

## Updating Large Models for Jacket Type Offshore Platforms from Limited Modal Data

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### Abstract

The marine industry requires continued development of new technologies in order to produce oil. Hence, an important requirement in design is to be able to compare experimental results from prototype structures with predicted results from a corresponding finite element model (FEM). In this context, model updating may be defined as the fit of an existing analytical model in the light of measured vibration test. After fitting, the updated model is expected to represent the dynamic behavior of the structure more accurately. In this way, this work presents a direct based updating study of a reduced scale four-story spatial frame jacket structure fabricated and tested at the Laboratory of Structural Dynamics. Also, an efficient model updating process was presented with limited modal data, which uses modal data in order to improve the correlation between the experimental and analytical models. The proposed technique is computationally efficient since it does not require iterations. It updates the mass and stiffness matrix such that they are compatible with the modal data of the observed modes.

**Keywords:** Offshore jacket platforms, model updating, experimental modal analysis, limited modal data

### Introduction

Jacket-type offshore platforms are by far the most common kind of marine structures and they play an important role in oil and gas industries in shallow and intermediate water depth such as Persian Gulf region. As jacket structures require more critical and complex designs, the need for accurate considerations to determine uncertainty and variability in analytical models, loads, geometry, and material properties has increased significantly. In this context, one way to verify the math model accuracy is by comparing the experimental results provided through the conduction of dynamic tests with those expected from a previous analytical analysis [1-4]. Model updating is becoming a common technique to improve the correlation between FEMs and measured data [5, 6]. A number of approaches to the problem exist, based on the type of parameters that are updated and the measured data that is used. This paper concentrates on improvement of dynamic matrices by using a direct updating technique along with empirical study. The direct techniques solve for the updated matrices by forming a constrained optimization problem. The excellence of direct techniques is that they are computationally straightway and efficient; they are not required addressing the problem of whether the solution converges because the result of the computation is unique. For example, Baruch considered the mass matrix to be exact and updated the stiffness matrix [7, 8]. A preliminary step estimated the mass normalized eigenvectors closest to the measured eigenvectors. Berman questioned whether the mass matrix should be considered exact, and updated both the mass matrix and the stiffness matrix [9, 10]. Baruch described these techniques as reference basis techniques, since one of three quantities (the measured modal data, and the analytical mass and stiffness matrices) is assumed to be exact, or the reference and the other two are updated [11]. Caesar extended this approach and produced a range of techniques based on optimizing a number of cost functions [12]. All the techniques described thus far share the feature that only one quantity is updated at a time. Wei updated the mass and stiffness matrices simultaneously, using the measured modal data as a reference [13, 15]. The constraints imposed were mass orthogonality, the equation of motion and the symmetry of the updated matrices. All of the techniques described above used real mode shapes and natural frequencies. The measured mode shapes were processed to produce the equivalent real modes. In this research the dynamics matrices are updated via Lagrange multiplier based techniques (direct technique) by using empirical investigation, so that the updated model reproduces the measured modal data. However, in model updating of an offshore jacket platforms using experimental modal analysis, there are two major challenges ahead: (i) the mismatching of measurement sensors and degrees of freedoms (DoFs) of the analytical model, namely the spatial incompleteness and (ii) the unavoidable corrupted measurements [16, 17]. In dealing with spatially incomplete situations, we can be used improved model reduction scheme.

Furthermore, to overcome modeling uncertainty problem the FEM updating process is applied by using results of the experiment on physical model of the offshore jacket platforms, when limited, spatially incomplete modal data is available. Here, an improved reduction technique associating the model updating process is utilized. The FEM updated provides a useful and less expensive way for studying the fixed marine structures. Thus, experimental programs are necessary to provide validation for the results and reduce the uncertainty of the values of the excitations for of fixed marine Structures.

