# **Response Amplitude Operators of Displacement, Velocity and Acceleration of TLP**

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### Abstract

Tension Leg Platform (TLP) is used in deep offshore industry for extracting oil and gas. Behavior investigating of TLP is important in design aspects. In this paper, hydrodynamic analysis of TLP is carried out by boundary element method (BEM) and dynamic equation of motion of TLP is derived and solved. Finally Response Amplitude Operator (RAO) of displacement, velocity and acceleration of TLP in different angle of incident wave have been obtained. Such a results are necessary for initial design and serviceability investigation.

Keywords: TLP, RAO, dynamic response, Displacement sensitivity, Acceleration sensitivity

#### Introduction

Primary designing of the platform comprises four cylindrical columns which are joined to each other by pontoons. these platforms are almost square or triangular, of course there is sea star platform is a member of tension-leg family. Floating body of tension-leg platforms is similar to semisubmersible platforms. Moring dynamic loads in other floating structures decreased mainly by inertia forces, Moring loads of tension-leg platform related to the first order wave forces on platform. Risers as extraction part are pipes which are used to extract oil. These risers stretch from deck to the sea bottom.

Downie et al. carried out an experimental study on a model truss spar in regular and irregular waves for four different heave plate (Two of the plate types were perforated and two were solid). In terms of the RAOs, the spar behaves similarly in surge and pitch in regular waves. However, there are significant differences in the heave response [1]. Bhattacharyya et al developed numerical model (finite element code) for a typical Sea Star platform at two water depths, 215m and 1000m and experimental work a scaled model corresponding to 215 m water depth for validation of the numerical model. The RAOs for the motions and tether tensions computed for both the prototypes using the Morison wave force model with a wave height of 2 m [2]. Zhang et al presented a new spar concept and studied its hydrodynamics both in operating and survival conditions by means of numerical simulation. Also in this article numerical and experimental results of RAOs with mooring lines for the surge, heave and pitch is investigated. Predicted RAOs are in excellent agreement with test measurements [3]. Zeng et al investigated 6-DOF coupled motions, time history of motions and wetted area, free surface and viscous drag effect and dynamic analysis of ISSC TLP in depth of 415 m in regular sea and extraction of horizontal plane motions RAOs [4]. Anitha et al presented a new geometric configuration which could be a better alternative to an existing configuration. Also in this paper a three column mini TLP is designed and added mass, radiation damping, transfer functions of wave force and RAOs of motions is investigated for three column mini TLP and compared with an existing four column mini TLP [5]. Tajali and Shafieefar carried out hydrodynamic analysis of a floating multi-body pier interacting with incident waves and presented results of wave-induced motions and structural responses are presented. Also examined the effect of relevant parameters on the pier hydrodynamic responses and computed RAO for a wide range of wave frequency and heading angle [6].

In this paper deals with RAOs of displacement, velocity and acceleration of TLP in different angle of incident wave.

## **Governing equations**

The equations of motion in frequency domain representing for surge, heave and pitch degrees of freedom can be written as:

$$\sum_{i=1}^{3} \left[ -\omega^{2} \left\{ M_{i} + A_{i}(\omega) \right\} + i \,\omega C_{i}(\omega) + K_{i} \right] X_{i}(\omega) e^{i \,\omega t} = F_{i}(\omega)$$
(1)

where  $[M_i], \omega, [A_i(\omega)], [C_i(\omega)], [K_i], [X_i(\omega)]$ , and  $[F_i(\omega)]$  are mass matrix, wave frequency, added mass matrix, damping matrix, stiffness matrix, displacement amplitude and wave force for degrees of freedom.

Response Amplitude Operator (RAO) is ratio of the response amplitude to the wave amplitude for linear systems that is plotted versus wave frequency or wave period:

$$x = X_{i}(\omega)e^{i\omega t}$$

$$RAO_{x} = \frac{X_{i}(\omega)}{\zeta_{a}}$$
(2)

where  $X_i(\omega)$  and  $\zeta_a$  are response and wave amplitude of i-th motion respectively. RAOs of velocity and acceleration are expressed as follow:

$$\dot{x} = \dot{X}_{i}(\omega)i \ \omega e^{i \omega t}$$

$$RAO_{\dot{x}} = \frac{\dot{X}_{i}(\omega)}{\omega \zeta_{a}}$$

$$\ddot{x} = \ddot{X}_{i}(\omega) \omega^{2} e^{i \omega t}$$

$$RAO_{\dot{x}} = \frac{\ddot{X}_{i}(\omega)}{\omega^{2} \zeta_{a}}$$
(3)
(4)

where,  $\dot{X}_{i}(\omega)$  and  $\ddot{X}_{i}(\omega)$  are velocity and acceleration amplitude of response.

