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2D Lattice Boltzmann Model Benchmarking for Low Reynolds Poiseuille Flow through Channel

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Abstract

A 2D Lattice Boltzmann BGK (LBGK) model on a D2Q9 lattice was constructed and tested for accuracy in an example physical situation. An introduction to the LBGK method is given followed by a brief outline of the computer model construction in MATLAB environment. Effects of viscosity and lattice number variation on velocity results were also examined. The LBM simulation results illustrate a good agreement with analytical solution of pressure driven flow through 2D symmetric channel.

Keywords: Lattice Boltzmann; pressure driven, Poiseuille flow, channel, analytical solution.

1. Introduction

In the last few years, we have witnessed a rapid development of the method known as the Lattice Boltzmann Equation (LBE). Although only in its infancy, the LBE method has demonstrated its ability to simulate hydrodynamic systems, magnetohydrodynamic systems, multiphase and multi-omponent fluids including suspensions and emulsions, chemical-reactive flows, and multicomponent flow through porous media. The obvious advantages of the LBE method are the parallelism of the method, the simplicity of the programming, and the capability of incorporating model interactions [1-4]. Historically, the models of the lattice Boltzmann equation directly evolve from the models of the lattice-gas automata (LGA) [1,2]. A number of LBE models in both two and three dimensional are derived [2,3]. The kinetic-model equation used in this paper is the BGK equation with a single relaxation time [1-4]. Although the BGK equation has its inherent limitations and shortcomings, it is sufficient to use the BGK equation for the purpose of studying hydrodynamics of simple fluids. The essential ingredients in any lattice Boltzmann models which are required to be completely specified are: (i) a discrete lattice space on which fluid particles reside; (ii) a set of discrete velocities (often going from one node to its nearest neighbors) to represent particle advection; and (iii) a set of rules for the redistribution of particles residing on a node to mimic collision processes in a real fluid [4-6]. In this article, the theory and construction of D2Q9 model are outlined in detail. Simulations were conducted under the MATLAB code for fluid moving through a symmetric 2D channel. The results obtained should allow one to assess the suitability of the model for implementation into a simulation.

2. The Lattice Boltzmann BGK Model

The Boltzmann model is constructed on a lattice space that contains fluid particles. Each of these particles is given a discrete set of velocities for traveling from one node on the grid to another. This represents particle advection

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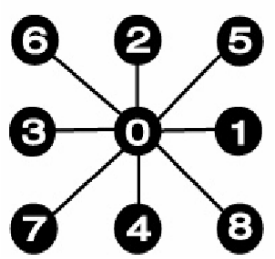

Figure 1: Node ordering on the DQ29 lattice

Koelmann (1991) Lattice Velocities:
C{0} = (0,0)
C{1,3}=(±c,0)
C{2,4}=(0,±c)
C{5,6,7,8}=(±c,±c)

Table 1: Different lattice velocities associated to relative position on grid. Compare with Figure 1.

The particles are redistributed on each node according to a set of rules that recover the collision process [5]. This model uses a D2Q9 lattice with nine discrete velocities that were originally outlined by Koelmann. Table 1 shows the velocity distribution rules from node point zero to the neighboring node points. Figure 1 displays the node ordering of the rules. The time evolution of the model is based upon particle distribution and collisions of the model. Time evolution is carried out during the propagation of the particles along the lattice points according to the rules described above. The time and space averaged microscopic movements of particles are modeled using molecular populations called the distribution function, which defines the density and velocity at each lattice node. The time dependent movement of fluid particles at each lattice node satisfies the following particle propagation equation [1-6]:

$$F_i(x+e_i,t+1) = F_i(x,t) - \frac{1}{t}(F_i(x,t) - F_i^{eq}(x,t)) \qquad (1)$$

Where F_i is the non-equilibrium distribution function, F_i^{eq} is the equilibrium distribution function, and e_i is the microscopic velocity at lattice node x at time t, respectively. The relaxation parameter τ, determines the kinetic viscosity ν of the simulated fluid according to:

$$t = \frac{(2n-1)}{6} \qquad (2)$$

Fig. 2a shows the orientation of the equilibrium distribution function components at time $t(F_i^{eq}(x,t))$, where each component points towards the direction of a microscopic velocity vector (e_i). The components at each node propagate to the neighboring nodes and produce the non-equilibrium distribution function for the next time step of the neighboring node $F_i(x+e_i,t+1)$.

