A Non-linear Spatial Sampling and Reconstruction for Seismic Exploration

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ABSTRACT

Seismic data are band-limited, therefore one can use only a partial set of frequency coefficients in the range of reflections band, where the signal-to-noise ratio is high and spatial aliasing is low, to reconstruct the original wavefield. Furthermore, low-frequency characteristics of the coherent ground rolls allows direct elimination of them during reconstruction by disregarding the corresponding frequency coefficients (usually bellow 10 Hz) via a frequency mask. In this paper, a non-linear algorithm is proposed which addresses some challenges of spatial sampling, reconstruction, and denoising in seismic exploration. Numerical experiments from both simulated and real field data are included to illustrate the effectiveness of the presented method.

Key words: Ground roll, Interpolation, Seismic signal processing

INTRODUCTION

Seismic data sets are generally irregularly sampled in spatial direction due to variable operating conditions during seismic surveys. This irregular sampling can limit the effectiveness of data processing and imaging algorithms. To overcome these problems, acquired traces need to be interpolated before being processed. Trace interpolation based on compressed sensing (CS) can be considered as a non-linear denoising procedure, furthermore, noise attenuation is an important step in seismic processing because seismic data are usually contaminated by some random and coherent (e.g. ground roll) noises. Obviously, there is a close relationship between CS and sparsity based denoising. It is common to perform trace interpolation and noise attenuation at separate processing steps; however, bringing them under a single formula which allows tackling them simultaneously in a single run is favorable.

There are several main observations that play a fundamental role in developing the proposed algorithm for simultaneous interpolation and ground roll attenuation. 1) In seismic exploration, the sampling strategy used for CS is different from that used in image processing. In the former, each sample is a trace (an N-dimensional vector) whereas in the latter each sample is a scalar or pixel (Donoho, 2006). 2) Seismic data are bandlimited and the information contained in each trace can be recovered with high-quality from an incomplete set (20-25 percent) of its Fourier spectrum in the range of wavelet bandwidth (Levy and Fullager 1981). 3) Ground roll noise is the main type of noise in land seismic surveys which is characterized by low frequency (0-10 Hz) (Askari and Siahkoohi 2008).

Based on the above observations, in this paper, a CS algorithm is developed for seismic exploration which addresses some challenges of acquisition and processing, simultaneously. The contributions of this paper are as follows: 1) Using non-convex regularizers instead of the conventional one-norm minimization for sparsity promotion in CS algorithm. 2) Using a frequency mask to additionally subsample the acquired traces in the $f-x$ domain. This brings useful advantages as 3) allowing better interpolation of spatially aliased seismic data compared to the case using full frequency samples or working in the $t-x$ domain. and 4) some processing steps such as seismic wavefield reconstruction and denoising can be replaced with a single non-linear reconstruction.

A NON-LINEAR SAMPLING FOR SEISMIC EXPLORATION

Let $\boldsymbol{d}=\boldsymbol{s}+\boldsymbol{n}$ be the noise contaminated seismic wavefield with $\boldsymbol{s}\in\mathbb{R}^{N_1\times N_2}$ the desired wavefield (signal) and $\boldsymbol{n}\in\mathbb{R}^{N_1\times N_2}$ the noise. The spectrum of a noise-free seismogram (a column of $\boldsymbol{s}$) falls off rapidly at high frequency because of the band-limited nature of the source function. It was shown by Levy and Fullager (1981) that only a partial set of the Fourier spectrum of a seismogram in the range of wavelet bandwidth, where the wavelet carries significant power, is sufficient for an accurate reconstruction of the seismogram. Generally, $\boldsymbol{n}$ includes both white random and coherent (e.g. ground roll) noise. The power of the random noise distributes among all frequencies while the ground roll noise is characterized by its low frequency (0-10 Hz).

Such information about the spectrum of seismograms can be used as a priori information to just invert the desired frequencies during interpolation. Let $\mathcal{L} \subset \{0,...,N_2 - 1\}$ be the set of indices corresponding to the spatial coordinate where the wavefield is to be sampled. Similarly, let $\mathcal{K} \subset \{0,...,N_1 - 1\}$ be the set of indices corresponding to the desired frequencies which are to be inverted. Then the wavefield is subsampled in the $f - x$ domain via

$$\boldsymbol{y}_k^l = \langle \boldsymbol{d}, \phi_k^l \rangle, \quad k \in \mathcal{K}, l \in \mathcal{L} \qquad (1)$$