## Case study

The research platform is a tension-leg platform named ISSC TLP shown in figure (8) with specifications given in table (1) as follow:

Draft	35 <i>m</i>	External radius of tendon	0.3 <i>m</i>
Displacement	$54.5 \times 10^{6} kg$	Internal radius of tendon	0.212 <i>m</i>
Mass	$40.5 \times 10^{6} kg$	Length of tendon	195
Roll motion moment of inertia	$82.37 \times 10^{9} kg.m^{2}$	Pre-tension in tendon	$1.1445 \times 10^7 N$
Pitch motion moment of inertia	$82.37 \times 10^{9} kg.m^{2}$	Young's modulus of tendon	$2.1 \times 10^{11} N/m^2$
yaw motion moment of inertia	$98.07 \times 10^9 kg.m^2$	Axial stiffness of tendon	$1.5 \times 10^8 \ N/m$
Center of gravity height	38 <i>m</i>	Number of tendons under leg	3

Table (1): specifications of tendons and ISSC TLP in depth of 230 m

## Numerical results

Numerical simulation for hydrodynamic analysis of TLP is conducted by ANSYS-AQWA commercial code. Figures (1 to 3) show RAOs of displacement, velocity and acceleration for approaching wave 0, 30 and 60 degrees respectively. It can be observed that RAO of displacement is relatively more than RAO of velocity and acceleration at wave frequency less than 1 rad/sec (low wave frequencies), this region displacement is dominant. Vice versa at wave frequency more than 1 sec (high wave frequencies) RAO of acceleration is relatively more than RAO of displacement and velocity (Acceleration sensitivity). Because according equation (3-4) for obtaining RAO of velocity and acceleration, RAOs of displacement and acceleration are multiply to wave frequency and square of wave frequency respectively. Also it is seen that in RAOs curve hollows and humps occur in different wave frequencies and these hollows and humps are same for RAO of displacement, velocity and acceleration.



Figure 1: Surge RAOs-Approaching wave 0 degree



—— Displacement RAO ------ Velocity RAO — — Acceleration RAO

Figure 2: Surge RAOs-Approaching wave 30 degree



— Displacement RAO ------ Velocity RAO — --- Acceleration RAO

Figure 3: Surge RAOs-Approaching wave 60 degree

Figure (4) presents RAO of displacement for wave approaching 0, 30 and 60 degrees. Figure (5) shows RAO of velocity for wave approaching 0, 30 and 60 degrees. Figure (6) illustrates RAO of acceleration for wave approaching 0, 30 and 60 degrees. It can be concluded that surge RAOs for wave approaching 0 degree is more than other waves approaching. It is seen that hollows and humps in RAOs are different for waves approaching.



Figure 4: Surge displacement RAOs-Approaching waves 0, 30 and 60 degrees



Figure 5: Surge velocity RAOs-Approaching waves 0, 30, 60 degrees



Figure 6: Surge acceleration RAOs-Approaching waves 60 degree

Figure (7-8) present RAOs of heave and pitch for wave approaching 0 degree. It can be observed for wave frequencies lower than 1, RAO of displacement is dominant (displacement sensitivity) and for wave frequency more than 1, RAO acceleration is dominant (acceleration sensitivity).



Figure 7: Heave RAOs-Approaching wave 0 degree



Figure 8: Pitch RAOs-Approaching wave 0 degree

Acceleration of heave and pitch motions are important, because natural frequency of the motions are high and can cause calm of humans are disturbed.

#### Conclusion

In this paper RAOs of displacement, velocity and acceleration for TLP are obtained. From numerical results, it is reported as follow:

- RAO of displacement is more than RAO of velocity and acceleration at low wave frequencies (acceleration sensitivity), vice versa RAO of acceleration is more than RAO of displacement and velocity at high wave frequencies (displacement sensitivity).
- 2) Surge RAOs for wave approaching 0 degree is more than other waves approaching.
- 3) Hollows and humps are same in RAOs of displacement, velocity and acceleration.

#### References

[1] Downie, M. J., Graham, J. M. R., Hall, C., Incecik, A., and Nygaard, I. (2000). An experimental investigation of motion control devices for truss spars. Marine structures, 13(2), 75-90.

[2] Bhattacharyya, S. K., Sreekumar, S., and Idichandy, V. G. (2003). Coupled dynamics of SeaStar mini tension leg platform. Ocean engineering, 30(6), 709-737.

[3] Zhang, F., Yang, J. M., LI, R. P., and Gang, C. H. E. N. (2007). Numerical investigation on the hydrodynamic performances of a new spar concept. Journal of Hydrodynamics, Ser. B, 19(4), 473-481

[4] Zeng, X. H., Shen, X. P., and Wu, Y. X. (2007). Governing equations and numerical solutions of tension leg platform with finite amplitude motion. Applied Mathematics and Mechanics, 28, 37-49

[5] Joseph, A., Mangal, L., and George, P. S. (2010). Coupled dynamic response of a three-column mini TLP. Journal of Naval Architecture and Marine Engineering, 6(2), 52-61.

[6] Tajali, Z., and Shafieefar, M. (2011). Hydrodynamic analysis of multi-body floating piers under wave action. Ocean Engineering, 38(17), 1925-1933.