Fig. 2b illustrates migration of distribution function components from the neighboring nodes (both F_i and $F_{eq\,i}$) and orientation of non-equilibrium distribution function components (F_i) of the current node.

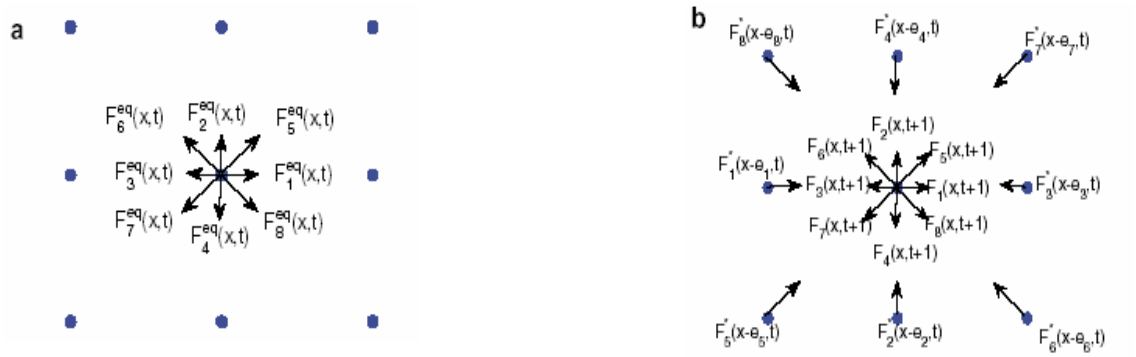

Figure 2. (a) Orientation of components of equilibrium distribution function calculated in the current time step, and (b) illustration of migration of distribution function components from the neighboring nodes

The equilibrium distribution function is given in the following form for the two-dimensional BGK model with nine microscopic velocity vectors (D2Q9) is: [6]

$$F_i^{eq} = w_i r[1+3(e_i.u)+\frac{9}{2}(e_i.u)2-\frac{3}{2}(u.u)] \qquad (3)$$

Where r is the density of the node, and w_i is the weight factor in the ith direction. For instance, the weight factors (w_i) for the D2Q9 LB model are: w_0 = 16/36 for rest particle, w_i = 4/36 ($1 \le i \le 4$) for particles streaming to the face connected neighbors and w_i =1/36 ($5 \le i \le 8$) for particles streaming to the edge-connected neighbors. The macroscopic properties, density and velocity (u) of the nodes are calculated using the following relations:

$$r = \sum_{i=1}^{i=Q} F_i \quad , \quad u = \sum_{i=1}^{i=Q} \frac{F_i e_i}{r} \qquad (4)$$

The most commonly used technique to calculate unknown components of the distribution function at solid nodes is the application of no-slip boundary condition [7]. Two kinds of no-slip boundary conditions exist. The first one, named as full bounce-back, assumes that the wall is located at the solid nodes whereas the second one, named as half-way bounce-back, assumes that the wall is located half-way between the pore and solid nodes [7].

3. Computational Implementation for Poiseuille channel flow and discussion

The mathematical method for Poiseuille flow in the 2D channel was implemented into MATLAB code. The LB algorithm developed in this study is almost straight, Figure 3 shows Flow chart of the LB algorithm.

