

## **Development of a Robust Intelligent Model to Determine Fracture Conductivity Based on Formation Lithology**

**M.R. Akbari**

Faculty of Petroleum Engineering, Amirkabir University of Technology, Tehran, Iran  
[akbariisf@gmail.com](mailto:akbariisf@gmail.com)

**M.J. Ameri**

Faculty of Petroleum Engineering, Amirkabir University of Technology, Tehran, Iran  
[ameri@aut.ac.ir](mailto:ameri@aut.ac.ir)

**M. Pournik**

Petroleum & Geological Engineering, University of Oklahoma, Oklahoma, USA  
[maysam.pornik@ou.edu](mailto:maysam.pornik@ou.edu)

### **Abstract**

Acid fracturing is one of the widely used techniques for stimulating well production. It is an alternative to proppant fracturing for limestone or dolomite formations. The success of acid fracturing is dependent on both the creation of effective fracture conductivity and fracture penetration. Although there has been a significant amount of studies on the acid fracturing process, most of these have concentrated on the acid penetration distance with only a few dealing with fracture conductivity. Accurate determination of this parameter is critical for an adequate design of fracturing jobs and project investment prospects. Due to the stochastic process inherent in acid fracturing, attempts at modelling have been met with challenges, particularly in predicting conductivity. In this study, an intelligent model was developed to predict acid fracture conductivity. Acid dissolving power and injection rate as the treatment parameters and rock embedment strength as the formation parameter are considered at different closure stresses, and ultimately, fracture conductivity was anticipated using the suggested model. The results showed an excellent match with the experimental data compared to common industrial models. Formation lithology played a substantial role in acid fracture conductivity and lumped models were not adequate to predict fracture conductivity.

**Keywords:** Well stimulation; Acid fracturing; Acid fracture conductivity; Intelligent modeling; Fracture conductivity correlations

## Introduction

Acid fracturing is typically conducted in carbonate reservoirs, which make up approximately ۷۰% of the worldwide hydrocarbon reserves. In such a treatment, acid or a fluid used on a pad ahead of the acid is injected down the well casing or tubing at rates greater than the rate at which the fluid can flow into the reservoir matrix. This injection yields a build-up in wellbore pressure sufficient to overcome compressive earth stresses and the formation's tensile strength. Failure then occurs, forming a crack (fracture). Continued fluid injection increases the fracture's length and width. A flow channel is created due to reaction of acid injected into the fracture and remains open when the well is put back into production (Williams et al., ۱۹۷۲; Economides et al., ۱۹۹۴). Optimization of live acid penetration distance and conductivity of created fracture are the main targets of designing acid fracturing treatments. Acid fracture conductivity is defined as a measure of the capacity for fluid flow through a fracture. The amount of rock dissolved, the fracture surface etching patterns, the rock strength and the closure stress impact fracture conductivity. Conductivity is created only if the less dissolved parts act like pillars to keep more dissolved parts open (Mou, ۲۰۰۹; Broaddus, ۱۹۶۸; Nierode, ۱۹۷۳; Anderson, ۱۹۸۹). In order to predict acid fracturing, there needs to be a model for conductivity which can accurately anticipate fracture conductivity against closure stress.

Acid fracturing conductivity models date back to the early ۱۹۷۰'s. In the past, there have been studies that predict the conductivity of acid fracturing correlated to the formation rock, mechanical properties and geometry of the fracture through experiments (Broaddus, ۱۹۶۸; Nierode, ۱۹۷۳; Anderson, ۱۹۸۹, Van Domelen, ۱۹۹۲; Beg, ۱۹۹۶; Ruffet, ۱۹۹۷; Navarrete, ۱۹۹۸; Gong, ۱۹۹۹). An acid fracture correlation consists of two parts: fracture conductivity at zero closure stress and the rate of conductivity change with closure stress. There are two possible ways to develop a correlation: theoretical and empirical. No conductivity correlation accurately predicts acid fracture conductivity, despite the theoretical and experimental work on the subject (Nierode, ۱۹۷۳; Gong, ۱۹۹۹; Pournik, ۲۰۰۷; Mou, ۲۰۰۹). Nierode and Kruk (۱۹۷۳) developed a correlation that is the most widely used in industry and requires experimental data on acid fracture conductivity from a laboratory test. The resulting conductivity is based on the volume of rock dissolved and the rock's mechanical strength ((Nierode, ۱۹۷۳). Nasr-El-Din et al. (۲۰۰۸) suggest that the correlations developed by Nierode and Kruk were lumped together rather than separated by lithology. They modified the correlation by graphing and evaluating the data again both as a lumped set and as individual sets by lithology. The recalculated correlations kept the same form as the original ones, but made the constants different and were more precise (Nasr-El-Din, ۲۰۰۸). Correlations since have attempted to include parameters that quantify the fracture surface's roughness, but these correlations consider idealized analytical solutions that generalize the mechanism of conductivity.