### The Model Updating Methodology

#### Lagrange Multiplier (Stiffness Matrix Updated)

The Lagrange multiplier based techniques (direct technique) generally consider one parameter set, either mass or stiffness to be correct, and the remaining two that is either mass or stiffness, and the modes, are updated by minimizing a cost function with the appropriate constraints imposed through Lagrange multipliers. Modes that are measured from

the structure will not necessarily be orthogonal to the analytical mass matrix since there are likely fewer transducers than DOF and because of imperfect measurements. For the direct techniques that assume that the mass matrix is correct, it is usually difficult to enforce orthogonality. In order to ensure the eigenvectors are orthogonal, the measured eigenvectors must be corrected. Baruch (1978) has derived a cost function,  $J$ , (1), in which the newly updated eigenvector matrix  $\varphi$  is to be minimized

$$J = \|N(\varphi - \varphi_m)\| = \sum_{i=1}^n \sum_{k=1}^m \left[ \sum_{j=1}^n [N]_{ij} ([\varphi]_{jk} - [\varphi_m]_{jk}) \right]^2 \quad (1)$$

where  $N = M_a^{1/2}$ ,  $M_a$  is the analytical mass matrix,  $\varphi_m$  is the measured eigenvector,  $[N]_{ij}$ ,  $[\varphi]_{ij}$ ,  $[\varphi_m]_{ij}$  are the  $(i,j)$  elements of the matrices  $N$ ,  $\varphi$ ,  $\varphi_m$ ,  $m$  is the number of measured eigenvectors,  $n$  is the number of DOF in the analytical model. Subjected to the orthogonality condition

$$\varphi^T M_a \varphi = I \quad (2)$$

The Lagrange Multiplier technique uses the constraint (2) to produce the augmented function to be minimized as [6]

$$J = \sum_{i,h,j=1}^n \sum_{k=1}^m \left\{ [N]_{ij} ([\varphi]_{jk} - [\varphi_m]_{jk}) [N]_{ih} ([\varphi]_{hk} - [\varphi_m]_{hk}) \right\} + \sum_{i,h=1}^n \gamma_{jh} \left\{ \sum_{j,k=1}^n ([\varphi]_{ji} [M_a]_{jk} [\varphi]_{kh} - \delta_{ih}) \right\} \quad (3)$$

Where the terms,  $\gamma_{jh}$ , are the Lagrange multipliers, which are cast into a matrix  $\Gamma$ , and the terms  $\delta_{ih}$  represent errors. The Lagrange Multipliers may be forced to be unique by introducing the constraint of symmetry so that

$$\Gamma = \Gamma^T \quad (4)$$

Differentiating the augmented function (3) with respect to each element of the corrected eigenvector matrix and the following expression is found

$$\varphi = \varphi_m [I + \Gamma]^{-1} \quad (5)$$

Which, when substituted back into the orthogonality condition, becomes

$$[I + \Gamma]^{-1} \varphi_m^T M_a \varphi_m [I + \Gamma]^{-1} = I \quad (6)$$

By pre and post multiplying by  $[I + \Gamma]$  and taking the square root, it becomes

$$[I + \Gamma] = [\varphi_m^T M_a \varphi_m]^{0.5} \quad (7)$$

Finally, substituting equation (7) into (5), the equation for the corrected eigenvector matrix is

$$\varphi_u = \varphi_m [\varphi_m^T M_a \varphi_m]^{-0.5} \quad (8)$$

If it is assumed that the analytical mass matrix is already correct and the eigenvectors are corrected to ensure orthogonality, the stiffness matrix can now be updated. Baruch (1978) found that the updated stiffness matrix can be found to minimize the cost function

$$J = \frac{1}{2} \|N^{-1} (K - K_a) N^{-1}\| \quad (9)$$

$$J = \frac{1}{2} \sum_{ij=1}^n \left[ \sum_{h,k=1}^n [N^{-1}]_{ih} ([K]_{hk} - [K_a]_{hk}) [N^{-1}]_{kj} \right]^2 \quad (10)$$

where  $N = M_a^{1/2}$ ,  $[N^{-1}]_{ij}$ ,  $[K]_{ij}$ ,  $[K_a]_{ij}$  are the  $(i,j)$  elements of the matrices  $N^{-1}$ ,  $K$ ,  $K_a$ , and is subject to the two constraints

