Numerical simulation of mass transfer in falling film with FEM approach

Barati, A.[*1], Daryabari, S. M. S.[1], Soltanian, G.[2], Moraveji, M.[1]
1. Chemical Engineering Group, Faculty of Engineering, Arak, I. R. Iran
2. Technology and Research Center of Polymer and Petrochemical (Pooyesh), Arak, I. R. Iran

Abstract

Falling film is important in many industries, but its importance is because of its classical concepts. Modeling and solving the falling film were always performed with simplifying assumptions. In this research falling film modeling and simulation performed with least simplifications. Mass transfer and Navier-Stokes equations coupled together to obtain real results.

Keywords: falling film, simulation, CFD, finite element, COMSOL

Introduction

Falling film is one of the classical mass transfer problems that found widely in wetted columns and heat pumps. The complex mass and momentum transfer phenomena occurring in the falling film has been the subject of a great amount of research ,especially within the last 25 years .Creating useful mathematical models of falling film has proven to be a difficult challenge due to the complex and coupled nature of the transport phenomena and for changing the driving force as the process progress. For modeling of falling film some assumption were used that make the simulation inaccurate. In this paper mass transfer and Navier-Stokes equations were coupled together to make an accurate model with minimum assumption.

Problem description

A thin liquid film, in fully developed laminar flow, is falling down over a vertical wall and is exposed to a gas, which flows either co-currently or counter currently with respect to the film; the schematic view is shown in Fig. 1 .Mass transfer of solute takes place from gas to liquid phase. Steady state conditions are assumed to prevail. A constant gas phase resistance is present and the concentration of solute in gas phase varies axially due to the absorption by liquid. Physical properties of both the fluids remain constant. Mass transfer for the differential control volume can be described by the following conservative equation (in two spatial dimensions and one temporal dimension):

$$\left(\frac{\partial C_A}{\partial t}+u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}\right)=\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right)$$

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[*] abbarati@yahoo.com

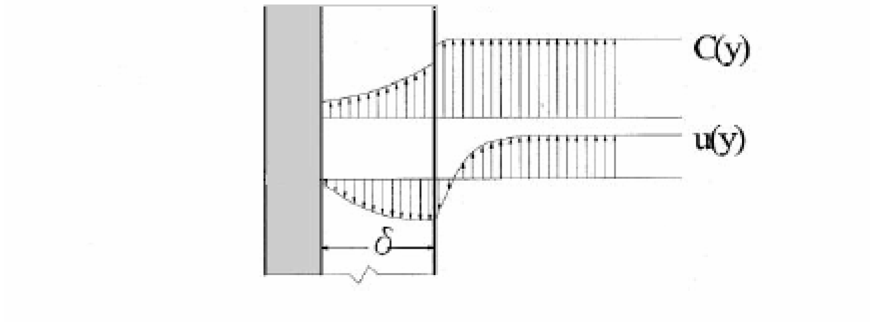

Fig. 1 Schematic diagram of gas absorption process in falling film

Assuming only two species, A and B, exist within the system and the mass densities of phases are constant. The above equation also assume negligible dissipation effects, pressure gradient and transport of mass due to energy flux [1].

Nakoryakov et al. [2,3] take a significant step forward in modeling the absorption process. They consider the case of steady absorption in smooth laminar falling film. The list of assumption they employ is quiet long, but may are still current practical. In order to simplify the problem following assumption are made:

- Vapor pressure equilibrium is instantaneously established at the liquid-gas interface. Its means the concentration of species A is equal to saturation concentration entire interface.
- All thermophysical properties of gas and liquid are constant.
- Diffusion in the stream-wise direction is negligible. This common assumption is well justified by comparing the rate of diffusion to the velocity in the stream-wise direction. This yields the following simplification:

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{AB}\frac{\partial^2 C_A}{\partial x^2}+D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

- Due to the low velocity associated with absorption, the transverse velocity, υ, is negligible, therefore:

$$u\frac{\partial C_A}{\partial x}=D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

- Also, because the anticipated rates are low compared to the solution flow rate, changes in the film thickness are negligible.
- The velocity and solution concentration in the falling film is uniform.