The objective of this work is to develop a precise model to estimate conductivity for acid fracturing treatment based on a new approach in function approximation modelling. Predicting conductivity is difficult because it is a function of the rock's strength, heterogeneities present in the rock, the transportation and dissolution of acid, the closure stress and other variables. The study uses a treatment parameter called dissolved rock equivalent conductivity (DREC) that stems from the total amount of rock dissolved by injected acid at zero stress. Then lets the conductivity decline as the fracture width reduces when closure stress is applied. DREC derived according to the cubic law indicates the acid dissolving power and the total volume of injected acid proportional to the geometry of the fracture created. Due to the complication of predicting acid fracture conductivity and also the capability of artificial neural networks in modelling, in this paper, a robust intelligent model based on neural networks is developed to predict acid fracture conductivity accurately. The input data are considered as a lumped set and as individual sets by lithology to provide a better understanding of the formation lithology effect. Ultimately, the resulting conductivity given by ANN models is compared to prior models.

## Development of Artificial Neural Network Model

An artificial neural network is defined as a powerful data modelling tool that is able to capture and represent complex input/output relationships. The true power of neural networks lies in their ability to represent both linear and non-linear relationships, as well as to learn of these relationships directly from the data being modelled. Traditional linear models are simply insufficient when it comes to modelling data that contain nonlinear characteristics (Haykin, ۲۰۰۴).

Due to the complication of acid fracture conductivity trends versus the effective parameters, an artificial neural network is an appropriate tool to obtain fracture conductivity. In order to generate a neural network model, Nierode and Kruk (۱۹۷۳) experimental data sets are applied as input data. One hundred and sixteen data sets including acid fracture conductivity in different closure stresses and various rock embedment strengths for different formation lithology (dolomite, limestone and chalk) are considered as modelling data. Table ۱ shows the range of experimental data used as the input variables in this study.

**Table ۱. The range of experimental data used as the input variables**

Parameters	Minimum	Maximum	Average	Standard Deviation
DREC (md-in)	۳۳۰۰۰	۵۱۰۰۰۰۰۰۰	۵۸۰۰۴۷۰۰۰	۱۲۶۰۴۹۶۰۲۸۷,۴۳
RES (Psi)	۵۰۶۰۰	۸۸۰۱۰۰	۳۵۰۱۷۰	۲۴۰۵۶۹,۳۰
Closure stress (Psi)	۰	۷۰۰۰	۲,۹۳۹	۲,۴۶۴,۵۴
Fracture Conductivity (md-in)	۱,۲	۷۴۰۰۰۰۰۰	۱۸۵۰۴۹۰	۷۴۴۰۳۳۵,۶۲

An artificial neural network model is developed based on the following algorithm, including seven main sections: ۱. Reading the data ۲. Preparing the data ۳. Dividing the data into training and test data ۴. Creating a new network ۵. Training the network ۶. Testing the network ۷. Saving the results and displaying the best results. Figure ۱ illustrates the main algorithm used for the development of the ANN models.

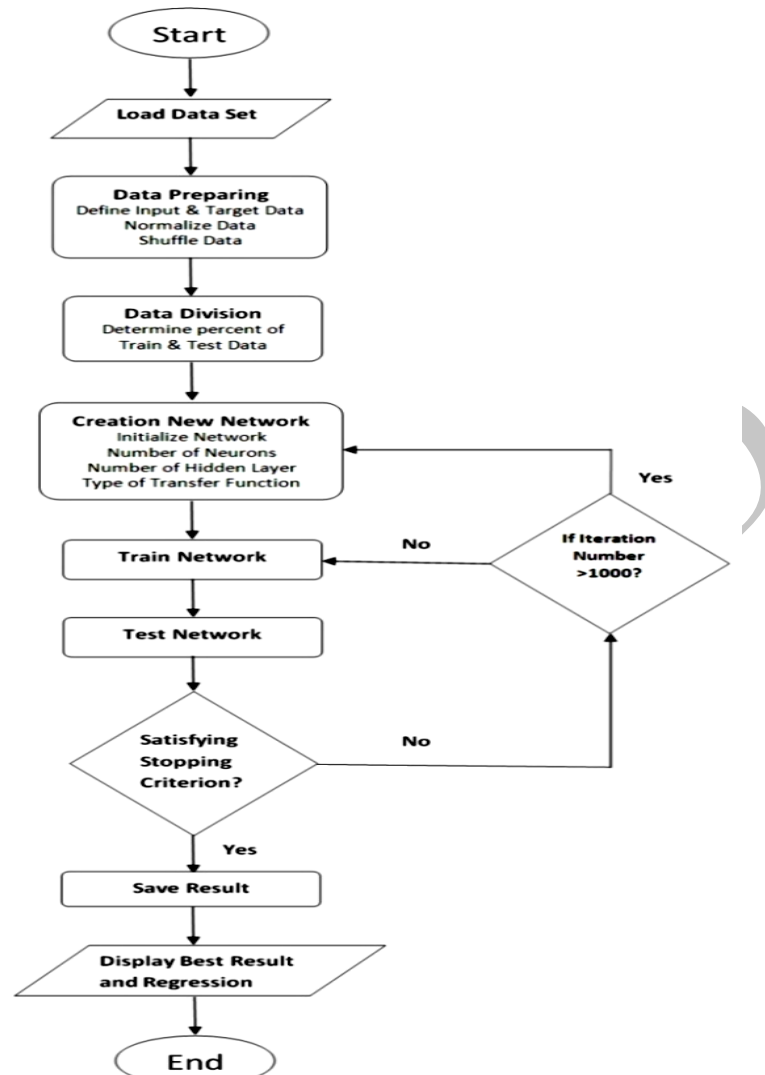


Fig 1. Main steps in the development of ANN models

In order to study of the effect of formation lithology on accuracy ANN developed models, three types of data sets, based on rock types are considered as input data. Three ANN models are developed according to each type of data set. The first model is generated based on all input data including the data of three rock types, called an ANN AD model. The second model is an ANN Limestone model which only considers limestone data and the last model is the ANN Dolomite model created based on only dolomite rock type data. Finally, the results of these three data sets are compared to reveal the effect of lithology on acid fracture conductivity. In this study, 80% of the data are selected for training and validation, and the remaining data are applied to test the network randomly. Different neural network architectures, train algorithms and transfer functions are tested to find a suitable model with minimum error. The average absolute relative error (AARE) and regression coefficient (R) are criteria for evaluating the capability of different models.