where $\boldsymbol{y}_k^l$ is the k th frequency sample at l th coordinate and $\phi_k^l \in \mathbb{C}^{N_1 \times N_2}$, forming the sampling basis, is defined as

$$\phi_k^l[n_1, n_2] = \begin{cases} \exp(\frac{-j2\pi k n_1}{N_1}), & if \quad n_2 = l; \\ 0, & otherwise, \end{cases} \qquad (2)$$

for $n_1 = 0,...,N_1 - 1$ and $n_2 = 0,...,N_2 - 1$, and $j^2 = -1$. Since $\boldsymbol{s}$ and $\boldsymbol{n}$ span distinct frequency bandwidths, for a properly defined index set $\mathcal{K}$, the frequency samples acquired via equation (1) correspond to $\boldsymbol{s}$ having very high SNR. Therefore equation (1) can be written in matrix form as $\boldsymbol{y} = \boldsymbol{\Phi}\mathrm{vec}(\boldsymbol{s})$ to be solved for $\boldsymbol{s}$. Where the vec operator vectorizes a matrix by stacking its columns. Needless to say, reconstruction of the desired wavefield $\boldsymbol{s}$ in this way will not be possible unless some a priori information about it is incorporated into the reconstruction. In this paper, the sparsity characteristics of $\boldsymbol{s}$ in the curvelet domain (Candes et al. 2006) is used as the priori information and the desired seismic wavefield is recovered via $\mathrm{vec}(\boldsymbol{s}) = \boldsymbol{\Psi\alpha}$, where

$$\boldsymbol{\alpha} = \arg\min_{\boldsymbol{\alpha}} \{\frac{1}{2} \| \boldsymbol{y} - \boldsymbol{\Phi\Psi\alpha} \|_2^2 + \tau \mathrm{Reg}(\boldsymbol{\alpha})\}, \qquad (3)$$

with

$$\mathrm{Reg}(\boldsymbol{x}) = \sum_{n=1}^{N} \varphi_q^p(x[n]). \qquad (4)$$

and

$$\varphi_q^p(u) = \begin{cases} \frac{1}{q}[1 - (|u|^p + 1)^{-q}], & \text{if} \quad q \neq 0; \\ \ln(|u|^p + 1), & \text{if} \quad q = 0, \end{cases} \qquad (5)$$

where $\tau > 0$, $0 < p \leq 2$, and $q \geq -1$. In problem (3), $\boldsymbol{y}$ contains the desired frequency samples of the acquired traces, $\boldsymbol{\Psi}$ is inverse curvelet transform, and $\boldsymbol{\Phi}$ is the sampling matrix. This problem can be solved via the algorithm presented in [and Hosseini 2011]. We use $p = q = 1$ for generating our results. The main properties of formulation (3) are: (1) It allows masking unwanted frequencies of the acquired data during processing. (2) Unmasked frequencies have very high SNR allowing a high quality reconstruction. (3) Since unnecessary high-frequency components are not allowed to incorporate into reconstruction the algorithm is more suitable for interpolation of spatially aliased seismic data comparing to the case working in $t - x$ domain. (4) Ground roll noise can simply by suppressed at field during data acquisition. (5) It performs interpolation and denoising processing steps, simultaneously.

APPLICATIONS TO SEISMIC DATA

In this section, to demonstrate the efficacy of the proposed method, numerical examples from seismic data processing are tested. A synthetic shot gather has been contaminated by both simulated dispersive ground roll and random noise to be used for evaluating the performance of the proposed method in simultaneous interpolation and denoising. The noisy wavefield has been 2-fold and 4-fold undersampled along spatial axis by uniformly and randomly removing its traces. The resulting incomplete wavefields are shown in different domains in Figures 1(a) and (b), respectively. Figures 1(c) and (d) shows the reconstructed sections for different undersampling patterns. The results demonstrated in Figure 1 show that the proposed algorithm has the capability of performing simultaneous seismic data interpolation and denoising, which is not possible when the formulation is performed in the $t - x$ domain.

The proposed algorithm has been used for testing its applicability on real world seismic signals. The used seismic record is shown in Figure 2(a). The original section has been undersampled by randomly removing 40% of its traces in spatial direction [Figure 2(b)]. The method has been used for simultaneous reconstruction and denoising. Figure 2(c) shows the recovered gather whose difference with the original data [Figure 2(a)] is shown in Figure 3(d). Again the method was able to reconstruct the noise free data as well as missing traces with high accuracy using only the noise-free coefficients of the partially known traces.