Nakoryakov et al. provide a solution using Fourier separation of variable techniques.
Pigford [10] assumes velocity profile for modeling falling film:

$$v_{\max}\left[1-\left(\frac{x}{\delta}\right)^2\right]\frac{\partial c_A}{\partial z}=\mathcal{D}_{AB}\frac{\partial^2 c_A}{\partial x^2}$$

Pigford solved this equation by analytical techniques as his PhD thesis.
Grossman [5,6] uses the essentially the same simplifications found in Nakoryakov. The main difference is that Grossman assumes the fully developed, laminar velocity profile:

$$u(y)=\frac{3}{2}\bar{u}\left[2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right]$$

Grossman uses two methods to solve the problem. First using Fourier method same as Nakoryakov. Second uses numerical technique base on finite difference methods. The results of numerical and analytical solutions are reported to be in excellent agreement.
Brauner et al. [7] develops new solutions to the vertical laminar falling film. Her assumption are the same as those of Grossman. The key difference is that the application of Flick's Law of diffusion at the interface is formulated without assuming infinite form of governing equation:

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

Brauner solves this equation by numerical technique and the results have good agreement with experimental data.
In falling film mass and momentum transport phenomena occurs instantaneously and affect each other. Using average velocity (Nakoryakov) or velocity profile (Pigford, Grossman and Brauner) cause error in modeling. In this research falling film modeled in this form:

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{AB}\frac{\partial^2 C_A}{\partial x^2}+D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

In this equation velocity and diffusion in two dimensions considered, but the key difference are values of *u* and υ that obtained from coupling with Navier-Stokes equations.

Solution Method

The governing equations are solved numerically with the commercial package, COMSOL Multiphysic 3.2 .This is designed to simulate systems of coupled non-linear partial differential equations (PDE) in one, two or three dimensions. The geometry of the storage is defined. The equations are written in partial differential form in line with program definitions, and initial and boundary conditions are determined. The mesh convergence was verified with refined mesh sizes in interface. 3067 elements mesh size were considered to be appropriate.
Navier-Stokes and mass transfer equations are used in this form:

$$\rho\frac{\partial \mathbf{u}}{\partial t}-\nabla\cdot\eta(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p = \mathbf{F}$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\nabla\cdot(-D\nabla c+c\mathbf{u}) = 0$$

Boundary conditions for simulation defined as below:

at y=0, $C_A = C_{A0}$, $u=u_0$
at x=0, $dC_A/dx=0$, u=0
at x= δ, $C_A = C_{Ai}$, u=slip

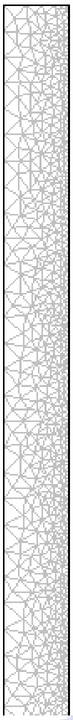

Fig. 2 meshes of falling film

Results and Discussion

Result of the simulation gives concentration and velocity profile at the same time. Fig. 2 describes typical concentration profile of CO_2 in different height. The liquid phase concentration shows variation in both y and z directions while, the gas phase concentration considered constant and equal to saturation gas concentration. Then the effect of falling film velocity and film thickness on concentration and velocity profile were discussed. Due to coupled solution of governing equations and minimum simplifying assumptions results are more accurate than previous works.