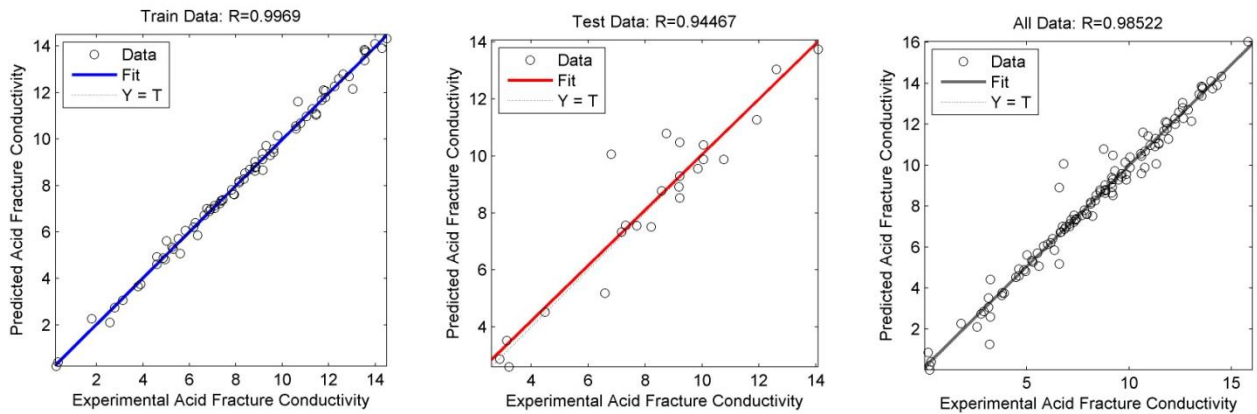
### Results and Discussion

Ultimately, a feed-forward back propagation network was prepared in which the weight and bias values of the network were updated according to the Levenberg-Marquardt algorithm. The best architecture of the network was selected for each model based on the average absolute relative error (AARE) and regression coefficient after being run 1000 times. Table 2 shows the specifications of the

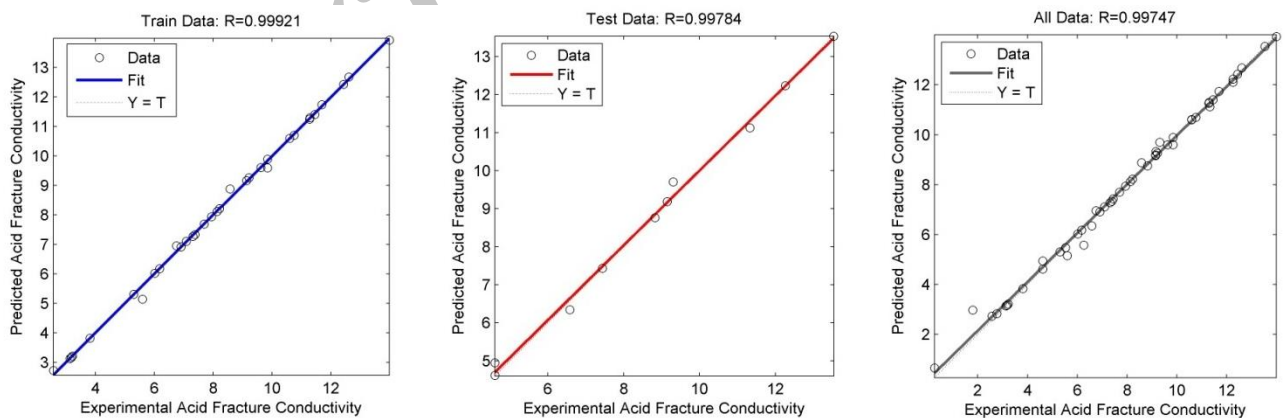
best ANN models in each data set for training and testing the data. All three purposed models have two hidden layers in which the best architecture of the network has ۱۳ neurons. Hyperbolic tangent sigmoid (Tansig) and log-sigmoid (Logsig) transfer functions have the highest accuracy for the dolomite and limestone data sets respectively. The acid fracture conductivity values predicted by the best ANN models for the three data sets are illustrated versus the experimental data in figures ۲ to ۴.