Figure 4 compares the velocity profile of the fluid for three different acceleration sets (0.1, 0.5, 1) at cross section of the channel (by simulations using 102×102 lattice resolution

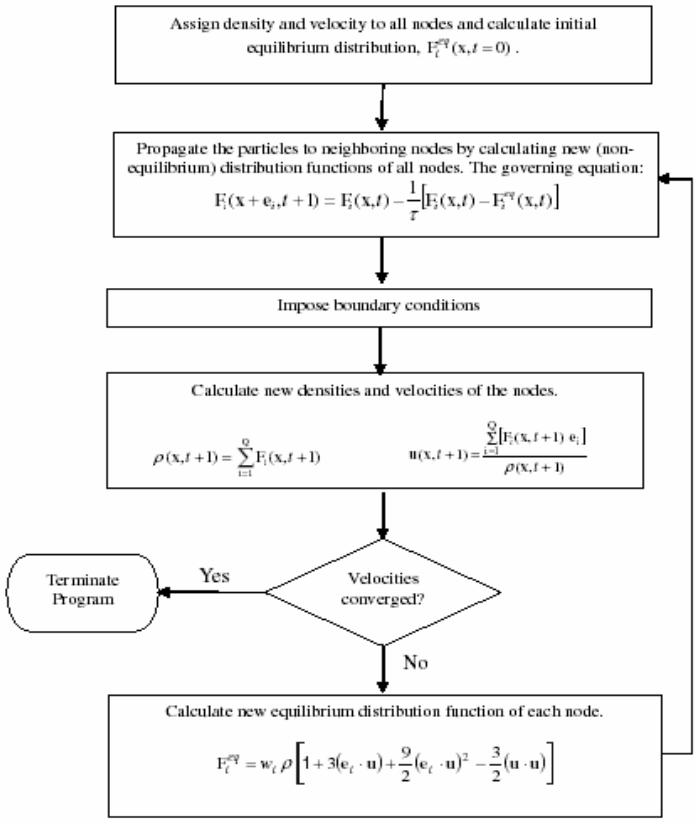

Figure. 3. Flow chart of the LB algorithm.

with the analytical equation. An excellent agreement exists between the results based on Poiseuille's Law and the D2Q9 model. Simulations were also carried out at four different resolutions (number of lattices): 152, 102, 66, and 52 lattice points along the channel width and three viscosities (relaxation time). As it is shown in Fig 5 the error decreases with an increase in resolution and further lattice number doesn't have any effect on LBM prediction. Also, the increase in viscosity of fluid in Fig 4 shows the expected trend of velocity changes similar to that of the analytical one. Figures 5 and 6 were accomplished for a similar case to Figure 4 just with different acceleration 1e-9.