$$K\varphi = M_a \varphi \Lambda \quad \text{and} \quad K^T = K \quad (11)$$

$\Lambda$  represent the eigenvalue matrix. The cost function is then differentiated with respect to the updated stiffness matrix and results in the following equation

$$M_a^{-1} (K - K_a) M_a^{-1} + 2\Gamma_\Lambda \varphi^T + 2\Gamma_k = 0 \quad (12)$$

where  $\Gamma_\Lambda$  and  $\Gamma_k$  are Lagrange Multipliers. By calculating the values of the Lagrange Multipliers, substituting them into equation (9), and then rearranging equation, the updated stiffness matrix can be found using the following equation

$$K_u = K_a - K_a \varphi_u \varphi_u^T M_a - M_a \varphi_u \varphi_u^T K_a + M_a \varphi_u \varphi_u^T K_a \varphi_u \varphi_u^T M_a + M_a \varphi_u \Lambda \varphi_u^T M_a \quad (13)$$

### Lagrange Multiplier (Stiffness and Mass Matrices Updated)

Berman and Nagy (1983) used a similar approach to the one presented by Baruch, however, they used it to update both the mass and stiffness matrices by assuming that the measured eigenvector matrix is correct. The advantage of this scheme is that it is not necessary to calculate the corrected eigenvectors because the mass matrix is updated in such a manner to ensure the orthogonality of the eigenvectors to the mass matrix.

Given the analytical mass matrix,  $M_a$ , and the measured eigenvector matrix,  $\varphi_m$ , the following cost function is created in which the updated mass matrix is found to minimize the function

$$J = \frac{1}{2} \left\| M_a^{-\frac{1}{2}} (M - M_a) M_a^{-\frac{1}{2}} \right\| \quad (14)$$

This function is also subject to the orthogonality constraint

$$\varphi_m^T M \varphi_m = I \quad (15)$$

The cost function  $J$  is minimized using the same steps as the cost function containing the corrected stiffness matrix. The result is

$$M_a^{-1} (M - M_a) M_a^{-1} + \varphi_m \Gamma \varphi_m^T = 0 \quad (16)$$

Combining this equation with that of the orthogonality constraint and the Lagrange Multiplier, the updated mass matrix can be found by adding an updating term, the second term in equation (16), to the analytical mass matrix as follows

$$M_u = M_a + M_a \varphi_m \bar{M}_a^{-1} (I - \bar{M}_a) \bar{M}_a^{-1} \varphi_m^T M_a \quad (17)$$

where,  $\bar{M}_a^{-1} = \varphi_m^T M_a \varphi_m$ .

The updated mass matrix can now be used to calculate the updated stiffness matrix. Since the eigenvector matrix is orthogonal to the newly updated mass matrix, the calculation for the updated stiffness matrix from the previous section can be used; however, the newly acquired updated mass matrix and the measured eigenvector matrix will appear in place of the analytical mass matrix and the corrected eigenvector matrix. So the equation for the updated stiffness matrix becomes

$$K_u = K_a - K_a \varphi_m \varphi_m^T M_u - M_u \varphi_m \varphi_m^T K_a + M_u \varphi_m \varphi_m^T K_a \varphi_m \varphi_m^T M_u + M_u \varphi_m \Lambda \varphi_m^T M_u \quad (18)$$

### Improved Reduction Algorithm due to Limited Modal Data

One of the simplest reduction schemes is static reduction. The full scale model may have certain nodal freedoms specified as master freedoms. The remaining freedoms are slave freedoms. For dynamic analysis purposes the mass, stiffness and loading on the slave freedoms are condensed to these master freedoms. In matrix notation the overall matrices may be partitioned into master, slave and cross coupling terms.

$$\begin{bmatrix} [M_{mm}] & [M_{ms}] \\ [M_{sm}] & [M_{ss}] \end{bmatrix} \begin{Bmatrix} \ddot{X}_m \\ \ddot{X}_s \end{Bmatrix} + \begin{bmatrix} [K_{mm}] & [K_{ms}] \\ [K_{sm}] & [K_{ss}] \end{bmatrix} \begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (19)$$