**Table ۲. Parameters of the best neural network models**

Model	ANN AD	ANN Dolomite	ANN Limestone
Transfer Function	Logsig	Tansig	Logsig
Train Algorithm	Trainlm	Trainlm	Trainlm
No. Hidden Layers	۲	۲	۲
No. neurons	۱۰ [۵ ۵]	۱۳ [۳ ۱۰]	۱۳ [۱۰ ۳]
ARE train data (%)	۳,۰۶۴	۰,۹۴۱۸	۰,۰۰۰۳۲
ARE test data (%)	۸,۵۴۱	۱,۸۸۷۵	۰,۶۵۵۴
Reg. coefficient for train data	۰,۹۹۶۹	۰,۹۹۹۲	۱
Reg. coefficient for test data	۰,۹۴۴۶	۰,۹۹۷۸	۰,۹۹۹۷



**Fig ۲. Values predicted by ANN AD vs. the experimental data for the training and test data**



**Fig ۳. Values predicted by ANN Dolomite vs. the experimental data for the training and test data**

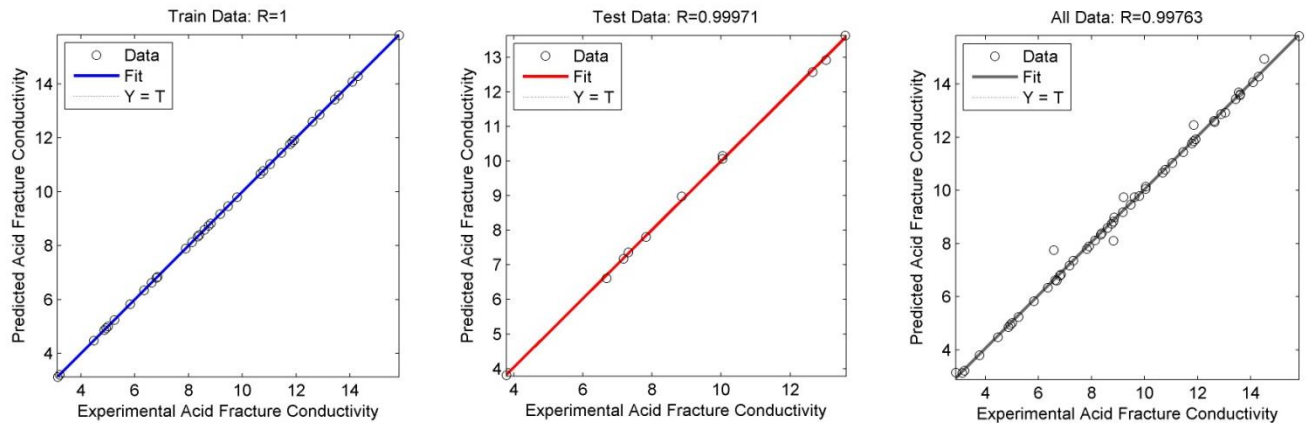


Fig ۴. Values predicted by ANN Limestone vs. the experimental data for the training and test data

As the results demonstrate, there is a perfect match for all the ANN models. However, the precision of the models in which the rock type data are entered exclusively are very high. The average relative error for the test data using the ANN Limestone and ANN Dolomite models are ۰,۶۵۵٪ and ۱,۸۸۷٪ respectively. It indicates that the developed ANN models which consider individual sets by lithology are much more accurate, as is shown in figures ۳ and ۴. In other words, it confirms that the rock type plays a significant role in the prediction of acid fracture conductivity and must be given considerable attention in modelling.

Due to the wide application of Nierode and Kruk's correlation in the industry, the results of ANN models in comparison with those of Nierode and Kruk's model are shown in figures ۵ to ۷. The results show that the acid fracture conductivity values predicted by all three ANN models have a better match to the experimental data compared to Nierode and Kruk's model. However, the match is more precise for the ANN Limestone model and then for the ANN Dolomite model.

