# 3D gravity COMPACT inversion based on new weighting functions

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**ABSTRACT**
We have developed a method to estimate the geometry, location and densities of anomalies coming from gravity data. The method is based on compact gravity inversion technique. Compact gravity inversion is simple, fast and user friendly but severely depend on the number of model parameter, i.e. by increasing the model parameter, and the anomalies tend to concentrate near the surface. To overcome this ambiguity the new weighting functions have introduced here which their elements are the density, depth, and kernel function of the model to produce compactness, depth and kernel weighting matrices respectively. Test with a 3D model obtained from Grav3D software 2005 show that the method can be performed successfully in deep gravity sources and the anomalies laid close together.

**Keywords:** gravity, inversion, kernel function, depth weighting, compactness, a priori information

## 1 INTRODUCTION

The geophysical inverse problem is an attempt to obtain information about the Earth interior from measurements of physical entities in the near surface region. When the inverse problem demands too much information from the geophysical data, its solution becomes either nonunique or unstable. Problems of this type are called ill-posed problems (Barbosa et al, 1994).

Many authors introduced solutions to overcome the problems in the three last decades. Last and Kubik (1983), in particular, developed a method explaining the observed anomaly by structures of minimum volume. Their strategy is to minimize the area (or volume in three dimensions) of the model. This is equivalent to maximizing its compactness. This method is severely depend on the number of model parameter (prisms), and with increasing number of the prisms, the anomalies tends to concentrate near the earth’s surface. Guillen and Menichetti (1984) applied an approach which includes the search for solutions minimizing the moment of inertia with respect either to the center of gravity or to an axis of a given dip line passing through it. The method works properly for a single gravity source, but the problem is dealing with multi-source and complicated anomalies which don’t lay in a point or one axe. Barbosa et al (1994) generalized the compact inversion method to compact along several axes using Tikhonov’s regularization. They improved the method offered by Guillen and Menichetti (1984) for multi-source and complicated anomalies. The newest gravity compact method is so called interactive inversion and developed by Silva and Barbosa (2006) which estimates the location and geometry of several density anomalies. They simplified their old method (Barbosa et al, 1994) for computational performing. The method is suitable for multi-source and even complicated anomalies depending on the quantity and quality of “a priori” information. Silva et al (2009, 2011) also developed interactive inversion successfully for 3D gravity data closest to prespecified geometric elements such as axes and points.

The generalized compact, and interactive inversion strongly need “a priori” information to yield an accurate estimation. A priori information is refer to geological information, well logs and previous inversions. In the case of a few information about gravity sources, modelling should be done with other inversion method such as Li and

Oldenburg (1998). The method introduced by Li and Oldenburg (1998), which largely used, can be applied without or with geological constrained (Williams, 2008) in two and three dimensional measurements. This method has many coefficients and functions that should be estimated refer to the kernel of potential field data and existence or absence of "a priori" information. This procedure is time consuming and makes the method rather unfriendly.

We developed a new approach for interpreting 3D gravity anomalies produced by multiple and complex gravity sources in any depths. New weighting functions have been used based on the depth of the prisms (Fig. 1), kernel matrix and compactness weighting.

## 2 METHODOLOGY

The model used here is based on 2-D model that is illustrated in Fig. 1. For this model the gravity effect at the *i*th data point is given by

$$g_i = \sum_{j=1}^{M} A_{ij} m_j + e_i \qquad (1)$$

$$i = 1, 2, \dots, N; \quad j = 1, 2, \dots, M$$

Where $m_j$ is the density of the *j*th prism, $e_i$ is the noise associated with *i*th data point, and $A_{ij}$ is kernel matrix which its elements representing the influence of the *j*th prism on the *i*th gravity value (last and Kubik, 1983).

The data equation can be written in matrix notation

$$g = Am + e \qquad (2)$$

The inversion method here is linear, like other linear geophysical inversion is then: given, the observed gravity data ($g$), find a density distribution "*m*" which explains "$g$", with a certain noise level.

The solution of the system in *k*th iteration, can be in the least-square problem in matrix notation (Menke, 1989):

$$\delta m^{k+1} = W_{m(k)}^{-1} A^T \left(A W_{m(k)}^{-1} A^T + \mu W_{e(k)}^{-1}\right)^{-1} \delta g^k \qquad (3)$$

$$m^{k+1} = m^k + \delta m^{k+1} \qquad (4)$$

Where $W_m$ is model weighting matrix, $W_e$ is a noise weighting matrix in *k*th iteration, which both matrices are diagonal, and $g^{(k)} = (g^{obs} - Am^{(k)})$ . Mu ($\mu$) is damping factor to get rid of matrix singularity, having the positive small value and depends on the noise level of the data points. The less value of damping factor refers to the less noise level of the data points.

We introduce a general weighting function include the compactness weighting, depth weighting and kernel weighting matrices in *k*th iteration:

$${W_{m(k)}}^{-1} = W_z W_{c(k)} W_a \qquad (5)$$

Noise weighting matrix $W_e$, can be simply written as Last and Kubik (1983):

$$W_{e(k)}^{-1} = diag\left(A W_{m(k)}^{-1} A^T\right) \qquad (6)$$

Where $W_z$ is depth weighting matrix and same as Li and Oldenburg (1998), $W_{c(k)}$ is compactness weighting function and explained by Last and Kubic (1983), and $W_a$ is kernel function as:

$$w_{a_j} = \sum_{i=1}^{N} A_{ij} \qquad (7)$$

$$W_a = diag(w_a) \qquad (8)$$

For the beginning, if there is no priori information, the reference model choose $m_o = 0$ then calculating $W_m$ (Eq. 5) and $W_e$ (Eq. 6), and choosing compactness factor. After that, having iteration procedure with least-square solution as Eqs. (3 and 4).