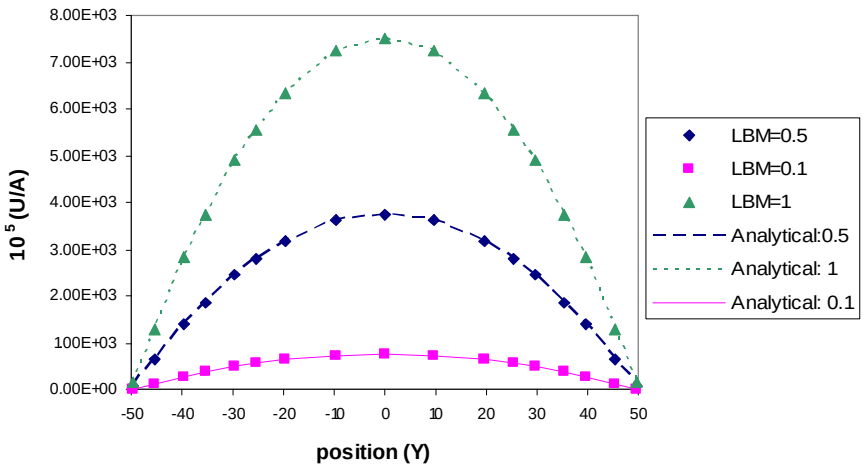

Figure 4. x-velocity profile for different velocities in laminar domain

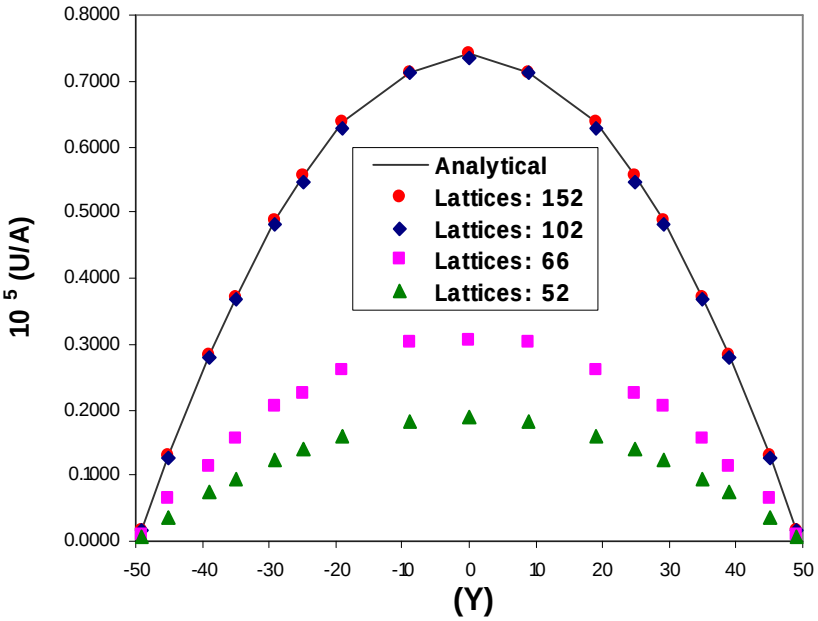

Figure 5. Effect of lattice points on velocity profile

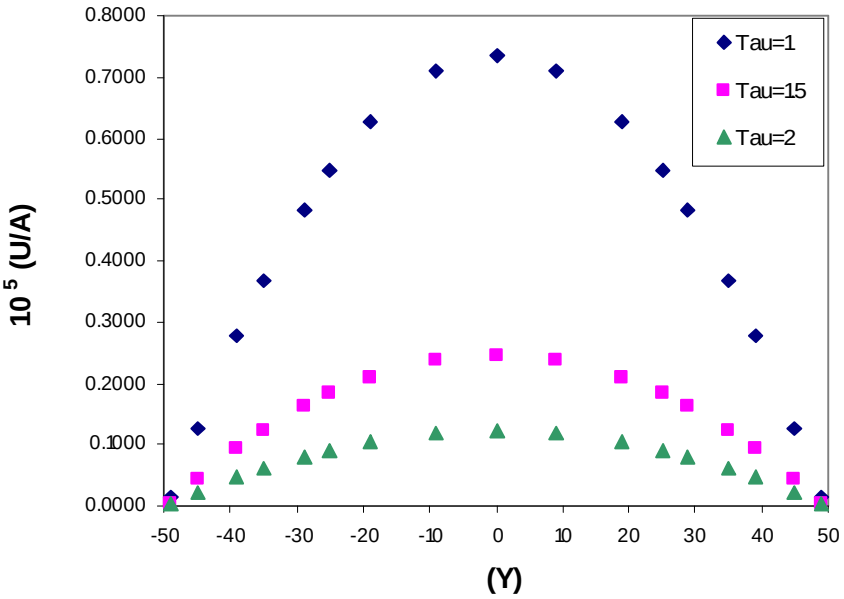

Figure 6. Effect of viscosity on velocity

Nomenclature

e_i	Microscopic velocity vector for D2Q9 model
$F_i(x, t)$	non-equilibrium distribution function at node x at time t
$F_i^{eq}(x,t)$	Equilibrium distribution function at node x at time t
$U(x, t)$	Momentum of particles at node x, at time t
$u (x, t)$	Macroscopic velocity at node x, at time t
x	Vector coordinates of the molecule
w_i	Weight factor for each direction around a node
ν	Kinematic viscosity
$\rho(x, t)$	Density at node x, at time t
τ	Relaxation time

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