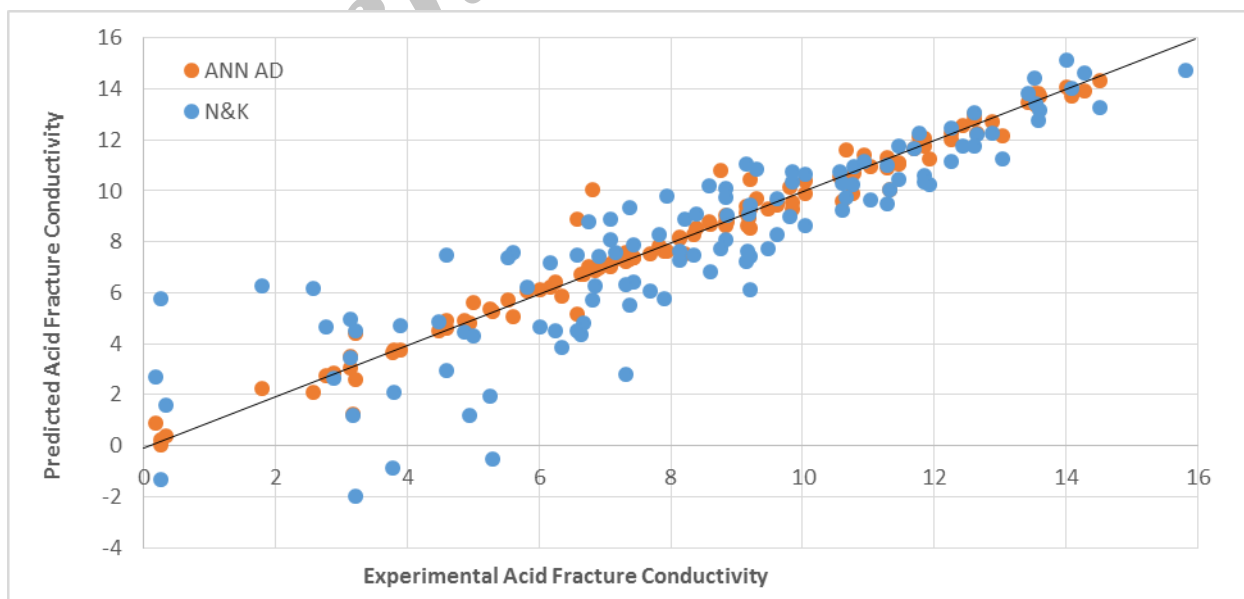
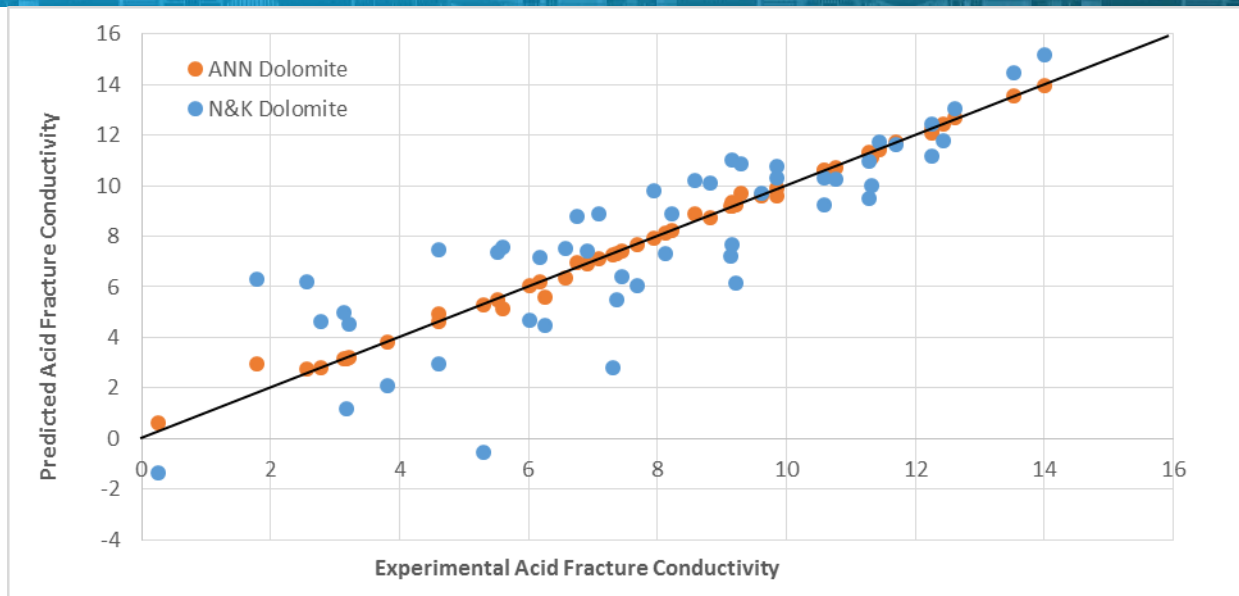
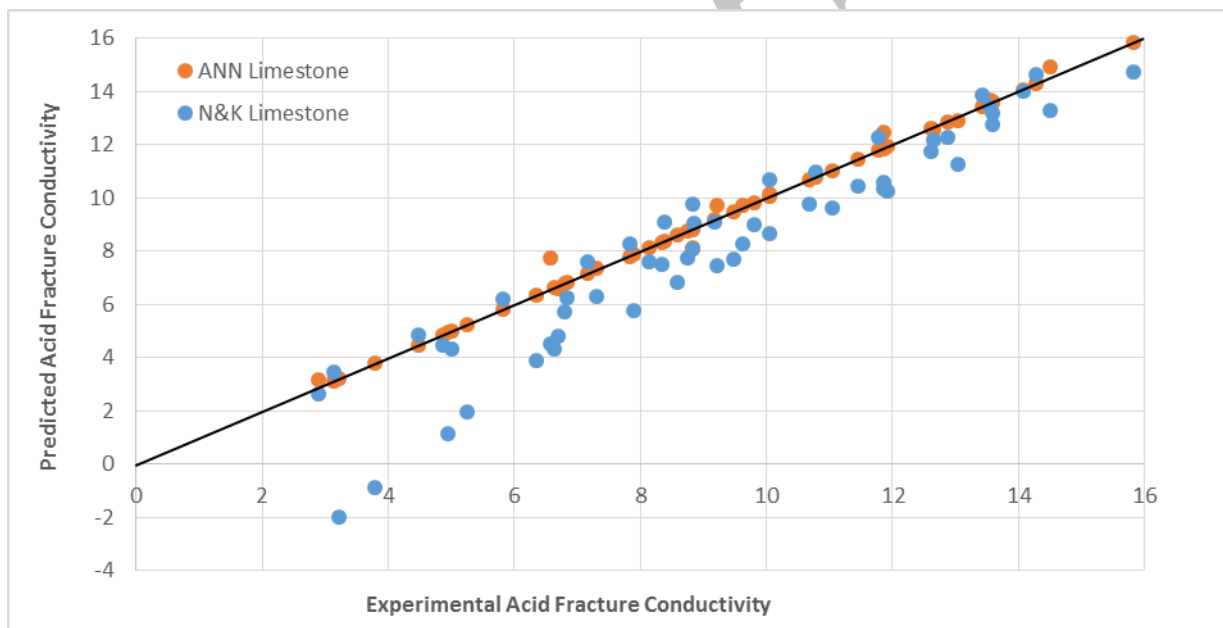


Fig ۵. The comparison between measured and predicted conductivities using the ANN AD model and the Nierode and Kruk model for all closure stresses.