Where, the subscripts  $m$  and  $s$  refer to the master and slave co-ordinates, respectively. The technique then ignores the inertia terms in the second set of equations. By eliminating the slave DOF, we obtain:

$$\begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = \begin{bmatrix} [I] \\ -[K_{ss}]^{-1} [K_{sm}] \end{bmatrix} \{X_m\} = [T_s] \{X_m\} \quad (20)$$

$T_s$  is Guyan transformation matrix and  $I$  is identify matrix. The reduced Guyan mass and stiffness matrices are then given by [18]

$$[M_R] = [T_s^T] [M] [T_s] \quad (21)$$

$$[K_R] = [T_s^T] [K] [T_s] \quad (22)$$

For larger marine structures, where it is necessary to reduce many slave DOF, this technique will not be as accurate as some of the more advanced approaches. Accordingly, improved reduction skill is probably the best practical process for solving large dynamic problems. Only the smallest frequencies are usually excited and for a typical jacket no more than 30 would normally be required. The process known as the Improved Reduction System (IRS) was presented by O'Callahan in 1989 (Friswell, 1995). This technique is an improvement over the Guyan static reduction scheme via introducing a term that includes the inertial effects as pseudo static forces. A transformation matrix  $T_i$  is applied to reduce the mass and stiffness matrices. It is defined as

$$[T_i] = [T_s] + [S] [M] [T_s] [M_R^{-1}] [K_R] \quad (23)$$

Where

$$S = \begin{bmatrix} [0] & [0] \\ [0] & [K_{ss}^{-1}] \end{bmatrix} \quad (24)$$

$M_R$  and  $K_R$  are the statically reduced mass and stiffness matrices. The new reduced mass and stiffness matrices can be obtained by

$$[M_{IRS}] = [T_i^T] [M] [T_i] \quad (25)$$

$$[K_{IRS}] = [T_i^T][K][T_i] \quad (26)$$

For this process, the rows and columns corresponding to the slave coordinates are eliminated from the mass and stiffness matrices one at a time; this allows the mass and stiffness matrices to adapt to the removal of a slave, and can possibly alter the DOF that will be removed. After each reduction, the DOF with the lowest  $\frac{K_{ii}}{M_{ii}}$  term is the slave which will be eliminated next [19].

### Description of The Physical Model And Test Set Up

Experimental modal tests were performed on a fixed jacket-type offshore platform modal. The measured responses were obtained from the shaker tests. Also, during the implementation of the test, the structural responses were acquired as the time series signals. Three dimensional views of the physical model and the FEM of the platform are shown in Figure 1(a). The structure, consisting of 46 steel tubular members with outer diameter 18 mm, wall thickness 2.5 mm for leg members and outer diameter 12mm, wall thickness 1.5 mm for other members, is fixed at the ground. The physical model was constructed of stainless steel pipes that were welded together using argon arc welding to ensure proper load transfer. The mass density of the members is  $\rho = 7850 \text{ kg/m}^3$  and the Young's modulus of steel is  $E = 2.07 \times 10^{11} \text{ pa}$ . There are 16 nodal points in the FEM, three translational DoFs ( $U_x, U_y, \theta_z$ ) at each node, thus total 48 translational DoFs. The test set up and instruments are illustrated in Figure 1(b). The excitation (based on white noise signals) was enforced by means of an electrodynamic exciter driven by a power amplifier (model 2706). The frequency sampling of the test setup was chosen to be 10 KHz, and the frequency rang was 0 to 200 Hz.