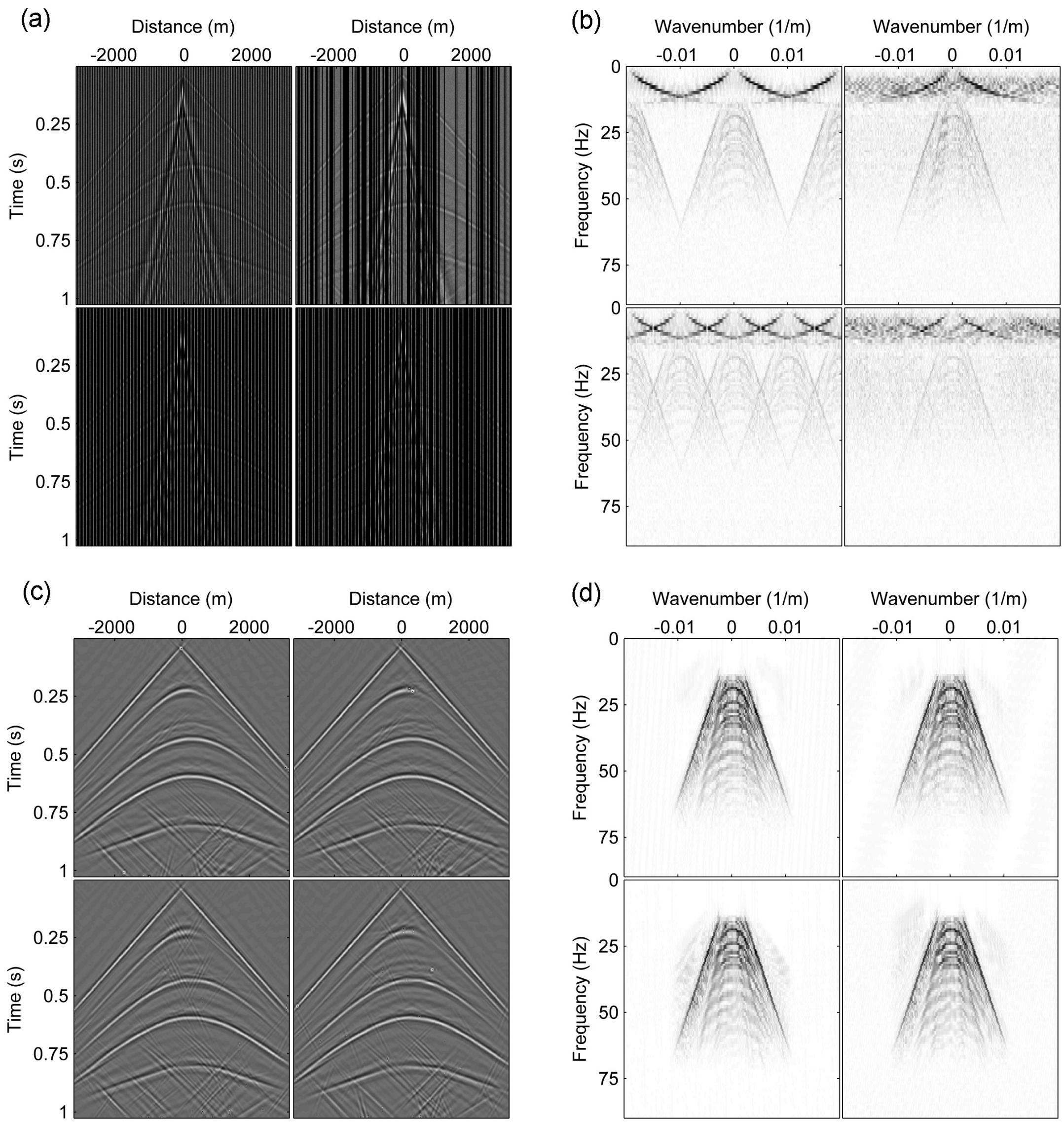

Figure 1. **(a) Shot gathers obtained from different (both random and uniform) undersamplings of a synthetic gather (a) and the corresponding f-k domain representations (b). (c) Reconstructed gathers via the proposed algorithm using frequency samples 20-75 Hz and the corresponding f-k domain representations (f).**

CONCLUSION

The results obtained from numerical tests showed that the desired seismic signals can accurately be reconstructed from a small number of frequency coefficients in the range of dominant frequency where the SNR is high. Comparing to the conventional $t-x$ domain formulation where all frequency samples are inverted, the proposed can generate much better reconstructions specifically when the acquired data are spatially aliased. Furthermore, applications on both synthetic and field data confirmed that the proposed method performs satisfactorily for simultaneous reconstruction and denoising.