**Fig ٦. The comparison between measured and predicted conductivities using the ANN Dolomite model and the Nierode and Kruk model for all closure stresses.**



**Fig ٧. The comparison between the measured and predicted conductivities using the ANN Limestone model and the Nierode and Kruk model for all closure stresses.**

In this study, the results of the three created models are compared with the results of the Nierode and Kruk (N&K) and the Nasr-EI-Din (NSD) models due to their convenience, the fact that there is no need for parameters relating to the fracture surface profiles and the similarity of the input variables. Figure ٨ illustrates the average relative error of the different models at various closure stresses. As the figure depicts, the ANN Limestone model and the ANN Dolomite predict acid fracture conductivity more accurately compared to other models. The accuracy of these models is demonstrated in this figure compared with prior models' at low, middle and high closure stresses. Since N&K presented a general correlation for the prediction of acid fracture conductivity for limestone and dolomite rock

types, the data are considered as a lumped set and as individual sets by lithology to provide a better understanding of the effect of rock type.

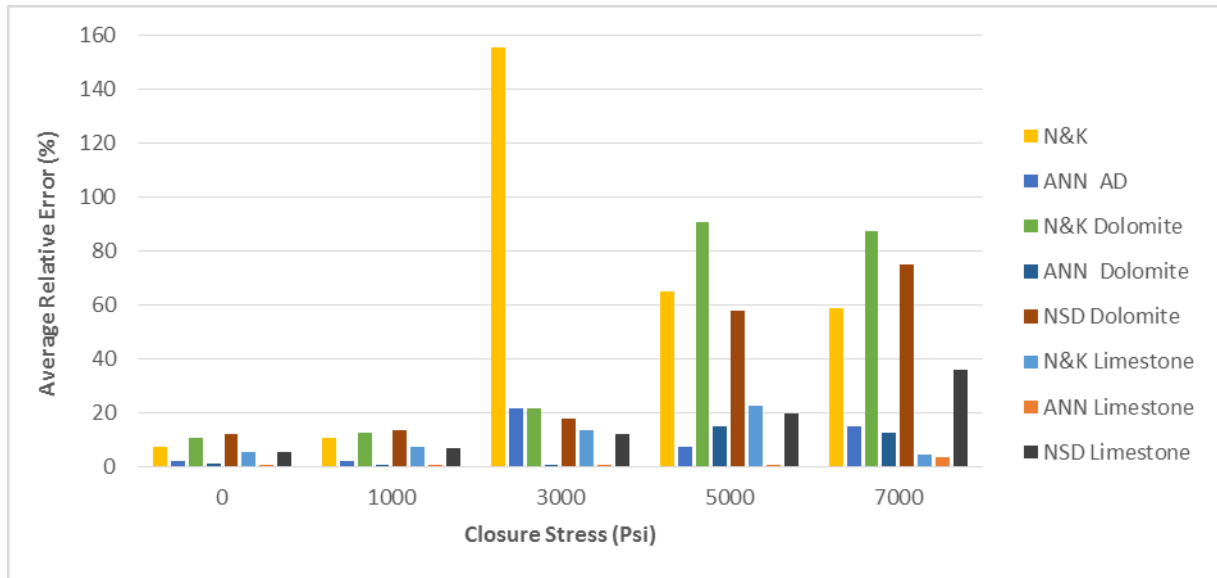


Fig 8. Average relative error of the predictive models at various closure stresses.

In order to evaluate the predictive capability of the generated models, the effect of different dissolved rock equivalent conductivity (DERC) as a treatment parameter and the rock embedment strength (RES) as formation strength parameter are investigated on a limestone sample. Table 3 shows the values of these parameters and the accuracy of each model for predicting acid fracture conductivity for Canyon limestone. The relative error of each model is based on an arithmetic average of different closure stresses from 0 to 7000 psi.

Tables 3. Accuracy of different models for predicting fracture conductivity for the Canyon Limestone

Model	N&K	NSD	ANN AD
Rock Type	Canyon Limestone	Canyon Limestone	Canyon Limestone
DREC (md-in)	4,6*10 <sup>-7</sup>	4,6*10 <sup>-7</sup>	4,6*10 <sup>-7</sup>
RES (psi)	30,700	30,700	30,700
Average Relative Error (%)	10,20	8,41	1,80

As table 3 shows, the ANN AD model is more accurate than the two other models. Figures 10 to 12 illustrate the predicted values of acid fracture conductivity by the N&K, NSD and ANN AD models at low, middle and high values of treatment parameter versus different closure stress for this sample.