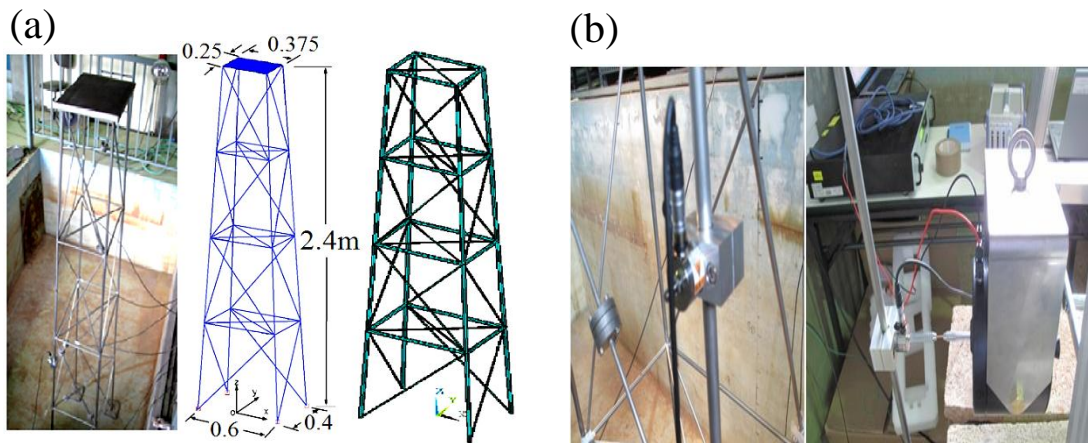


Figure 1: (a) The geometrical properties of the physical model and (b) Connections between amplifier, exciter, load cell and structure

### Numerical and Experimental Modal Analysis

Modal analysis is the procedure of identifying the intrinsic dynamic properties of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate an analytical model for its dynamic behavior. In this paper, the *Block Lanczos* method has been applied for solving the modal analysis. Modal testing is an experimental method utilized to derive the modal model of a linear time-invariant vibration system. The theoretical basis of the method is secured upon establishing the relationship between the vibration response at one location and excitation at the same or another location as a function of excitation frequency. In the present article, *modal assurance criterion (MAC)* method is applied for updating of the platform model. The modal assurance criterion (*MAC*), which is also known as mode shape correlation coefficient, between analytical mode  $\varphi_i$  and experimental mode  $\varphi_j$  is defined as:

$$MAC(\varphi_i, \varphi_j) = \frac{|\varphi_i^T \varphi_j|^2}{(\varphi_i^T \varphi_i)(\varphi_j^T \varphi_j)} \quad (27)$$

A *MAC* value close to 1 suggests that the two modes are well correlated and a value close to 0 indicates uncorrelated modes [20, 21 and 22].

### Results and Discussion

#### Improvement of Stiffness Matrix

The jacket platform was modeled using 3-D FE software, *ANSYS*, modal analysis was performed. For the implementation of the proposed technique, initially the mass and stiffness matrices were extracted by *ANSYS* software under *SUBSTRUCTUR* analysis (see Figure 2). With limited transducers, it is only possible to estimate the lower modes. Mode shapes of the numerical and experimental modal analysis are shown in Figure 3; also frequencies of numerical, experimental and updated model along with *MAC* value are listed in Table 1. In this case the corrected stiffness matrix becomes similar to Figure 4. It is apparent that the updated stiffness matrix is now filled and no longer physically represents the model. Finally, it can be concluded that there is perfect correlation between the numerical and

experimental modal vectors. This means that, *MAC* value is close to 1 and the numerical and experimental models have appropriate correspondence.