## 3 INVERSION OF SYNTHETIC DATA

To illustrate the efficiency of the proposed approach, the algorithm was tested using a synthetic example obtained from Grav3D 2005, it consists of five blocks of different density contrasts in a uniform background with the density contrast at most 1.2 gr/cc. There is one large dipping dyke to the left that extends to a large depth. Four smaller blocks of various shapes are located at shallower depths to the right (Fig. 1).

The inversion was started using a model with the cubic voxel size of 50m and 5% data noise added. The model with damping factor of 6 after 30 iterations are depicted in Figs. 2. This model were imposed minimum and maximum bounds ( $0 \leq m_j \leq 1.2$ ). The model is coincident with the real anomalies, but the large dipping dyke is shown properly at shallower depth and smoothly. All five anomalous blocks are imaged. The Recovered model is shown in three cross-sections in Fig. 2. The depth extent of the large dipping block is also smaller. This is expected since the area of the data is limited. Over all, this is a good representation of the true model, and the inversion utilizing the method has performed rather reliable and true.

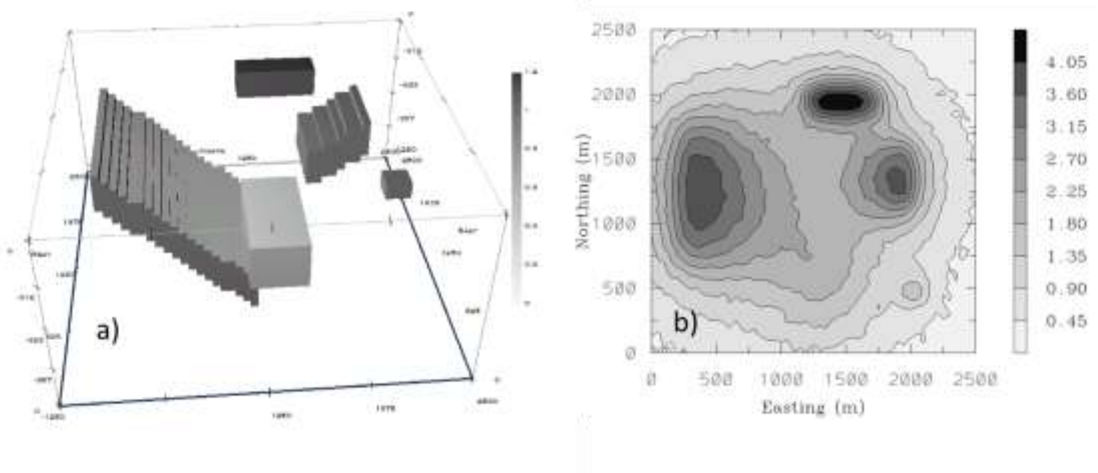


**Figure 1.** Synthetic example: five blocks of different density contrasts in a uniform background with the density contrast at most 1.2 gr/cc (Grav3D V. 3.0, UBC-Geophysical Inversion Facility, 2005).

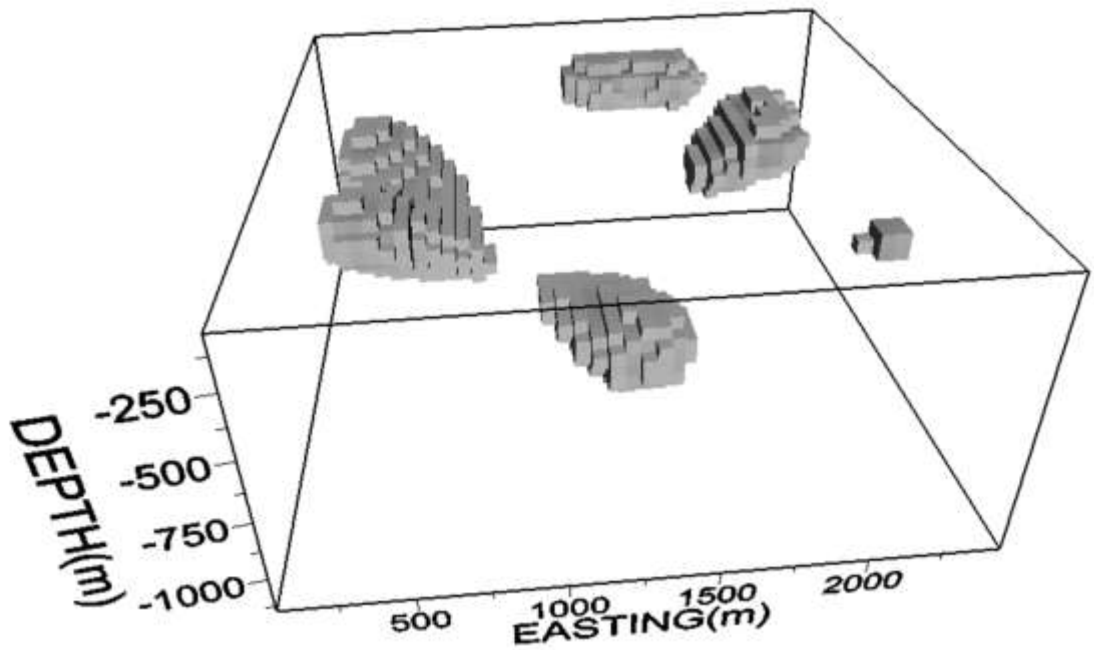


**Figure 2.** Inversion result of the example in Fig. 1, with the cubic voxel size of 50m and cutoff 0.17 gr/cc.

## 4 CONCLUSION(S)

According to synthetic example modelled by present algorithm, the method can be applicable even in complex and multi sources in any depth and with any geometry, and has sufficient sensitivity and accuracy to model noisy data.

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