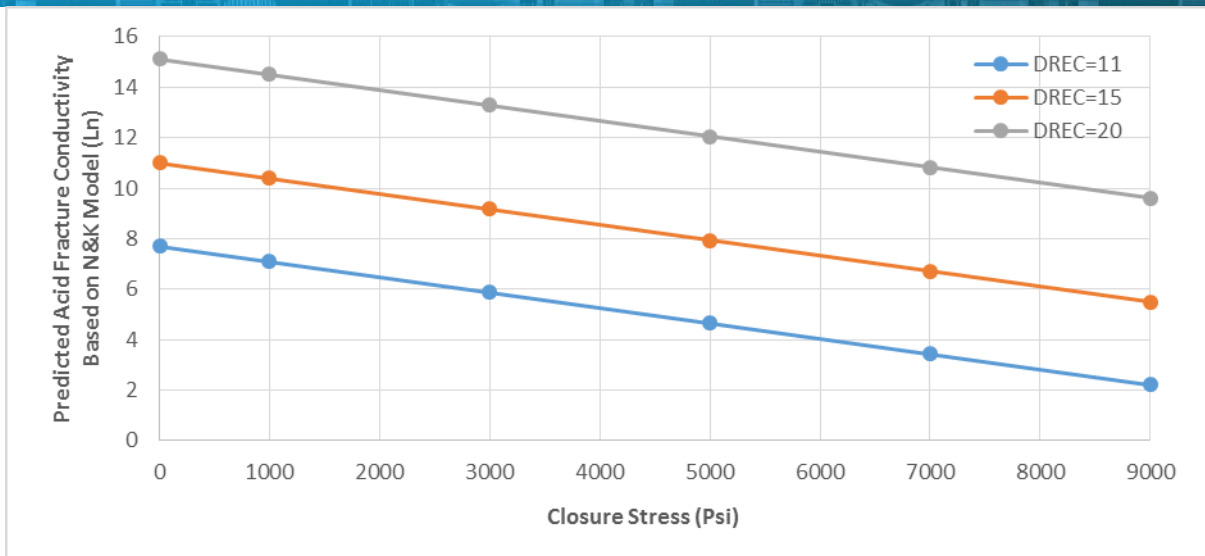


Fig 10. Acid fracture conductivity predicted by the N&K model for different treatment parameters versus various closure stresses.

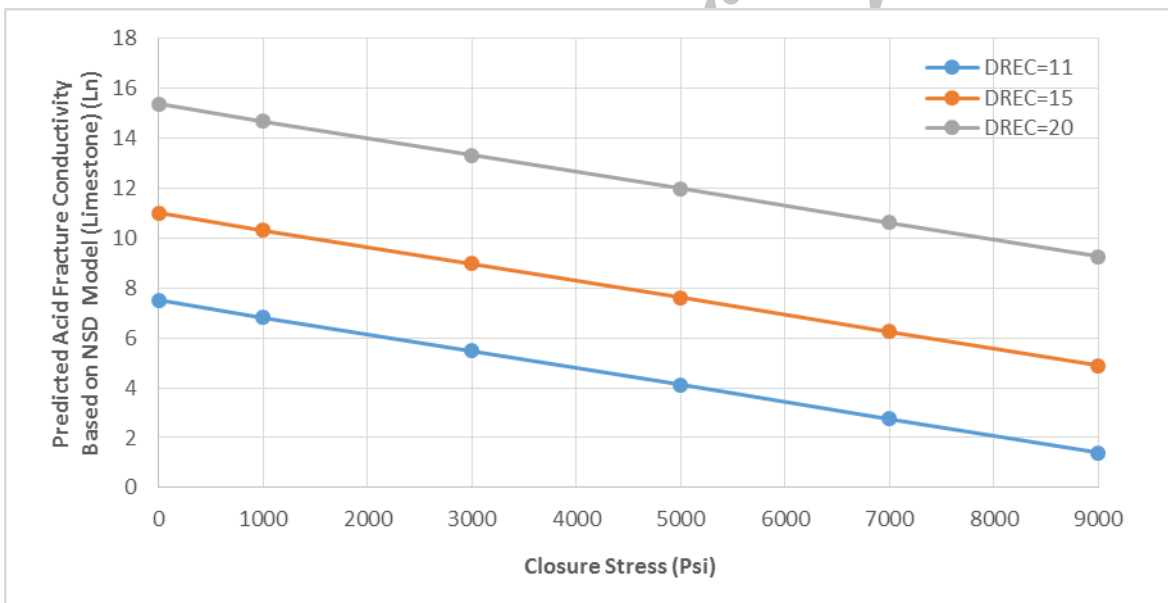


Fig 11. Acid fracture conductivity predicted by the NSD model for different treatment parameters versus various closure stresses.