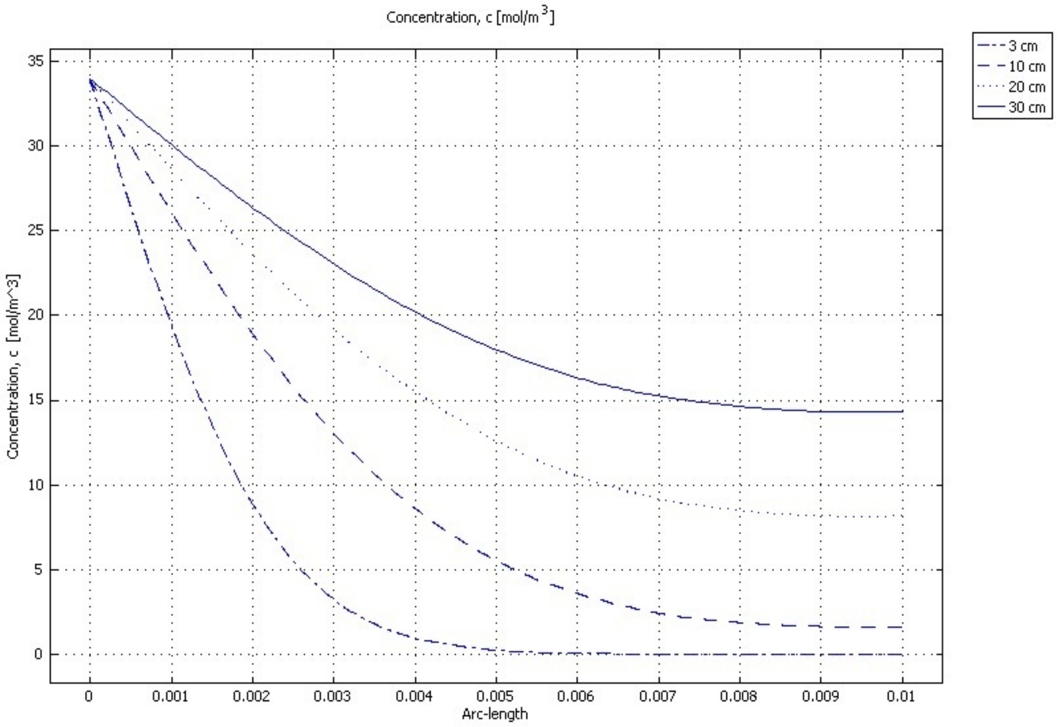

Fig. 3 Concentration profile of CO_2 in different height

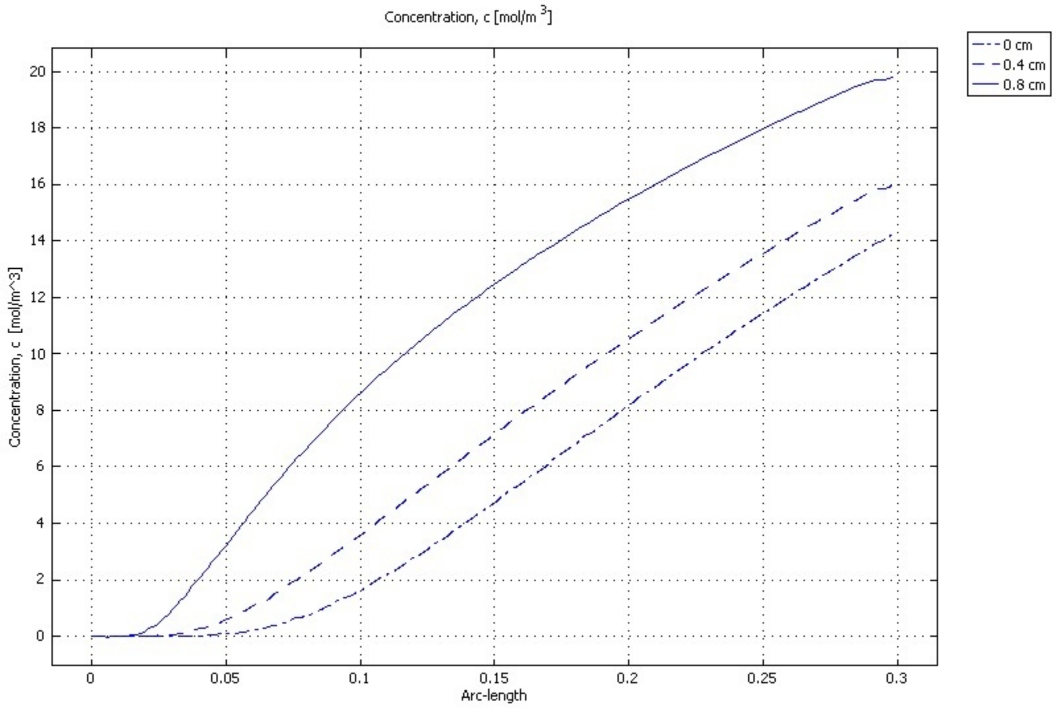

Fig 4. CO_2 concentration profile in various distances (δ) from wall

