Plesset and Zwick [4] and other researchers which is somehow

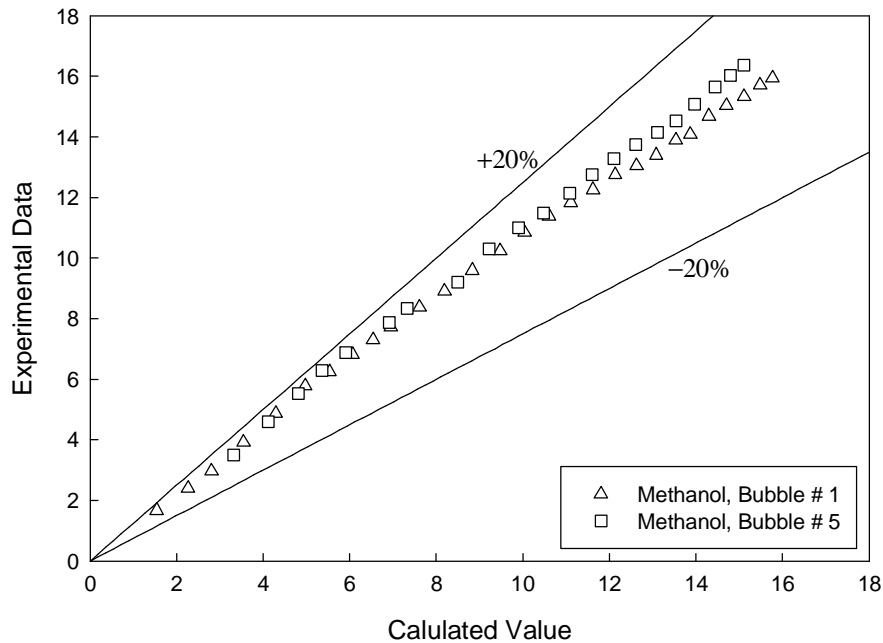


Figure 6. The error band for present model at 135 mmHg and $\Delta T = 13.9^\circ\text{C}$

Nomenclature

a = constant in equation (12)

C_p = specific heat

$$Ja = \frac{(T_\infty - T_{sat}) r_L C_{PL}}{L r_v} = \text{Jakob number}$$

k = thermal conductivity

L = latent heat

m = constant in equation (13)

n = constant in equation (13)

P_v = vapor pressure inside the bubble

r = radial position within the liquid

R = bubble radius

R_o = initial radius

t = time

T = temperature

T_v = vapor temperature inside the bubble

u = radial outward velocity

v = velocity

a = thermal diffusivity

$\Delta T = (T_\infty - T_{sat})$ = superheat degree

ρ = density

S = surface tension

Subscripts – Superscripts

c = characteristic

d = departure

L = liquid

v = vapor

∞ = infinite or far field

$+$ = dimensionless

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