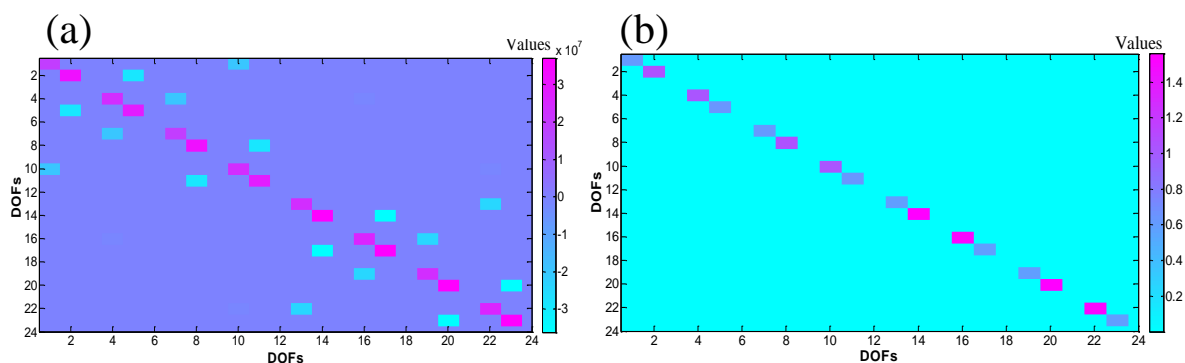


Figure 2: The initial dynamic matrix of platform model (a) Stiffness (b) Mass

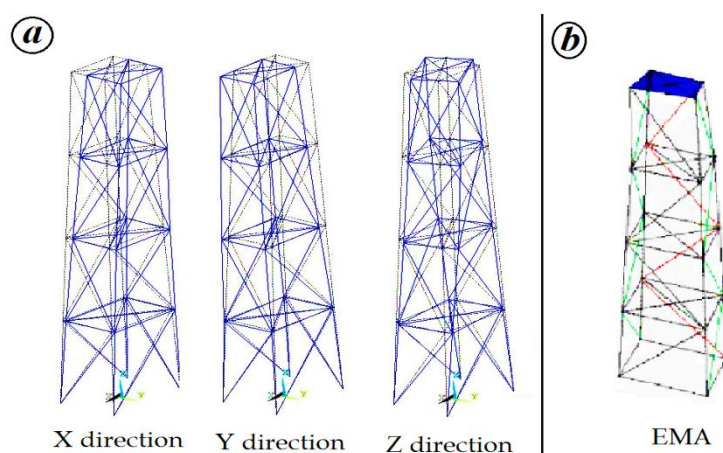


Figure 3: The first mode shape using (a) numerical modal analysis, (b) experimental modal analysis.

Table 1: The first four natural frequencies (Stiffness Matrix Updated)

Mode no.	Natural frequencies (Hz)			Differences (%)	MAC
	Numerical Analysis	Experimental Result	Updated model		
1	67.29	58.34	58.83	0.85	0.994
2	91.46	94.13	93.61	0.50	0.992
3	100.8	106.21	106.85	0.60	0.990
4	125.1	130.28	131.04	0.58	0.992

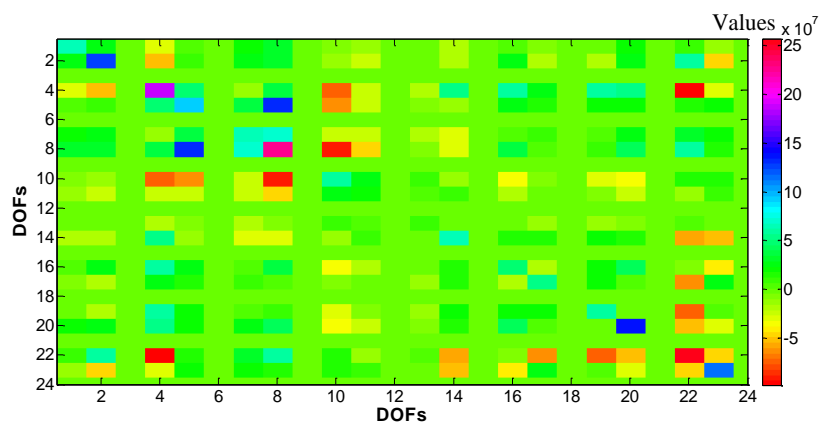


Figure 4: Change in stiffness (Lagrange Multiplier-Stiffness Matrix Updated method)

### Improvement of Stiffness and Mass Matrices

Using equations (17) and (18), the updated mass and stiffness matrices will be similar to Figure 5. Again, the updated matrices become completely filled for the second case. However, since both the mass and stiffness matrices are allowed to be perturbed, they are closer to physically representing the system. The results for the first 4 modes are presented in Table 2. Also, the Changes of those matrices are shown in Figure 5. The results are similar to the direct technique (stiffness), which is to be expected, since they are both based on similar optimization procedures, the only difference is in the matrices being updated. However, it is noted that the mass matrix is no longer diagonal; since the stiffness matrix is already not a physical representation it is more beneficial to update only the stiffness matrix.