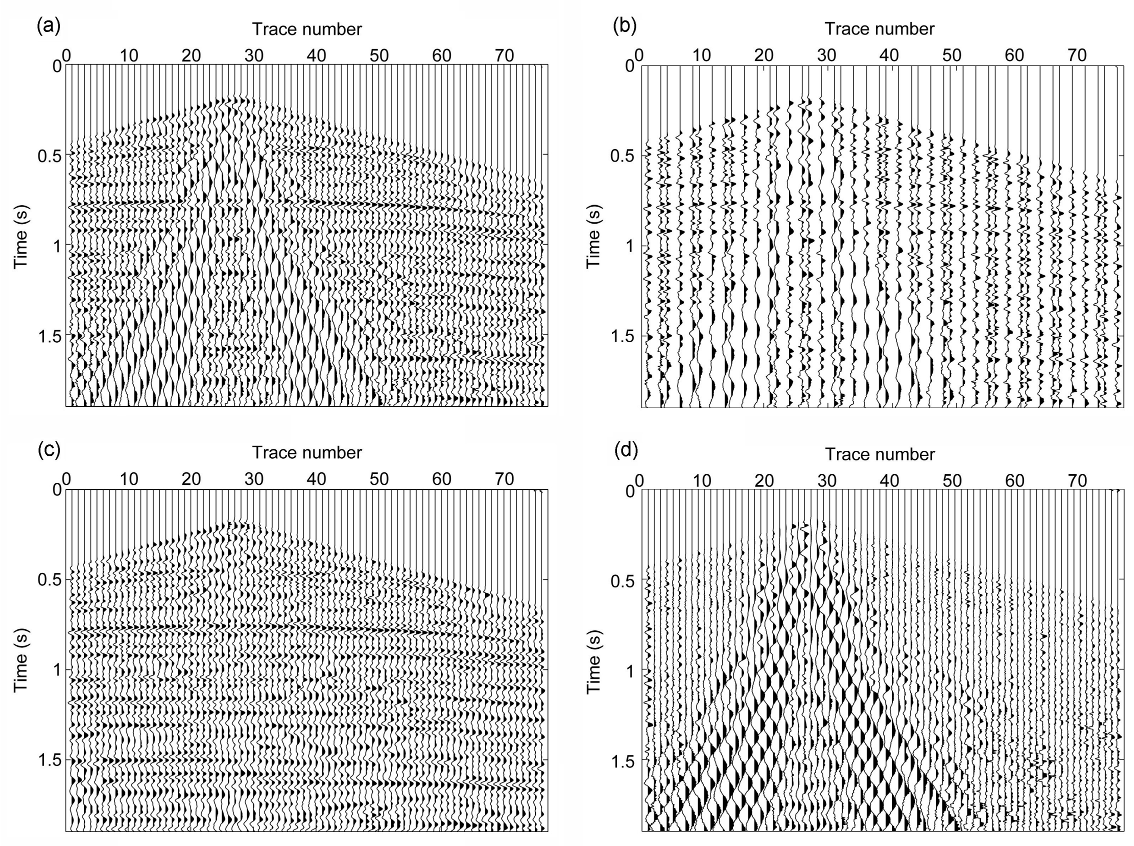

Figure 2. Simultaneous interpolation and denoising of field seismic data. (a) original shot gather with coherent ground roll and some random noise. (b) the gather after randomly removing 40% of its traces. (c) reconstruction obtained via the proposed algorithm by inverting only the frequency samples 10-100 Hz of the incomplete record shown in (b). (d) difference between (c) and the original record shown in (a). The results are shown after applying automatic gain control.

REFERENCE

Askari R., and Siahkoohi H. R. 2008. Ground roll attenuation using the S and x-f-k transforms, *Geophysical Prospecting*, **56**, 105-114.

Candés E. J., Demanet L., Donoho D. L., and Ying L. 2006. Fast discrete curvelet transforms, *SIAM Multiscale Modeling and Simulation*, **5**, 861-899.

Donoho D. L. 2006. Compressed sensing, *IEEE Trans. Inform. Theory*, **52**(4), 1289-1306.

Gholami A., and Hosseini S. M. 2011. A general framework for sparsity-based denoising and inversion, *IEEE Transactions on Signal Processing*, **59**, 5202-5211.

Levy S., and Fullager P. K. 1981. Reconstruction of a sparse spike train from a portion of its spectrum and application to high-resolution deconvolution, Geophysics, 46(9), 1235-1243.