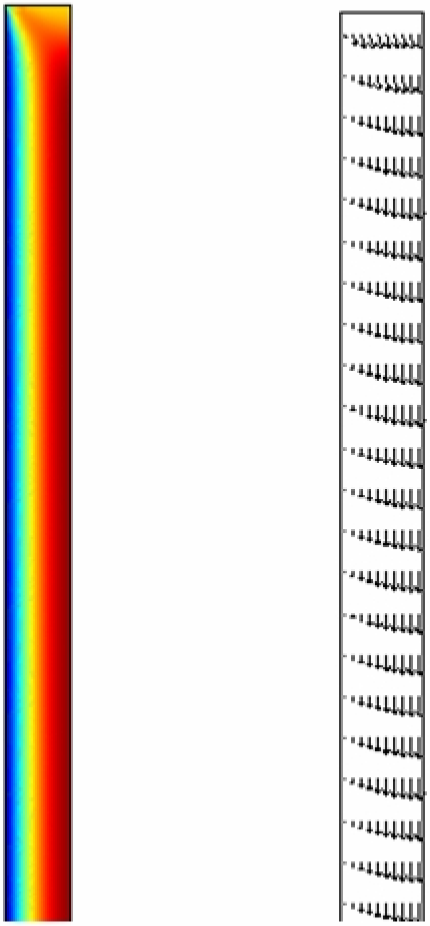

Fig 5. Liquid velocity counters and vector

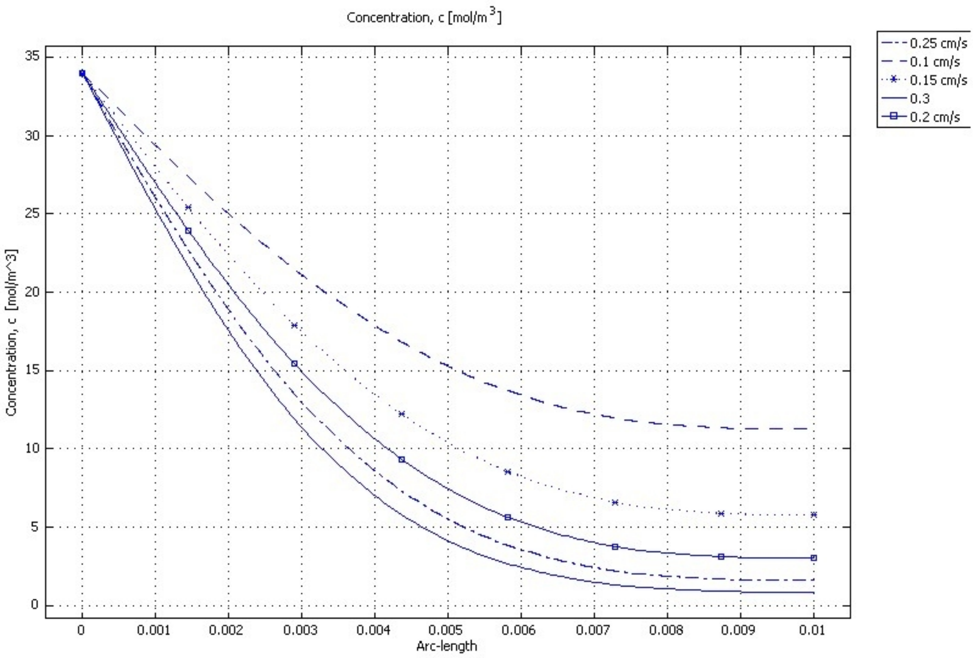

Fig 6.Effect of various inlet of velocity on CO_2 concentration

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