Table 2: The first four natural frequencies (Lagrange Multiplier - Stiffness and Mass Matrices Updated)

Mode no.	Natural frequencies (Hz)			Differences (%)	MAC
	Numerical analysis	Experimental result	Updated model		
1	67.29	58.34	58.59	0.4	0.995
2	91.46	94.13	93.84	0.3	0.992
3	100.8	106.21	106.6	0.4	0.994
4	125.1	130.28	129.87	0.4	0.991

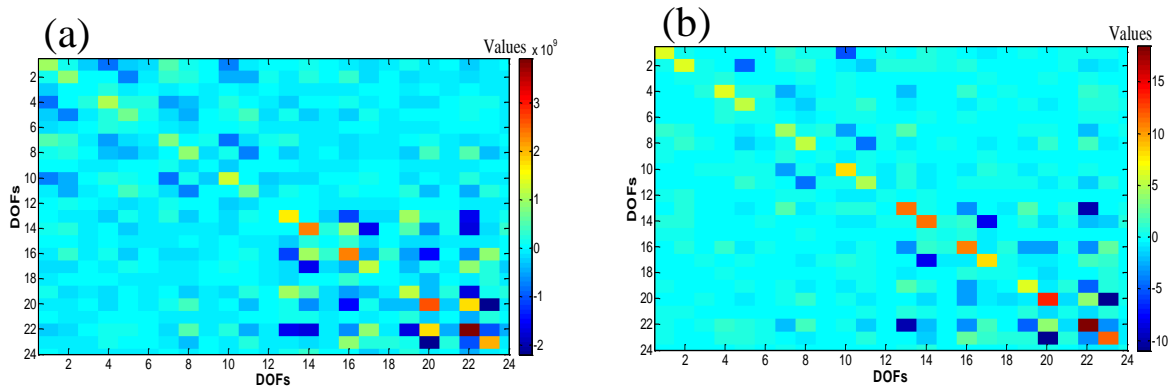


Figure 5: Changes in (a): stiffness and (b): Mass (Lagrange Multiplier- Stiffness and Mass Matrices Updated)

### Summary and Conclusions

FE matrix updating has received a considerable amount of attention by the engineering community and as a result, there now exist a voluminous work on this problem. In this research the ability of empirical investigation of the jacket platform model updating are considered. Also, an efficient model updating technique was presented with incomplete modal data, which uses modal data in order to improve the correlation between the experimental and analytical models. An example with incomplete modal data of a typical reduced scale four-story spatial frame of the jacket platform was carried out showing that the methodology was able to correct update both mass and stiffness matrices and reproduce correctly the tested data. The mode shapes are not required to be measured at all DOF. The proposed technique is computationally efficient since it does not require iterations. It updates the mass and stiffness matrix such that they are compatible with the modal data of the observed modes. The Lagrange multiplier techniques reproduce the measured eigen-system, however, the results are not physically meaningful, or in other words cause the updated system to lose its physical representation. This is a potential problem for situations where the stiffness and/or mass of a specific DOF are needed, such as in damage detection. These techniques are advantageous for systems that contain measured eigenvalue and eigenvectors for every DOF, especially if the physical representation of the mass and stiffness matrices is not of importance.

The FEM updating provides a practical and less expensive way for studying the behavior of fixed offshore platforms. However, an experimental program can be used to validate a FEM. Through experimentations one can reduce the uncertainty of the fixed offshore platforms.

## List of Symbols

FEM	Finite element model	DOF	Degree of freedom
$M_a$	Analytical mass matrix	$\varphi_u$	Corrected eigenvector matrix
$M_R$	Statically reduced mass matrix	$T_s$	Guyan transformation matrix
$M_u$	Updated mass matrix	$S$	Slave
$K_a$	Analytical stiffness matrix	$m$	Master
$K_R$	Statically reduced stiffness matrix	$\Gamma_k, \Gamma_\Lambda$	Lagrange Multipliers
$K_u$	Updated stiffness matrix	$\Lambda$	Eigenvalue matrix
$\varphi_m$	Measured eigenvector	$\delta_{ih}$	Errors
$I$	Identify matrix	$\gamma_{jh}$	Lagrange multipliers

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