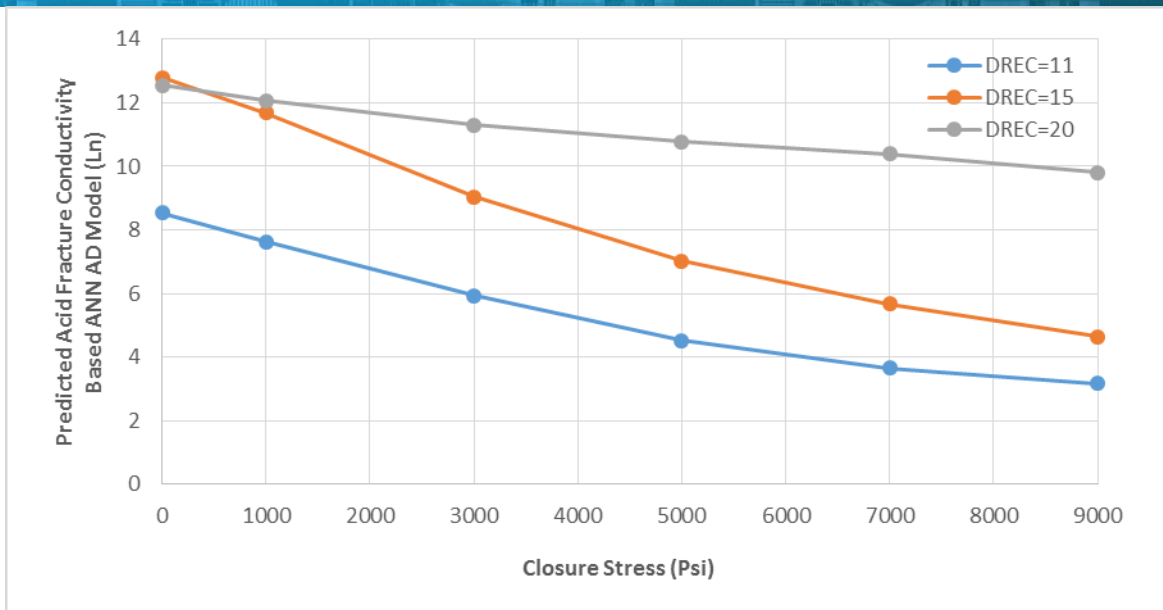


Fig ۱۲. Acid fracture conductivity predicted by the ANN AD model for different treatment parameters versus various closure stresses.

There is a general trend that states that as the closure stress increases the fracture conductivity decreases. The N&K and NSD models predict the trend similarly. The fracture conductivity predicted by both models decreases versus the increase of the closure stress with the same and constant rates for different values of DREC. While, the ANN AD model considers the role of DREC, in particular. As figure ۱۲ shows, in the low dissolved rock equivalent conductivity, fracture conductivity decreases naturally as the closure stress increases. Whereas, the rate of decrease in the middle dissolved rock equivalent conductivity increases sharply and in the high dissolved rock equivalent conductivity the trend goes down again. Therefore, it can be concluded that there is an optimum dissolved rock equivalent conductivity value for achieving high fracture conductivity. In other words, at the low closure stress, the middle values of treatment parameter create conductivity the same as (or even more) the high values of treatment parameter. While at the high closure stress, to achieve proper fracture conductivity, the treatment parameter must be raised dramatically, which affects the expense of the acid fracturing jobs directly. For instance, when the dissolved rock equivalent increases from ۱۱ to ۲۰, acid fracture conductivity increases only ۵۰% at low closure stress. While at high closure stress the fracture conductivity increases by ۲۰۰% for this range, which reveals the significant role of formation closure stress in the design and implementation of acid fracturing treatments.

## Conclusions

Acid fracture conductivity is a crucial parameter in acid fracturing design. The prediction of this parameter has been a serious challenge for scientists and researchers over the past ۴۰ years. By considering the capabilities of intelligent modelling techniques, in this study a feed-forward multi-layer perceptron artificial neural network with a back-propagation algorithm was developed to predict acid fracture conductivity. One hundred and sixteen of experimental data, including dissolved rock equivalent conductivity as a treatment parameter and rock embedment strength as a formation strength parameter, were inputted at different closure stresses to enhance the universality of the ANN AD model. The results showed an excellent fit between the predicted values and the experimental data. The comparison of the fracture conductivity predicted by the ANN AD model with the other common models revealed that the accuracy of the neural model is very high. However, the precision of all the

models decreased as the closure stress increased which reflects the complication and the difficulty of the acid fracturing design in the high closure stress. In order to investigate the effect of lithology, two ANN models based on limestone and dolomite data were developed separately. The results of these ANN models were also more accurate compared to other common models and even to the ANN AD model. The ANN Limestone model predicts fracture conductivity  $\lambda_0$  and  $\lambda_0$  times more precisely than the N&K and NSD models. In addition to the high accuracy of the ANN models, they can anticipate the behavior of the fracture conductivity versus the different closure stresses, which differ from the common models. While the current models predict this trend linearly, the ANN model illustrated that the behavior is different at low, middle and high closure stresses. The results of this study showed, formation lithology plays a substantial role to predict acid fracture conductivity. Since most carbonate reservoir rocks are not pure dolomite or limestone, it is recommended that the percentage of limestone and dolomite are considered as input data and the effect of mineralogy is investigated quantitatively in the next studies.

## Nomenclature

**DREC** Dissolved rock equivalent conductivity

$m_i$  Measured fracture conductivity

$\bar{m}$  Mean of the measured fracture conductivities

$n$  Number of data

$p_i$  Predicted fracture conductivity

$\bar{p}$  Mean of the predicted fracture conductivities

**RES** Rock embedment strength

**WK<sub>f</sub>** Fracture conductivity

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### Appendix

Calculations of absolute relative error (ARE) and regression coefficient (R) as a criteria for evaluating the capability of ANN developed models.

$$ARE = \left| \frac{WKf_P^{predicted} - WKf_P^{measured}}{WKf_P^{predicted}} \right| \times 100 \quad (1)$$

$$R = \frac{\sum_{i=0}^n (m_i - \bar{m})(p_i - \bar{p})}{\sqrt{\sum_{i=0}^n (m_i - \bar{m})^2 (p_i - \bar{p})^2}} \quad (2)$$

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